Homework Set 3

1). The velocity field for a flow is given by:

$$v_1 = Cx_1 x_2 / t \ \text{ft/sec} \qquad v_2 = -C(x_2^2)/2t \ \text{ft/sec} \qquad v_3 = 0$$

Consider a differential fluid element, like we did in handout #7. C is a constant equal to 1 ft^{-1}.
a). Is the fluid element being rotated? (calculate the rate of rotation, if any, in terms of $\mathbf{v}$).
b). Is the fluid element being shear strained? (calculate the rate of shear strain, if any, in terms of $\mathbf{v}$).
c). Is the fluid element being dilated or compressed? (calculate the rate of dilatation, if any, in terms of $\mathbf{v}$).
d). Is the flow steady state (yes/no)? How can you tell?

2). Consider the same velocity field as in problem 1.
a). Calculate the velocity gradient tensor $\nabla \mathbf{v}$.
b). What is the acceleration of a fluid element, moving with the flow, at the point $x_1 = 5$ ft, $x_2 = 2$ ft, and at time $t = 10$ sec?
c). What part of this acceleration is due to: 1) unsteady nature of the flow, and 2) motion through a velocity gradient?

3). A liquid film flows down a solid wall under the action of gravity (see Figure). Given the following:

i). the stress tensor $\boldsymbol{\sigma}$ is (Newtonian fluid)

$$\sigma_{ij} = -p\delta_{ij} - (2/3)\,\mu\,(\frac{\partial v_k}{\partial x_k})\,\delta_{ij} + \mu\,(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$$

ii). the flow is incompressible
iii). the pressure p in the film is constant
iv). the viscosity μ of the liquid is constant
v). the viscosity of air can be approximated as equal to zero
vi). gravity points in the negative x_2 direction
vii). the thickness d of the film may be assumed constant
viii). the flow is steady state

Calculate the velocity profile across the film.
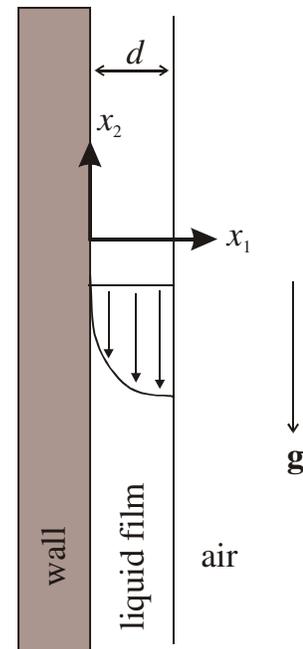
(Hint: you need to decide which velocity components are zero and which are not. Then, what coordinate variables do the nonzero components depend on? Then, apply the proper differential balance equation and solve.)

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4). Consider the same flow situation as in Problem 3 (a liquid film flowing down a solid wall).

a). Derive an expression for the dissipation function Φ. Note that you will need the velocity profile calculated in problem 3.
b). Calculate an expression for the rate of dissipation of mechanical energy to heat (i.e. the dissipated power) per unit area of the wall.
c). Take $d = 0.01$ ft, standard value for g, and the liquid to be water at atmospheric pressure and 20 °C. Under these conditions, density of water $\rho = 1.937$ slugs/ft^3 and viscosity of water $\mu = 2.107$ lb$_f$ s/ft^2. With these data, what is the numerical value (in English units) of the dissipated power per unit area of the wall?

5). Consider the same flow as in problem 3 (film flowing down a vertical wall). In addition to information provided previously, you are also given that:
i). The fluid obeys Fourier's Law, with a constant heat conductivity κ.
ii). There is no heat transfer by radiation or externally-coupled source of heat within the fluid.
iii). Also recall that the flow is steady state, and the fluid is incompressible.
iv). The temperature at the interface between the liquid film and air is fixed at T_o (temperature of the surrounding atmosphere).
v). There is no heat transfer (i.e. $\mathbf{q} = \mathbf{0}$) into the wall (this means that the wall is modeled as an adiabatic barrier - by definition, an "adiabatic" barrier is one across which no transfer of heat can occur).

a). What is the temperature profile across the liquid film? In other words, what is $T(x_1)$, where T is the temperature?
b). Determine an expression for the heat flux $\mathbf{q}$ from the liquid film into the air. Is the heat flowing out of, or into the liquid film?
c). Compare the expression for the heat flux from part b) to the expression for the dissipated power from part b) of problem 4. How do the two expressions compare? Do the answers make physical sense?

6). A liquid passes through a sudden enlargement as shown in the following figure. The flow is incompressible, frictionless, steady state, the walls of the container are adiabatic (that is, no heat flows across them), and parameters can be identified as indicated in the Figure. For a fixed position along the pipe, the parameters (pressure, velocity, density) can be assumed uniform across the pipe cross-section (note that Port 2 is sufficiently far downstream to avoid any complications due to the enlargement that may invalidate this assumption). In addition, the pressure acting at port 1 can be considered uniform with a value p_1 across the entire *enlarged* section, not just the actual opening (of area A_1) across which flow occurs.

Derive an expression for the change in internal energy per unit mass of fluid, $u_2 - u_1$, in terms of v_1, A_1, and A_2 only. You will need to apply all three balance laws. Note: what is the force of the walls on the fluid if the fluid is truly frictionless?
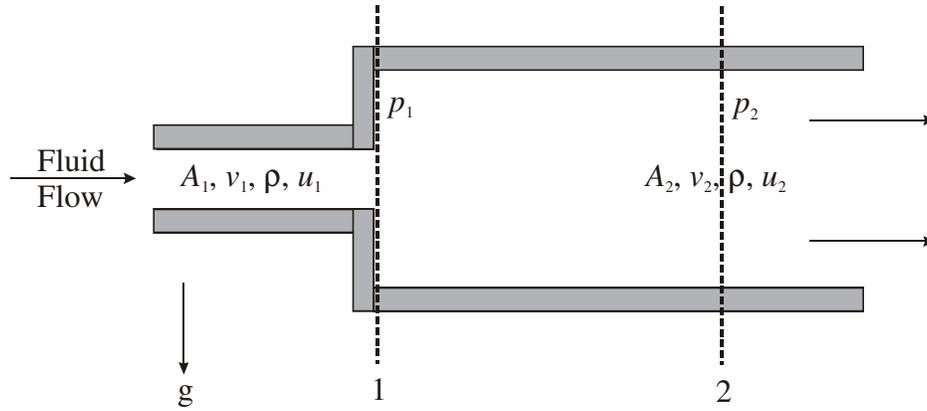
7). A slab of thickness L is subjected to microwave radiation that causes volumetric heating to vary according to

$$\dot{q}(x) = \dot{q}_0 [1 - (x/L)]$$

where $\dot{q}_0$ has a constant value of 180 kW/m³ and the slab thickness, L, is 0.06 m. The thermal conductivity of the slab material is 0.6 W/m • K. You can assume that the extent of the slab along the y and z directions is infinite.

The boundary at $x = L$ is perfectly insulated, while the surface at $x = 0$ is maintained at a constant temperature of 320 K.

(a) Determine an expression for $T(x)$ in terms of x, L, k, $\dot{q}_0$, and T_0.
(b) Where, in the slab, will the maximum temperature T_{max} occur?
(c) What is the value of T_{max}?