On Randomized and Deterministic Schemes for Routing and Sorting on Fixed-Connection Networks

Torsten Suel

Bell Laboratories, 700 Mountain Avenue, Murray Hill, NJ 07974

Abstract. We give a high-level description of some fundamental randomized and deterministic techniques for routing and sorting on fixed-connection networks such as meshes, hypercubes or point-to-point networks. On the randomized side, we focus on the techniques of randomized routing and random sampling and their use in many algorithms, while our presentation of deterministic algorithms uses the example of the Columnsort algorithm to highlight techniques such as local sorting and deterministic sampling. We then demonstrate that there is a close relationship between the randomized and deterministic techniques presented, and illustrate how this relationship can be used to transform randomized into deterministic algorithms and vice versa. Our main objective here is to provide a more unified perspective on many of the algorithms in the literature, and we do not attempt to provide a complete survey of routing and sorting problems on parallel machines.

1 Introduction and Motivation

Routing and Sorting are fundamental primitives in the design of parallel algorithms for fixed-connection networks such as meshes, hypercubes, and many other classes of topologies. Routing and sorting are also intimately related; for example, many routing problems can be solved by sorting the packets on their destination addresses, while many sorting algorithms rely on routing schemes for their efficient implementation. Not surprisingly, there is a vast literature on routing and sorting problems on interconnection networks, see, e.g., [11], and we will not attempt to give a complete survey of the field.

Our goal here is to describe some basic algorithmic techniques for these problems, and to highlight an interesting relationship between the most common randomized and deterministic approaches. This relationship has not only been at the basis of several new technical results obtained over the last few years, but we also believe that it provides a new and insightful perspective on many known results in the area. The technical material that we present is drawn from work of the author and his collaborators in [7, 8, 16] as well as from that of many other colleagues. The high-level perspective given here, however, has not yet been presented in its full generality.

The basic techniques and observations presented here apply to very general classes of fixed-connection networks, including meshes, hypercubic networks, and also topology-less point-to-point networks as assumed by the *Postal* or *LogP* models. Many of the more technical results, however, are presented for meshes and tori, and do not fully extend to low-diameter networks such as the hypercube.

2 Overview

We start out by presenting some basic randomized techniques. Many of the randomized algorithms for routing and sorting in the literature can be seen as descendants of the basic two-phase routing scheme proposed by Valiant [17]. The fundamental idea in Valiant's routing scheme is to first route each packet to a randomly chosen intermediate destination, and then from there to its final destination, thus reducing the routing of a "worst-case" permutation to that of two consecutive random (or average) permutations.

In the case of parallel sorting algorithms, this scheme is often combined with the use of random sampling [13, 14]. An example is the well-known Sample Sort algorithm, which essentially uses random sampling (plus some local steps) to reduce sorting to routing. These two fundamental techniques have been used in the design of literally hundreds of randomized sorting algorithms for many classes of interconnection networks, often in quite intricate ways, and in many cases they provide the best upper bounds known.

On the deterministic side, many routing and sorting algorithms proceed by repeatedly sorting subsets of the keys or packets, and using the results of these sorts to assign routing paths, intermediate destinations, or approximate ranks to the packets. Maybe the simplest and purest example for this approach is the Columnsort algorithm of Leighton [10], which will be described and discussed in detail.

After describing these basic randomized and deterministic techniques, we illustrate them with the example of the h-relation ruuting problem on point-to-point networks. We describe a very simple and practical deterministic algorithm for this problem based on the total-exchange communication pattern [1] that was independently discovered by several groups of researchers [2, 7, 8, 9, 15]. We use the randomized and deterministic algorithms for h-relation routing to point out an interesting relationship between randomized two-phase routing and the Columnsort algorithm.

We then present a number of results on routing and sorting on meshes that show how this relationship between the most common randomized and deterministic approaches can be used to transform many randomized algorithms (including, e.g., those in [3, 4, 5, 6, 12]) into deterministic ones. The transformation technique is very general, and we are currently not aware of any randomized result for a sorting or static routing problem on the mesh that cannot be matched by a deterministic bound.

This situation is different for low-diameter networks such as the hypercubic networks, where the best randomized results continue to outperform their deterministic counterparts. We will discuss the reasons for this by revisiting the

h-relation routing problem. Finally, we give some open problems and concluding remarks.

References

1. D. Bertsekas, J. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*, Prentice-Hall, 1989.
2. D. Helman, D. Bader, J. JáJá, 'Parallel Algorithms for Personalized Communication and Sorting with an Experimental Study,' *Proc. 8th Symp. on Parallel Algorithms and Architectures*, pp. 211–222, ACM, 1996.
3. C. Kaklamanis, D. Krizanc, L. Narayanan, Th. Tsantilas, 'Randomized Sorting and Selection on Mesh Connected Processor Arrays,' *Proc. 3rd Symposium on Parallel Algorithms and Architectures*, pp. 17-28, ACM, 1991.
4. C. Kaklamanis, D. Krizanc, 'Optimal Sorting on Mesh-Connected Processor Arrays,' *Proc. 4th Symposium on Parallel Algorithms and Architectures*, pp. 50-59, ACM, 1992.
5. C. Kaklamanis, D. Krizanc, S. Rao, 'Simple Path Selection for Optimal Routing on Processor Arrays,' *Proc. 4th Symposium on Parallel Algorithms and Architectures*, pp. 23-40, ACM, 1992.
6. M. Kaufmann, R. Rajasekaran, J. Sibeyn, 'Matching the Bisection Bound for Routing and Sorting on the Mesh,' *Proc. 4th Symp. on Parallel Algorithms and Architectures*, pp. 31–40, ACM, 1992.
7. M. Kaufmann, J. F. Sibeyn, T. Suel, 'Derandomizing Algorithms for Routing and Sorting on Meshes,' *Proc. 5th Symp. on Discrete Algorithms*, pp. 669–679 ACM-SIAM, 1994.
8. M. Kaufmann, J. F. Sibeyn, T. Suel, 'Beyond the Worst-Case Bisection Bound: Fast Sorting and Ranking on Meshes,' *Proc. 3rd European Symp. on Algorithms*, pp. 75–88, Springer LNCS 979, 1995.
9. M. Kunde, 'Block Gossiping on Grids and Tori: Deterministic Sorting and Routing Match the Bisection Bound,' *Proc. 1st European Symp. on Algorithms*, LNCS 726, pp. 272–283, Springer, 1993.
10. F. T. Leighton, 'Tight Bounds on the Complexity of Parallel Sorting,' *IEEE Transactions on Computers*, C-34(4), pp. 344–354, 1985.
11. F. T. Leighton, *Introduction to Parallel Algorithms and Architectures: Arrays, Trees and Hypercubes*, Morgan Kaufmann, 1991.
12. S. Rajasekaran, T. Tsantilas, 'Optimal Routing Algorithms for Mesh-Connected Processor Arrays,' *Algorithmica*, 8, pp. 21-38, 1992.
13. J. H. Reif, L. G. Valiant, 'A logarithmic time sort for linear size networks,' *Journal of the ACM*, 34, pp. 68–76, 1987.
14. R. Reischuk, 'Probabilistic Parallel Algorithms for Sorting and Selection,' *SIAM Journal of Computing*, 14, pp. 396–411, 1985.
15. R. Shankar, S. Ranka, 'Performing Dynamic Permutations on a Coarse-Grained Parallel Machine,' *Proc. First International Workshop on Parallel Processing*, 1994.
16. T. Suel, Improved bounds for routing and sorting on multi-dimensional meshes. In *Proceedings of the 6th Annual ACM Symposium on Parallel Algorithms and Architectures*, pages 26–35, June 1994.
17. L. G. Valiant, 'A Scheme for Fast Parallel Communication,' *SIAM Journal on Computing*, 11, pp. 350–361, 1982.