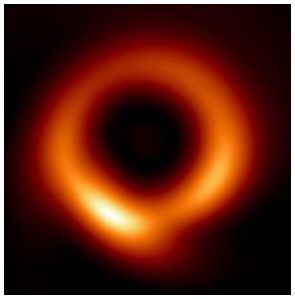
15. Black Hole Entropy as Entanglement Entropy

1. Motivations

$$S_{\mathrm{BH}} = \mathrm{Area}(\text{event horizon})/4$$



$$S_{\mathrm{vN}}(\rho_A) = -\mathrm{Tr}(\rho_A \log \rho_A)$$

The degree of mixedness of the reduced density operator ρ_R of a subsystem of a bipartite composite system.

Is $S_{\mathrm{BH}} = S_{\mathrm{vN}}(\rho_A)$?

Is the entanglement entropy of a black hole proportional to the surface area of its event horizon?

What does a black hole have to do with quantum entanglement?

- Black holes are described by general relativity (GR), which is a (*non-quantum!*) theory that describes types of spacetimes.

Ex: Schwarzschild spacetime describes Schwarzschild black holes.

- Quantum mechanics (in particular, quantum field theory QFT) describes matter, not spacetime.

- What happens if we place a quantum field in a black hole spacetime?

  - *As of yet, no entirely consistent theoretical description.*

On-Going Scandal in 21st Century Theoretical Physics: GR is inconsistent with QFT!

  - *One attempt: In a particular approach to reconciling GR and QFT called the AdS/CFT correspondence, there is a formula that computes entanglement entropy of a quantum field that is formally identical to black hole entropy!*

Many other attempts: e.g., string theory, causal set theory, loop quantum gravity, etc.

2. The AdS/CFT Correspondence...

Maldacena (1996)

- A "duality" map between quantities in two types of theories:

A d-dim theory of gravity in an asymptotically anti-de Sitter (AdS) spacetime

$\Leftrightarrow$

AdS/CFT duality map

A $(d-1)$-dim conformal field theory (CFT)

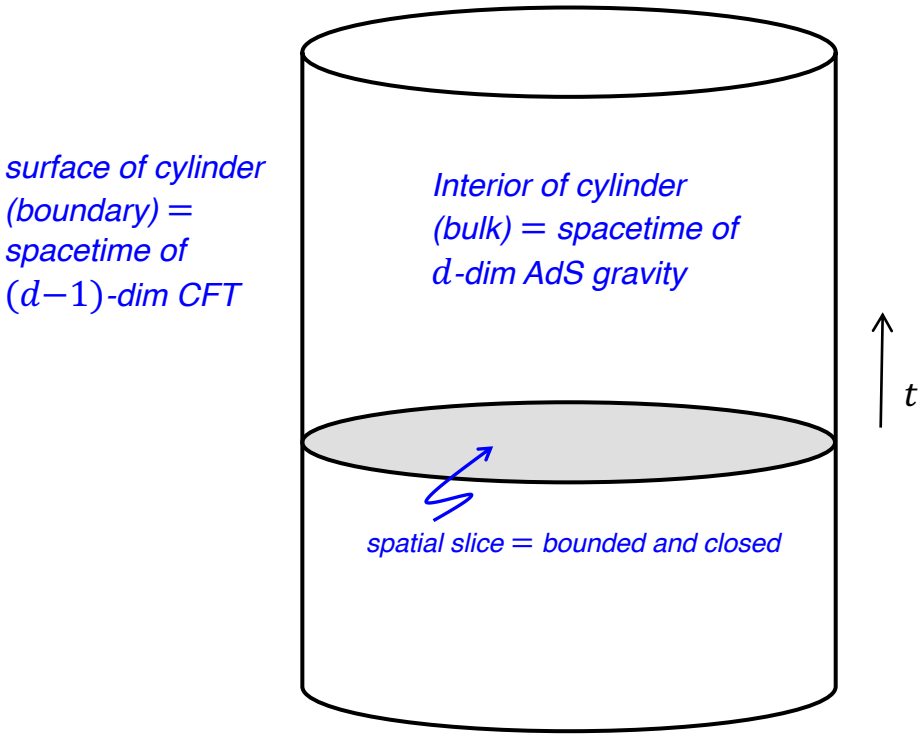
- *AdS spacetime = very simple spatially closed general relativistic spacetime*
- *"Asymptotically AdS" = looks like AdS spacetime at large distances*

very simple type of quantum field theory

- *Idea*: The CFT lives on the boundary of the "bulk" theory of gravity...

- *Suggests*: A way to compute values of quantities in a theory of gravity using quantities in a quantum field theory.

- *More provocatively*: Our universe (as described by GR) is encoded in a hologram (as described by a quantum theory in one less dimension).

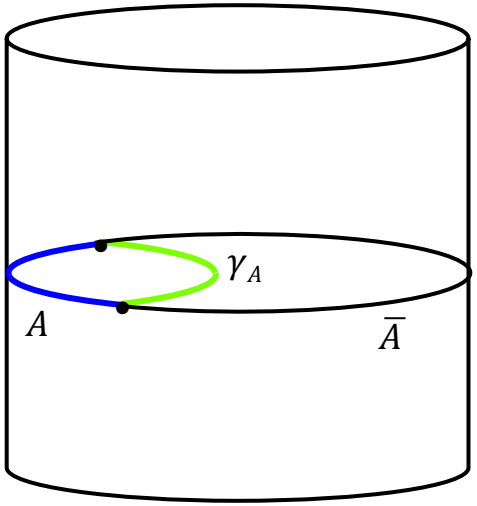


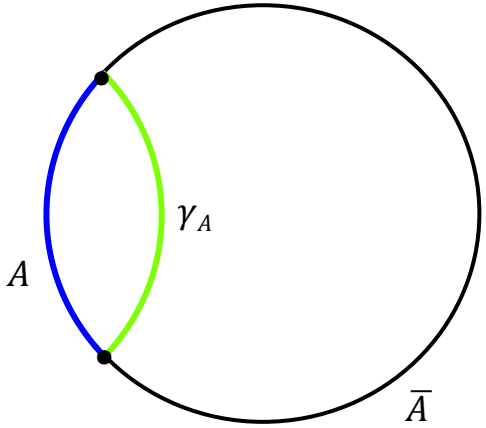
2. ...and the RT Formula

Ryu & Takayanagi (2006)

A formula that can be derived using the AdS/CFT map!

- *Set-up*:
  - Take a spatial slice of the AdS/CFT "cylinder".
  - Divide boundary of slice (a circle) into regions A and $\bar{A}$.
  - Connect endpoints of A with a minimal area surface γ_A through the bulk.
  - Consider a boundary system decomposable into two subsystems localized in A and $\bar{A}$.





Ryu-Takayanagi (RT) Formula

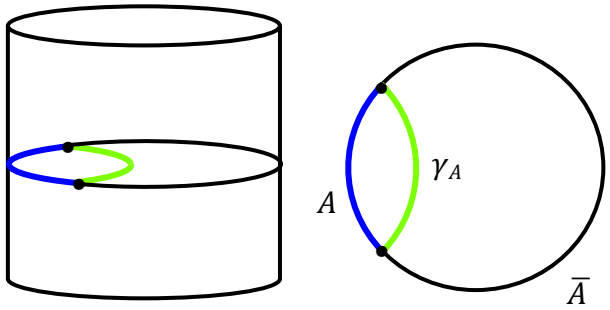
$$S_{\mathrm{vN}}(\rho_A) = \mathrm{Area}(\gamma_A)/4$$

The entanglement entropy of the boundary subsystem in A is given by the area of the minimal area bulk surface γ_A defined with respect to A.

ρ_A is the density operator of the boundary subsystem localized in A

Ryu-Takayanagi (RT) Formula

$S_{\mathrm{vN}}(\rho_A) = \mathrm{Area}(\gamma_A)/4$



- *Question #1*: *So what?*
  - So recall the definition of entanglement entropy:

$$S_{\mathrm{vN}}(\rho_A) = -\mathrm{Tr}(\rho_A \log \rho_A)$$

Trace of the product of a matrix and its logarithm: nasty!

In principle, it's much more easy to calculate the area of a surface!

- *Question #2*: *What does this have to do with black hole entropy?*

3. The RT Formula and Black Hole Entropy

Ryu-Takayanagi (RT) Formula

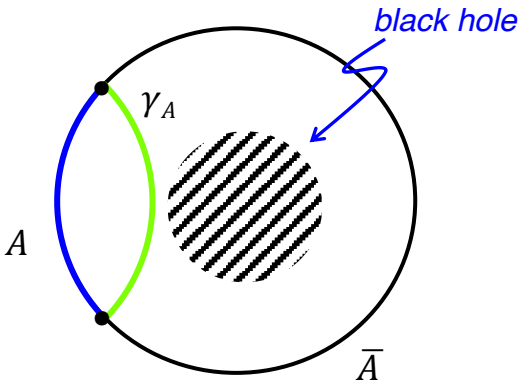
$S_{\mathrm{vN}}(\rho_A) = \mathrm{Area}(\gamma_A)/4$

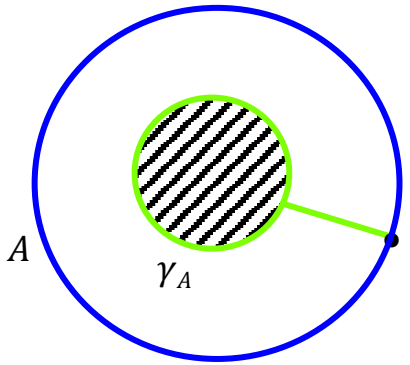
Black Hole Entropy

$S_{\mathrm{BH}} = \mathrm{Area}(\text{event horizon})/4$

Formally identical! *But: Not that surprising...*

- *One motivation for RT formula*:
  - Place a black hole in the AdS bulk.
  - Now take the limit in which A encompasses the entire boundary.
  - In this limit, γ_A becomes the event horizon!
  - *In other words*: Black hole entropy is a special case of the RT formula.





Suggests: Black hole entropy is the entanglement entropy of a subsystem far from the event horizon (localized in a boundary region A) with respect to a subsystem inside the event horizon.

Black Hole Entropy

$S_{\mathrm{BH}} = \mathrm{Area}(\text{event horizon})/4$

The entanglement entropy of a subsystem far from the event horizon with respect to a subsystem inside the event horizon?

- *Assumes the AdS-CFT correspondence*:
  - At present still just a conjecture.
  - The spacetime that accurately describes our universe is much more complex than an asymptotically AdS spacetime.
  - The quantum field theories that accurately describe the matter content of our universe are much more complex than a simple CFT.
- *But!*
  - AdS-CFT at least points in a possible direction of a quantum theory of gravity that reconciles GR and QFT.
  - The RT Formula, it turns out, can be used as a solution to the infamous *Black Hole Information Loss Paradox*!