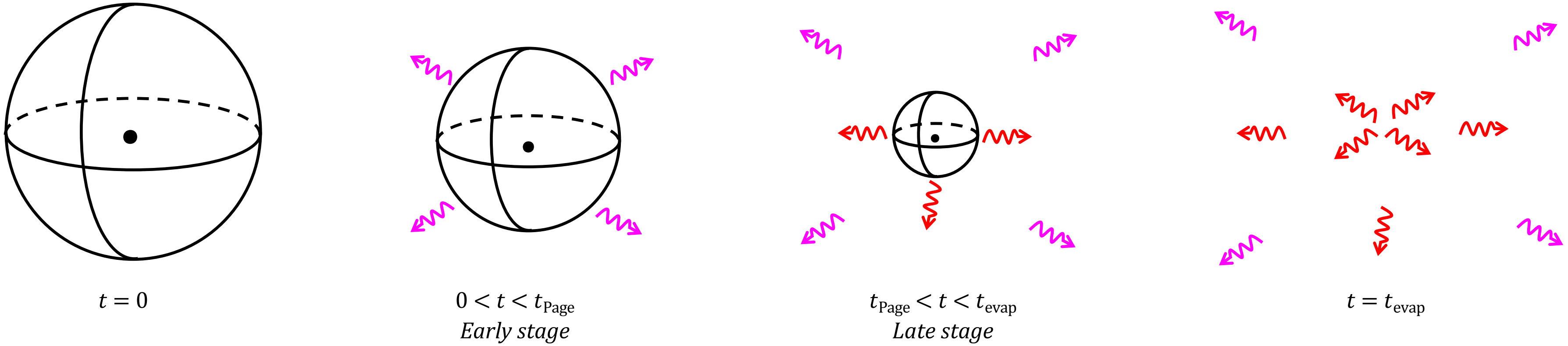
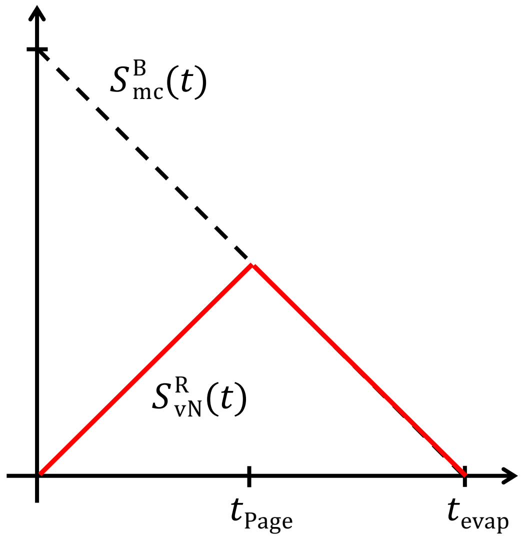
17. A Resolution of the Page Time Paradox?

- *Recall*: The Page Time Paradox is a disagreement over how the entanglement entropy of Hawking radiation $S^{\mathrm{R}}_{\mathrm{vN}}(t)$ behaves:

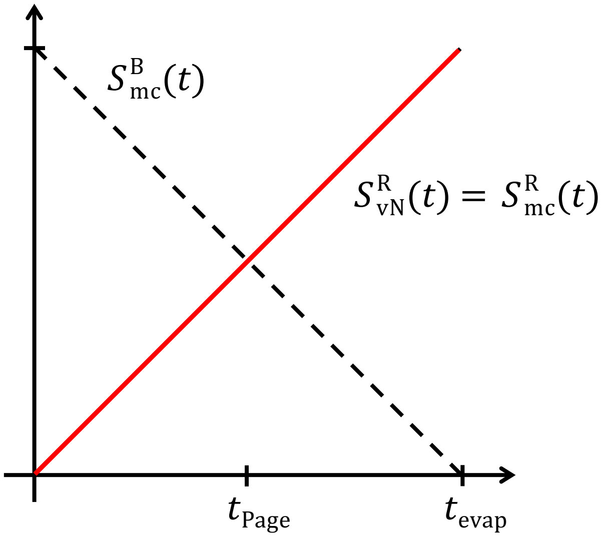


During early stage: Page and Hawking agree:
Early stage radiation R-modes are in thermal states, and are entangled with black hole B-modes; so as more and more R-modes are emitted, $S^{\mathrm{R}}_{\mathrm{vN}}(t)$ increases.

During late stage: Page and Hawking disagree:
- *Hawking*: Late stage R-modes are still in thermal states, so $S^{\mathrm{R}}_{\mathrm{vN}}(t)$ continues to increase.
- *Page*: Late stage R-modes are max. entangled with early stage R-modes; so they cannot be in thermal states; and as more and more B-modes become late stage R-modes, $S^{\mathrm{R}}_{\mathrm{vN}}(t)$ decreases.



The "Page curve" according to Page (1993) and statistical mechanics: Info is not lost!



Hawking's (1995) prediction according to quantum field theory: Info is lost!

Claim: The Page curve can be derived using the RT formula from the AdS/CFT correspondence.

Penington (2020); Almheiri *et al*. (2020)

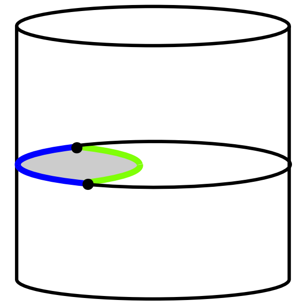
1. Set-Up

RT Formula

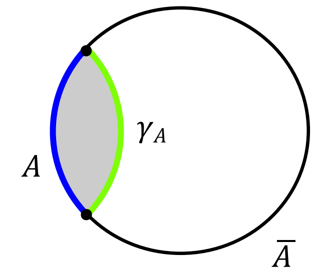
$$S_{\mathrm{vN}}(\rho_A) = \mathrm{Area}(\gamma_A)/4$$

The entanglement entropy of a boundary quantum subsystem on A with respect to a subsystem on $\bar{A}$ is given by the area of the minimal area bulk surface γ_A that has the same boundary as A.

surface of cylinder (boundary) = spacetime of quantum system



interior of cylinder (bulk) = spacetime of gravity system

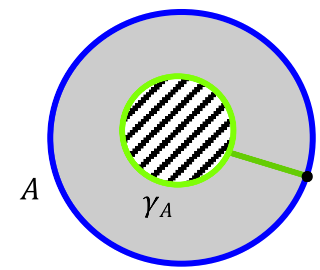


A and γ_A have the same boundary, and hence enclose a bulk surface (grey region)

Black Hole Entropy

$$S_{\mathrm{BH}} = \mathrm{Area(event\ horizon)}/4$$

The entanglement entropy of a subsystem far from the event horizon with respect to a subsystem inside the event horizon?



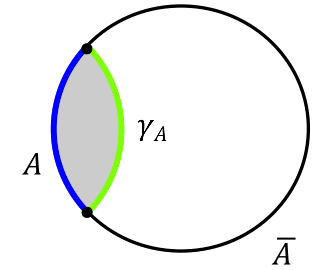
A and γ_A have the same boundary (the empty set!) and enclose a bulk surface (grey region)

Subsystem far from the horizon = Hawking radiation?

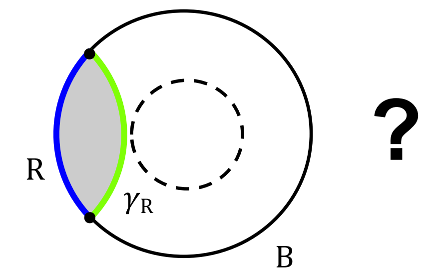
Subsystem inside horizon = black hole degrees of freedom?

Suggests: We can use the RT formula to calculate the entanglement entropy of Hawking radiation!

- *Note*: The RT formula gives the entanglement entropy of one subsystem $\mathcal{H}_A$ of a bipartite quantum system $\mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$ located on the boundary $A \cup \bar{A}$ of a bulk spacetime.

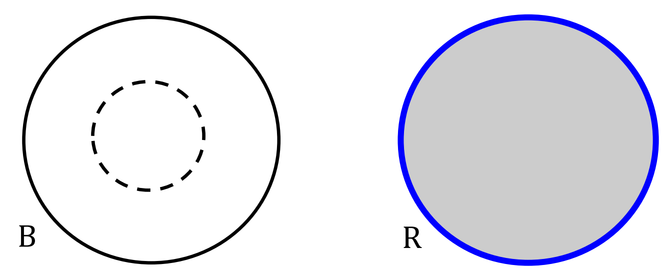


- *Page's Model*: An evaporating black hole is a bipartite quantum system $\mathcal{H}_\mathrm{B} \otimes \mathcal{H}_\mathrm{R}$ of black hole $\mathcal{H}_\mathrm{B}$ and radiation $\mathcal{H}_\mathrm{R}$.
  - *But*: *In what sense are the $\mathcal{H}_\mathrm{B}$ and $\mathcal{H}_\mathrm{R}$ subsystems localized on the boundary of a bulk spacetime?*
  - *And*: *Can the bulk spacetime be identified as an evaporating black hole spacetime?*



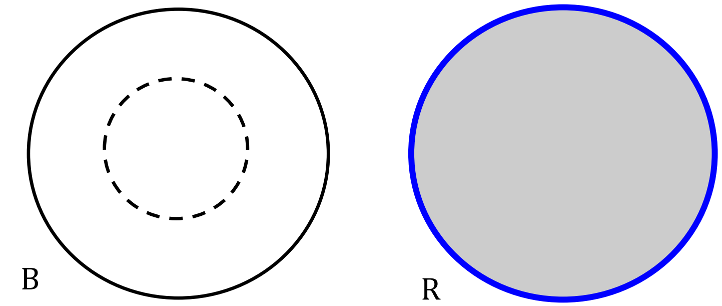
Initial Problem: AdS spacetimes are spatially closed: Any radiation that makes it to the boundary gets reflected back into the bulk!

- *Solution*: Two bulk spacetimes with separate boundaries:
  - Evaporating black hole AdS bulk spacetime that corresponds (under AdS/CFT) to a boundary quantum system $\mathcal{H}_\mathrm{B}$ located on the entire boundary B.
  - Auxiliary radiation bulk spacetime that corresponds (under AdS/CFT) to a boundary quantum system $\mathcal{H}_\mathrm{R}$ located on the entire boundary R.



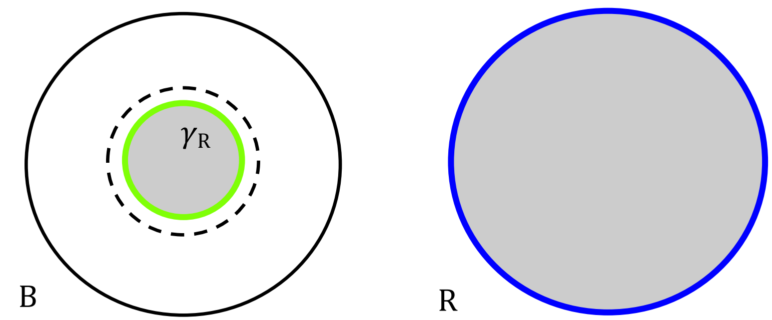
2. RT Formula Predictions

Claim 1. For $0 < t < t_{\text{Page}}$, the "minimal area" bulk surface with the same boundary as R is the empty surface $\gamma_{\text{R}} = \emptyset$.



Technical derivation in general relativity...

Claim 2. For $t_{\text{Page}} < t < t_{\text{evap}}$, the "minimal area" bulk surface with the same boundary as R is a closed surface just inside the event horizon.



<u>*Consequence of Claim 1*</u>:

$S^{\text{R}}_{\text{vN}}(t)$ increases for $0 < t < t_{\text{Page}}$!

Due to "quantum corrections" to the RT formula...

<u>*Consequence of Claim 2*</u>:

$S^{\text{R}}_{\text{vN}}(t)$ behaves like the event horizon (decreases) for $t_{\text{Page}} < t < t_{\text{evap}}$!

Reproduces the Page Curve!

3. An Explanation of the Page Curve?

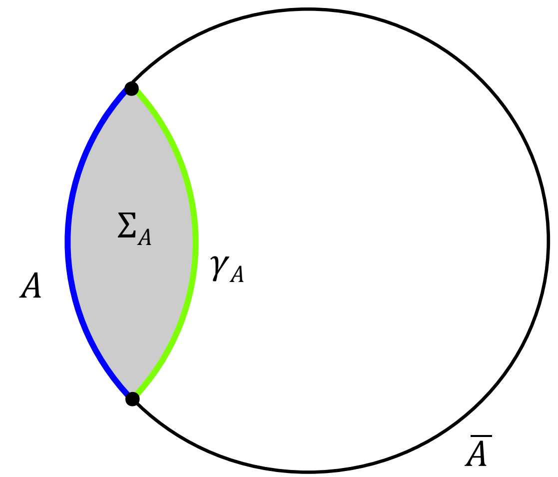
RT Formula

$S_{\text{vN}}(\rho_A) = \text{Area}(\gamma_A)/4$

Entanglement Wedge Reconstruction (EWR)

Physical quantities defined on a boundary region A correspond to bulk quantities in the "entanglement wedge" $W[A] \equiv D_{\text{bulk}}[\Sigma_A]$ of A.

The bulk domain of dependence of Σ_A



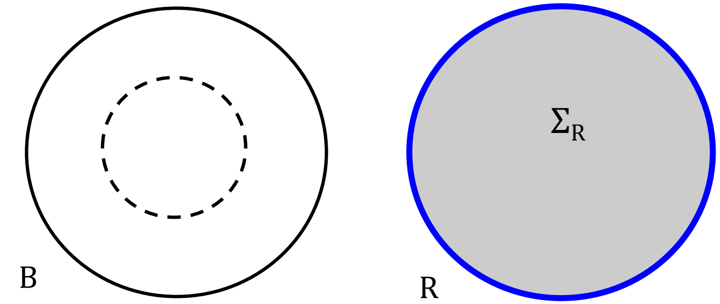
A and γ_A have the same boundary, and hence enclose a bulk surface Σ_A

- *Can Show*: The RT formula holds *if and only if* EWR holds.

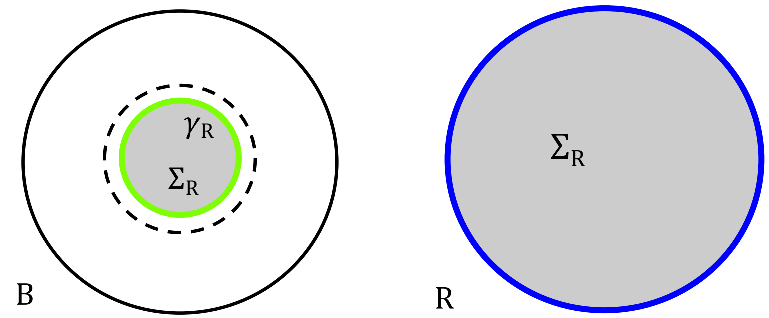
- *And this means*: When the RT formula holds, physical quantities in the grey bulk region Σ_A can be represented by physical quantities in the blue boundary region A.

So...

Claim 1. For $0 < t < t_{\text{Page}}$, the "minimal area" bulk surface with the same boundary as R is the empty surface $\gamma_R = \emptyset$.



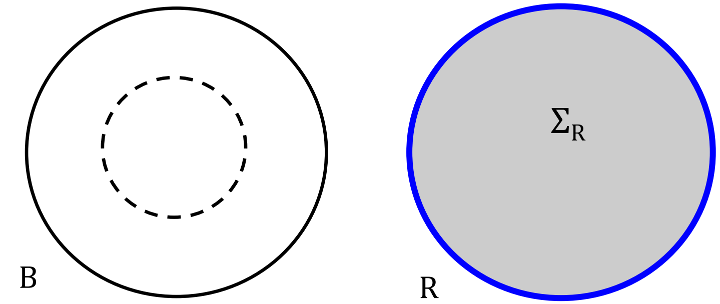
Claim 2. For $t_{\text{Page}} < t < t_{\text{evap}}$, the "minimal area" bulk surface with the same boundary as R is a closed surface just inside the event horizon.



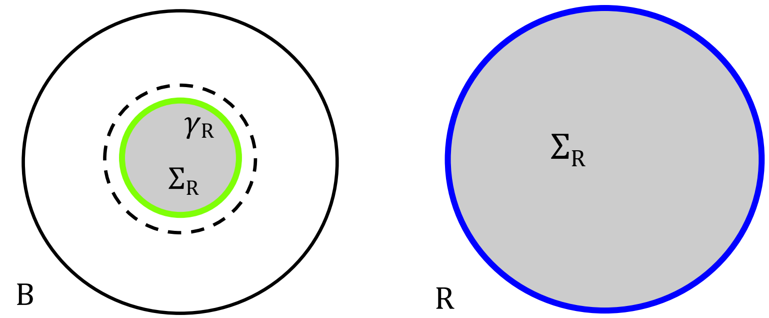
- *According to EWR*: After t_{Page}, the radiation subsystem R corresponds to bulk quantities in the grey regions.
  - *Everything in the bulk radiation spacetime ("far" from the black hole).*
  - *And degrees of freedom in the interior of the event horizon (inside the black hole)!*

So...

Claim 1. For $0 < t < t_{\mathrm{Page}}$, the "minimal area" bulk surface with the same boundary as R is the empty surface $\gamma_{\mathrm{R}} = \emptyset$.



Claim 2. For $t_{\mathrm{Page}} < t < t_{\mathrm{evap}}$, the "minimal area" bulk surface with the same boundary as R is a closed surface just inside the event horizon.



- *According to EWR*: After t_{Page}, the radiation subsystem R corresponds to bulk quantities in the grey regions.

- *Possible interpretations*:
  - After t_{Page}, early stage R-modes (far from the black hole) represent/encode interior black hole degrees of freedom (i.e., "info" escapes the interior).
  - After t_{Page}, a wormhole connects regions far from the black hole to the black hole interior.

Bizzare, but suggested by one method of deriving the RT formula!

4. Recap: A Bestiary of Entropies in Contemporary Physics

Def. 1 (*Thermodynamic entropy*). The **thermodynamic entropy** $S_{\text{TD}}(\sigma_2)$ of a state σ_2 of a physical system is the ratio of the change in heat δQ_R to temperature T of a reversible process that connects an initial state σ_1 to σ_2:

$$S_{\text{TD}}(\sigma_2) \equiv \int_{\sigma_1}^{\sigma_2} \frac{\delta Q_R}{T} + \text{const.}$$

Def. 2 (*Boltzmann entropy*). The **Boltzmann entropy** $S_{\text{Boltz}}(\Gamma_M)$ of a macrostate Γ_M of size $|\Gamma_M|$ is given by:

$$S_{\text{Boltz}}(\Gamma_M) \equiv \ln|\Gamma_M|$$

Def. 3 (*Gibbs entropy*). The **Gibbs entropy** $\boldsymbol{S_{\text{Gibbs}}(\rho)}$ of an ensemble distribution ρ is the ensemble average of the quantity $-\ln\rho$:

$$S_{\text{Gibbs}}(\rho) \equiv -k\int_{\Gamma} \rho(x)\ln\rho(x)dx$$

Def. 4 (*Shannon entropy*). The **Shannon entropy** $S_{\text{Shan}}(X)$ of a random variable X with possible values $\{x_1, \ldots, x_\ell\}$ and probability distribution $\{p_1, \ldots, p_\ell\}$ is given by:

$$S_{\text{Shan}}(X) \equiv -\sum_i p_i \log_2 p_i$$

Def. 5 (*von Neuman entropy*). The **von Neumann entropy** $S_{\text{vN}}(\rho)$ of a density operator state ρ is given by:

$$S_{\text{vN}}(\rho) \equiv -\text{Tr}(\rho \ln \rho)$$

Def. 6 (*Entanglement entropy*). The **entanglement entropy** S_A of a subsystem A of a bipartite system AB is defined as the von Neumann entropy of ρ_A:

$$S_A \equiv S(\rho_A) = -\text{Tr}(\rho_A \ln \rho_A)$$

Def. 7 (*Black hole entropy*). The **Bekenstein-Hawking entropy** of a black hole is given by

$$S_{\text{BH}} \equiv \text{Area(horizon)}/4$$

Def. 8 (*RT Formula*). The entanglement entropy entropy S_A of of a subsystem localized on a boundary region A of an AdS spacetime is given by:

$$S_A \equiv \text{Area}(\gamma_A)/4$$

where is the minimal area bulk surface with the same boundary as A.