Homework Set 1

(1). Find the work done in moving an object through a displacement of $-\boldsymbol{\delta}_1 + 2\,\boldsymbol{\delta}_2 - 5\,\boldsymbol{\delta}_3$ meters if the applied force is $\mathbf{F} = 2\,\boldsymbol{\delta}_1 + 4\,\boldsymbol{\delta}_2 - \boldsymbol{\delta}_3$ Newtons.

(2). Find a unit normal to the surface $2x^2y + 2xz = 3$ at the point $\mathbf{r} = \boldsymbol{\delta}_1 + 2\,\boldsymbol{\delta}_2 - 0.5\,\boldsymbol{\delta}_3$ (i.e. the point at which $x = 1$, $y = 2$, and $z = -0.5$).

(3). Find the gradient of the scalar (let's say pressure) field $P = x^2yz + 4x^2z^2$ Pascals at the point $\mathbf{r} = 1\,\boldsymbol{\delta}_1 - 2\,\boldsymbol{\delta}_2 + 1\,\boldsymbol{\delta}_3$ meters and in the direction of $2\,\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2 - 2\,\boldsymbol{\delta}_3$.

(4). If a force field $\mathbf{F}$ is given by $\mathbf{F} = (3x^2 + 6y)\,\boldsymbol{\delta}_1 - 14yz\,\boldsymbol{\delta}_2 + 20xz^2\,\boldsymbol{\delta}_3$, evaluate the line integral

$$\int_{(0,0,0)}^{(1,1,1)} \mathbf{F}\cdot d\mathbf{r} \qquad \text{(Recall that the differential vector } d\mathbf{r} = dx\,\boldsymbol{\delta}_1 + dy\,\boldsymbol{\delta}_2 + dz\,\boldsymbol{\delta}_3\text{)}$$

from the point (0,0,0) to the point (1,1,1) along the following two paths:

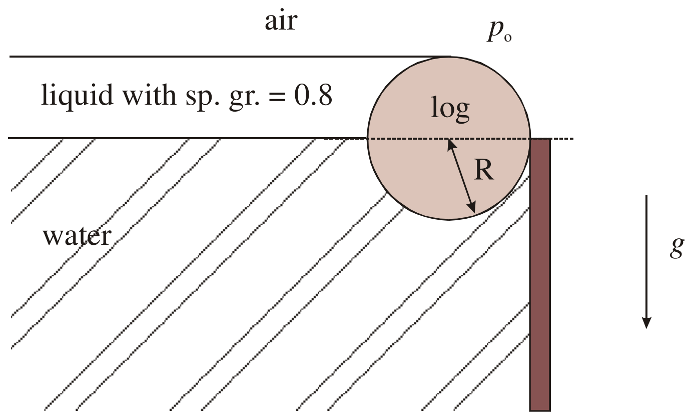
(a) the straight lines from (0,0,0) to (1,0,0), then to (1,1,0), and then to (1,1,1).
(b) the straight line directly from (0,0,0) to (1,1,1). (Hint: parametrize this line by using $x = t$, $y = t$ and $z = t$, and let t vary from 0 to 1).
(c) Is this force field conservative? Why or why not?

(5). Is the velocity field $\mathbf{v} = 2xyz\,\boldsymbol{\delta}_1 + x^2\,\boldsymbol{\delta}_2 - yz^2\,\boldsymbol{\delta}_3$ m/s conservative? Is it incompressible?

(6). Calculate the volumetric flowrate (in m^3/s) of a fluid across the surface B. Here, the fluid velocity $\mathbf{v} = 12x_3\,\boldsymbol{\delta}_1 - 8\,\boldsymbol{\delta}_2 + 2x_2\,\boldsymbol{\delta}_3$ m/s and the surface B is that part of the plane $2x_1 + 3x_2 + 6x_3 = 12$ m which is located in the first octant of the CCS system (the result is given by calculating the surface integral $\iint_B \mathbf{v}\cdot\mathbf{n}\,dB$).

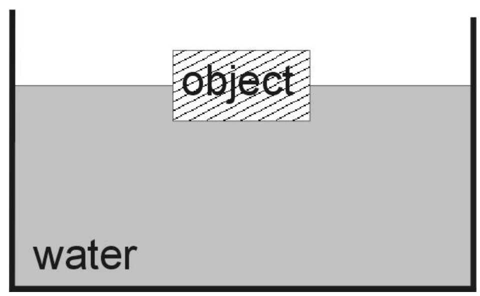
(7). A long, cylindrical log is pressed against a vertical barrier as depicted. The entire system is in a standard gravitational field g (32.174 ft/sec^2), pointing down, and surrounded by air at pressure p_o. $R = 1$ ft.

a). Calculate, in lb_m/ft^3, the density of the log.
b). Calculate, in lb_f/ft, the horizontal force exerted by the log on the barrier per one foot of the log into the page.



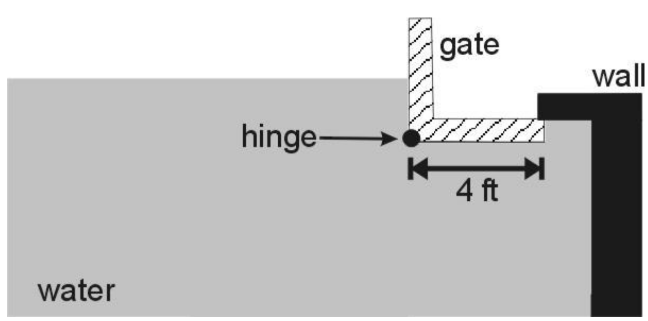
(8). An object with a specific gravity of 0.5 is floating on water in a container as shown in the below figure. If the container, liquid, and object are suddenly accelerated upward at 10 ft/s^2, what will be the position of the object relative to the water surface?

Write down and simplify the relevant equations to show the desired result. You can use the following parameters. The object dimensions are W (width), L (length), and H (height). The density of water is ρ_w, and the specific gravity of the object is $s_o = 0.5$. Gravitational acceleration is g, and the imposed upward acceleration is $a = 10$ ft/sec^2.



(9). If the water rises on the left side of the gate in the figure below, the gate will eventually rotate open. At what depth above the hinge will this occur? Neglect effects of gravity and assume the hinge is frictionless.

Hint: You want to perform a ______________ balance. Note that the gate is *not* in a vacuum. The air pressure may be assumed to be the same everywhere.



(10). A thin glass tube of radius a is inserted into the free surface of a liquid (see fig. below). The liquid rises a distance H into the tube. The angle θ depends only on the material of the tube and the identity of the liquid. If θ is positive, as shown in the figure, the liquid is said to wet the surface.

For this problem, the meniscus at the free liquid surface in the tube can be assumed to be approximately spheroidal (i.e. forms a portion of a spherical surface, whose radius depends on a and θ). For a given positive value of θ, surface tension T of the air-liquid interface, and a, find an expression for H. Gravity g is present and points downward.