CBE 6333, R. Levicky

Homework Set 4

1). How would you set up the problem of an accelerating plate, discussed on pages 3-5 of Handout 10, at short and long times? "Long times" means sufficiently long for a velocity perturbation, imposed at the plate, to propagate across the liquid in the gap. To "setup a problem" means the fully simplified differential equation, plus all boundary and initial conditions, from which the velocity profile in the liquid gap could be calculated. The problem only asks you to set up the problem; i.e. you do not need to solve it.

2). When using similarity analysis to predict the performance of a liquid pump, the parameters that one wants to typically predict are the power consumed by the pump P, the pressure difference Δ produced across the pump, and the efficiency η of the pump. For the pumping of an incompressible fluid using a centrifugal pump, you estimate that the performance is likely to depend on the fluid density ρ, angular velocity of the pump impeller ω, the mean diameter of the impeller D, the fluid viscosity μ, and the fluid volumetric flowrate Q.

So, you start with the following set of relations,

$$P = f_1(\rho, \omega, D, Q, \mu)$$
$$\Delta = f_2(\rho, \omega, D, Q, \mu)$$
$$\eta = f_3(\rho, \omega, D, Q, \mu)$$

a). Apply the Buckingham Π Theorem to the above expressions to convert them to a dimensionless form (i.e. to relations entirely expressed in terms of dimensionless groups).

b). How would you adjust the dimensionless relations derived in a) if you are given, in addition, the following information: "Experimental data show that viscosity is not important to determining the performance of a centrifugal pump."

3). A centrifugal pump tested at 1000 rpm delivers 5 ft^3/s of incompressible water flow against a pressure difference of 12,490 lb_f/ft^2. The power necessary to run the pump under these conditions is 200 hp.

a). Calculate the efficiency of the pump. The efficiency is the actual power delivered to the fluid (you need to calculate this) divided by the total power supplied to the pump (200 hp).

b). A geometrically similar pump of three times the diameter is made to run at 500 rpm. Find the flow rate, pressure difference, and power for the same efficiency.

For this problem, assume that the pump behaves according to results of 2b). Note that the density of water is 1.94 $slug/ft^3$.

4). In designing a thermally insulated container unit, tests were made on a prototype model of dimensions 3 ft × 3 ft × 4 ft. Initially, the interior as well as the surroundings of the unit were at 70 ^{o}F.

Then, an incandescent light bulb heat source was placed inside the unit and used to generate a constant heat output. With the bulb turned on, and the external temperature maintained at 70 °F, the difference between the temperature of the inside surfaces of the unit and the outside temperature of 70 °F was measured as given below.

Data:

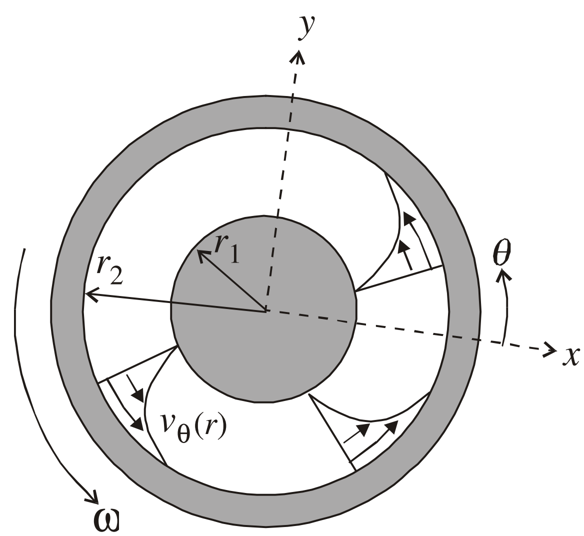
Time, hours	**0**	**0.5**	**1.0**	**2.0**	**3.0**	**5.0**	**10**
Temperature difference, °F	**0**	**15.9**	**22.5**	**30.0**	**35.5**	**42.0**	**48.3**

Rate of heat generation by bulb, per unit area of wall = 6.09 BTU/(hr ft^2)
Thickness of insulation = 2 inches
Thermal conductivity of insulation = 0.0209 BTU/(ft hr °F)
Thermal heat capacity of insulation = 4.85 BTU/(ft^3 °F)

a). Plot the temperature difference as a function of time in terms of dimensionless groups so that the data can be applied to other problems of a similar nature. One group should contain the temperature rise but not the time, and the other should contain the time but not the temperature rise.

b). Determine the temperature rise that would be observed at 15 minutes if the insulation was only 1 inch thick (but all other settings were identical to those listed above).

5). A steady-state, laminar flow of an incompressible, constant viscosity Newtonian fluid occurs in the gap between two cylinders (see fig. at right). The flow is driven by the rotation of the external cylinder at an angular velocity ω while the inner cylinder is stationary. Gravity points down (into the page).



a). Find an expression for the velocity profile v_θ. The following identity may prove useful: $d/dr\,[1/r\; d(rv_\theta)/dr\,] = d^2v_\theta/dr^2 + 1/r\; dv_\theta/dr - v_\theta/r^2$.

b). What is the expression for the *torque* per unit area exerted by the outer cylinder on the fluid? The torque is measured relative to the origin in the center of the inner cylinder.

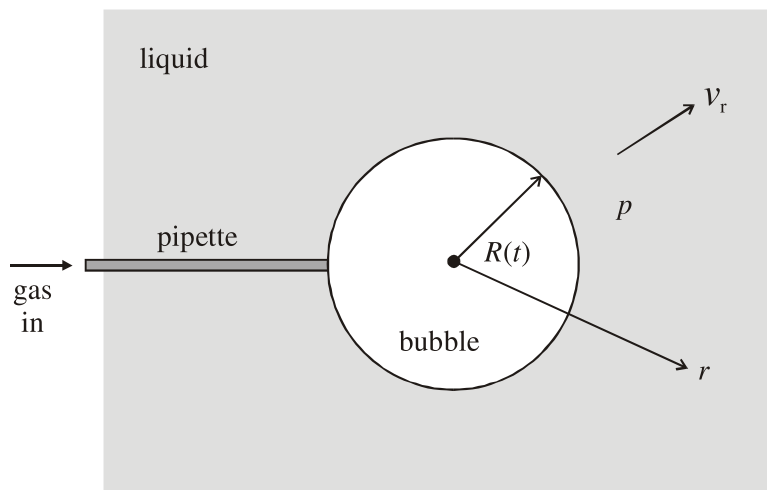
c). How could you use this device to measure viscosity?

6). In problem 5, given that pressure at $r = r_1$ equals $p(r_1, z) = p_o - \rho g z$, what is the expression for the pressure $p(r, z)$ at an arbitrary radial position $r_1 \le r \le r_2$?

7). A bubble of gas is situated at the tip of a small glass pipette (see the below figure). Surrounding the bubble is a constant viscosity, incompressible Newtonian liquid. The bubble is being slowly inflated through the pipette. The bubble radius is given by a known (i.e. measured) function $R(t)$ where t is time.

The bubble can be assumed to be perfectly spherical.
All effects of gravity can be neglected.
The pipette itself can be assumed not to affect the liquid flow in any way.
The bubble center remains at the coordinate origin at all times.
Since the bubble radius R is a function of time this is *not* a steady-state problem.

a). Is there any buoyancy force on the bubble (why or why not)?
b). Derive an expression for the velocity v_r inside the *liquid* (i.e. we need an expression for v_r when $r > R(t)$). Express v_r as a function of r, $R(t)$, and derivatives of $R(t)$.
Hint: what equations does v_r appear in? Any of these may be helpful for solving for it.
c). Derive an expression for the pressure p inside the liquid in terms of r, $R(t)$, and derivatives of $R(t)$. Note that at $r = \infty$, $p = p_\infty$.
d). What is the physical reason why the pressure p in the liquid is different from p_∞ in the vicinity of the bubble?



8). At time $t = 0$, solute A starts to diffuse across a membrane of thickness L from left to right (see figure below). Before that time, there is no A anywhere in the membrane. The partitioning coefficient, defined as the ratio of concentration of A in the membrane to that in the external solution, is K. D is the diffusion coefficient of A in the membrane. The concentrations of A in the external solutions are fixed at the values shown in the figure. Transport by virtue of bulk convection can be neglected, since A remains sufficiently dilute at all times and there is no forced or free convection. In addition, the total concentration of all species in the membrane and the diffusion coefficient are to be treated as constant.

a). Perform an order-of-magnitude estimate of the time required for A to diffuse across the membrane.

b). Derive the solution to the concentration profile of A, $C_A(x,t)$, in the membrane for times much longer than the time scale estimated in part a).

c). Using a "combination of variables" approach, derive $C_A(x,t)$ in the membrane for times much shorter than the time scale estimated in part a).