Homework Set 2

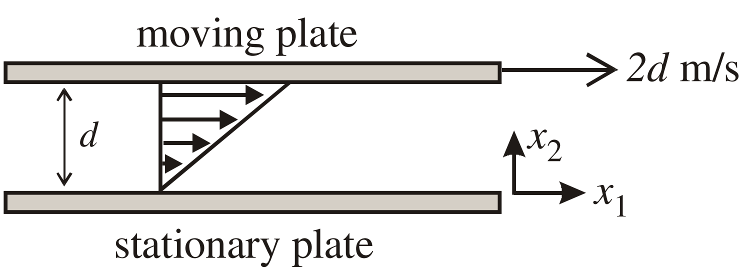
1). In all the expressions below:

A and **B** are 2nd rank tensors.
c and **d** are vectors (1st rank tensors).
∇ is the del operator.
r is the result of the indicated operation.

Determine the rank of r. Also, using Cartesian notation give the *full* expression for each of the components of r in the below equations (ex. if $\mathbf{c} \bullet \mathbf{d} = r$, then the full expression for r is $r = c_1 d_1 + c_2 d_2 + c_3 d_3$). "Cartesian notation" means that the Cartesian coordinate system is being used, so that, for instance, the ith component of the del operator is equal to $\partial/\partial x_i$ where x_i is one of the cartesian coordinate variables.

a). $\mathbf{c} \bullet \mathbf{A} \bullet \mathbf{d} = r$
b). $\nabla \bullet (\mathbf{B} \bullet \mathbf{A}) = r$
c). $\nabla \mathbf{c} \bullet \mathbf{d} = r$

2). The velocity **v** for the flow shown in the figure is given by:



$v_1 = 2x_2$ m/s

$v_2 = 0$

$v_3 = 0$

The flow is steady, incompressible, and laminar.

a). If the stress tensor $\boldsymbol{\sigma}$ is given by the following expression, calculate all of the Cartesian components of $\boldsymbol{\sigma}$ (i.e. σ_{11}, σ_{12}, σ_{13}, σ_{21}, σ_{22}, σ_{23}, σ_{31}, σ_{32}, σ_{33}). This expression holds for so-called Newtonian fluids. Note that if **v** is in m/s, then the pressure p would be in Pascals and the viscosity μ in kg/(m s). Also, note the use of the summation convention and the Kronecker delta. Express your results in terms of p, μ and the numerical values of the velocity derivatives.

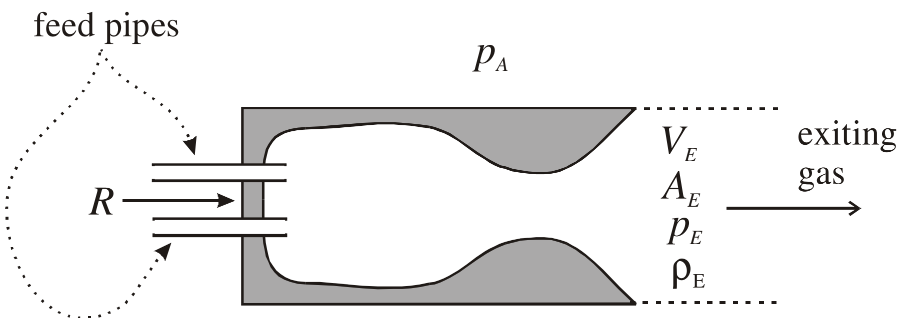
$$\sigma_{ij} = -p\delta_{ij} - (2/3)\,\mu\,(\frac{\partial v_k}{\partial x_k})\,\delta_{ij} + \mu\,(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$$

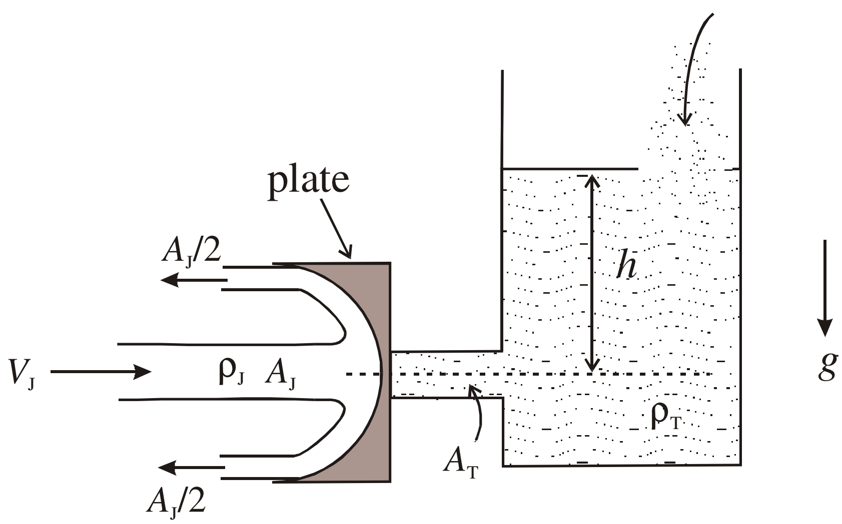
b). What are the stresses exerted by the fluid on the stationary *and* the moving plates for all three coordinate directions x_1, x_2 and x_3? Classify the stresses as shear or normal.

c). Compare the values of the stress component σ_{21} calculated for the stationary and moving plates. Does your answer make physical sense?

3). Consider the reactor in the figure. A steady flow of liquids A and B comes in through the feed pipes on the left. In the reactor, the liquids mix and combust to form a gas. The gas exits through the nozzle on the right side of the reactor with a velocity V_E, a density ρ_E, and pressure p_E at the point at which the area of the nozzle is A_E. The process is at steady state (no time dependence). Also note that:
1). The reactor walls (except at the exit) are surrounded by atmospheric pressure p_A.
2). Neglect any effects due to pressure and stresses in the *feed* pipes, or due to the velocity of the liquids entering through the *feed* pipes.

In terms of the given parameters, what is the restraining force R necessary to keep the reactor stationary?

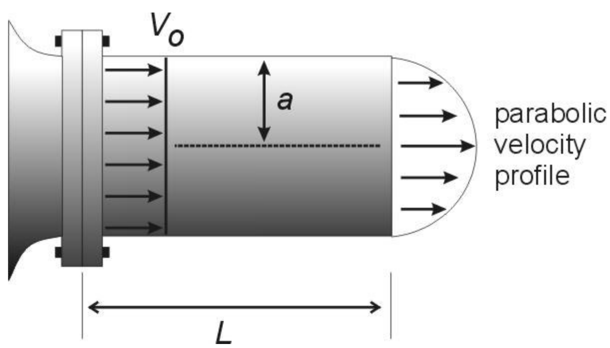




4). A jet of liquid of density ρ_J, velocity V_J, and cross-sectional area A_J pushes a plate against the outlet of a tank as shown in the figure. The jet hits the plate, and is split into two equivalent return jets, each with an area $A_J/2$. The plate seals a tank outlet as shown. The tank outlet is square and has a total cross-sectional area A_T. The tank is being filled with a liquid of density ρ_T. Note that the outlet will open once the pressure in the tank has risen sufficiently. Gravity acts downward. Weight of the plate is negligible (i.e. don't worry about any vertical forces on the plate). All of the fluids are incompressible. What is the maximum value of the depth h that can be attained inside the tank before the outlet opens?

5). A circular pipe of radius a ft and length L ft is attached to a smoothly rounded outlet of a liquid reservoir by means of flanges and bolts as shown in the below figure. At the flange section, the velocity is uniform over the cross section with magnitude V_o. At the outlet, which discharges into the atmosphere, the velocity profile is parabolic because of the friction in the pipe. What force must be supplied by the bolts to hold the pipe in place?

Additional notes: the liquid is incompressible. The fluid velocity at the walls of the pipe is zero. The zero velocity (no-slip) condition holds both at the inlet and outlet of the pipe, but at the inlet the boundary layer is so thin you can neglect its presence and effectively assume that the velocity is uniform across the pipe cross-section as drawn in the figure. The pressure at the outlet of the pipe is p_2, at the inlet it is p_1. The problem is asking for the *horizontal* component of the force supplied by the bolts. Finally, the pipe is cylindrical; you will have to perform an integration in cylindrical coordinates. After choosing your control volume, the first question to ask is: How would you write the general form of the velocity profile at the outlet, given that it is a parabola?



6). Derive an expression for the angular velocity ω (radians per second) for the sprinkler shown below. Water enters the sprinkler from below, along the central vertical duct, and then passes outward through the two arms, thus making the sprinkler rotate. The vertical duct exerts a torque $a\omega$ on the sprinkler that opposes rotation. The angular velocity has reached a steady state (i.e. is not changing with time), and the flow properties are uniform across the pipe cross-sections (as in Handout 5 examples). The outlets of the arms are angled at 90° relative to the rest of the arm.

Given:
R: radius of the sprinkler
a: torque coefficient associated with the sprinkler rotation (the sprinkler is not frictionless)
Q: volumetric flowrate of water through the sprinkler
ρ: density of water
A: cross-sectional area of the sprinkler arms used for water flow