06. Information and Maxwell's Demon

1. Dilemma for Information-Theoretic Exorcisms

Two Options:

(S) (*Sound*). The combination of object system and demon forms a canonical thermal system.

(P) (*Profound*). The combination of object system and demon does not form a canonical thermal system.

Dilemma:

- If (S), then the 2nd Law applies and no appeal to the notion of "information" is necessary.
- If (P), then one needs a new physical postulate to explain why the 2nd Law applies phrased in terms of information and entropy.

Concerning (P): "The issue is whether a valid principle concerning the entropy costs of information aquisition and processing can defeat demonic devices." (Earman & Norton 1999.)

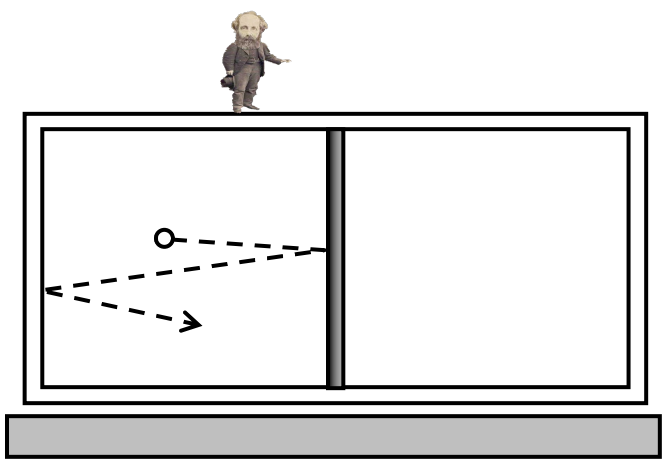
2. Two Approaches to Information-Theoretic Exorcisms.

Approach 1: Szilard's Principle

> *Gaining* information that allows us to discern between n equally likely states is associated with a minimum increase in entropy of $k \ln n$.

Leo Szilard
(1898-1964)

- *Recall*: In Szilard's (1929) one-molecule engine, to obtain information about which side the molecule is located requires an increase in entropy of $k \ln 2$.

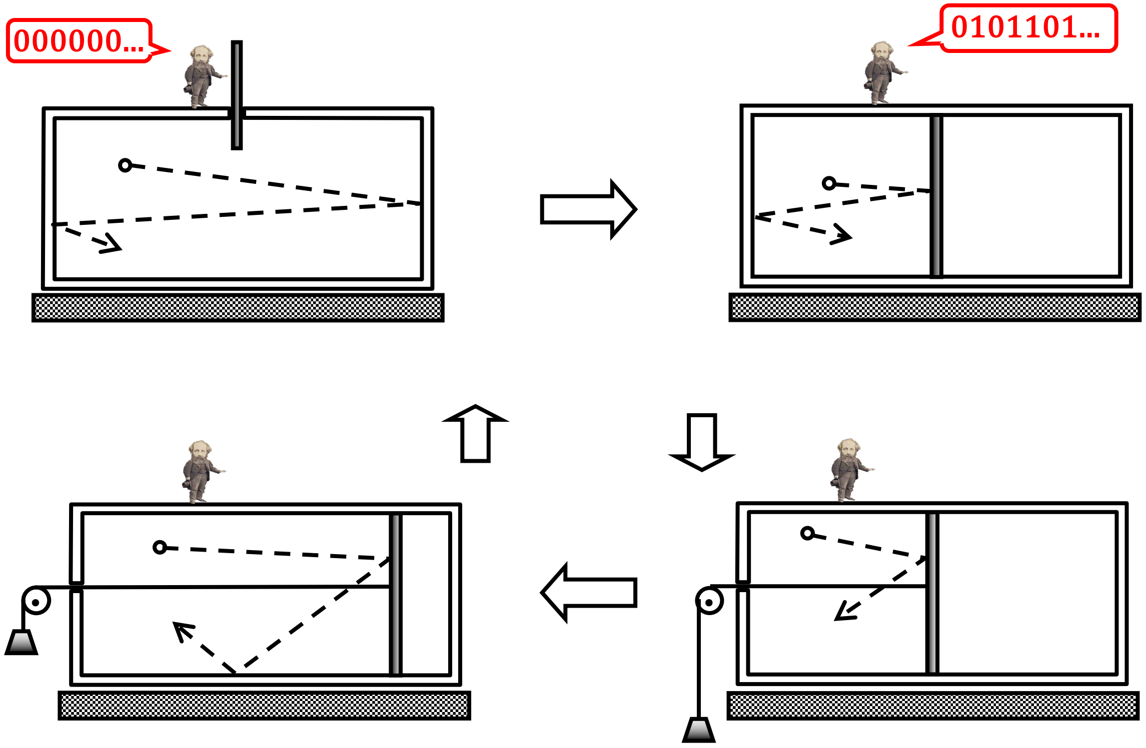


Approach 2: Landauer's Principle

> *Erasing* information that allows us to discern between n equally likely states is associated with a minimum increase in entropy of $k \ln n$.

Rolf Landauer
(1927-1999)

- The demon works in a *cycle*.
- At some point, it must aquire information.
- At some later point, it must erase this information in order to return to its initial state.



(a) Brillouin and "Negentropy"

Brillouin (1953) "The Negentropy Principle of Information"

- Consider a thermodynamical system in a macrostate Γ_D corresponding to W $(= G(D))$ equiprobable microstates (arrangements).

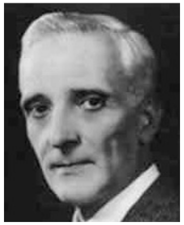
- *Then*: The Boltzmann entropy is given by

$$S_{\text{Boltz}}(\Gamma_D) = k\ln W + \text{const.}$$

$$S_{\text{Boltz}} = k\ln|\Gamma_D| = k\ln(G(D)\delta w^N)$$
$$= k\ln(G(D)) + Nk\ln(\delta w)$$
$$= k\ln(G(D)) + \text{const.}$$

Let the information I associated with a process that reduces the number of microstates from W_0 to W_1 be given by

$$I = k\ln W_0/W_1 = k\ln W_0 - k\ln W_1$$



Leon Brillouin
(1889-1969)

- *Motivation*: A *reduction* in the number of microstates corresponds to a positive value of I.

Shannon's intuition: A decrease in the probability of a macrostate (i.e., a decrease in the number of its microstates) should increase its information content!

- *Now note*: A *reduction* in the number of microstates corresponds to a *decrease* in S_{Boltz}:

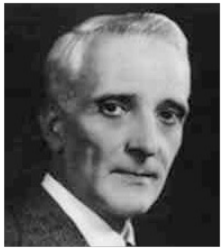
$$S_{\text{Boltz}}(\Gamma_1) - S_{\text{Boltz}}(\Gamma_0) = k\ln W_1 - k\ln W_0 < 0$$

Final macrostate with W_1 microstates

Initial macrostate with W_0 microstates

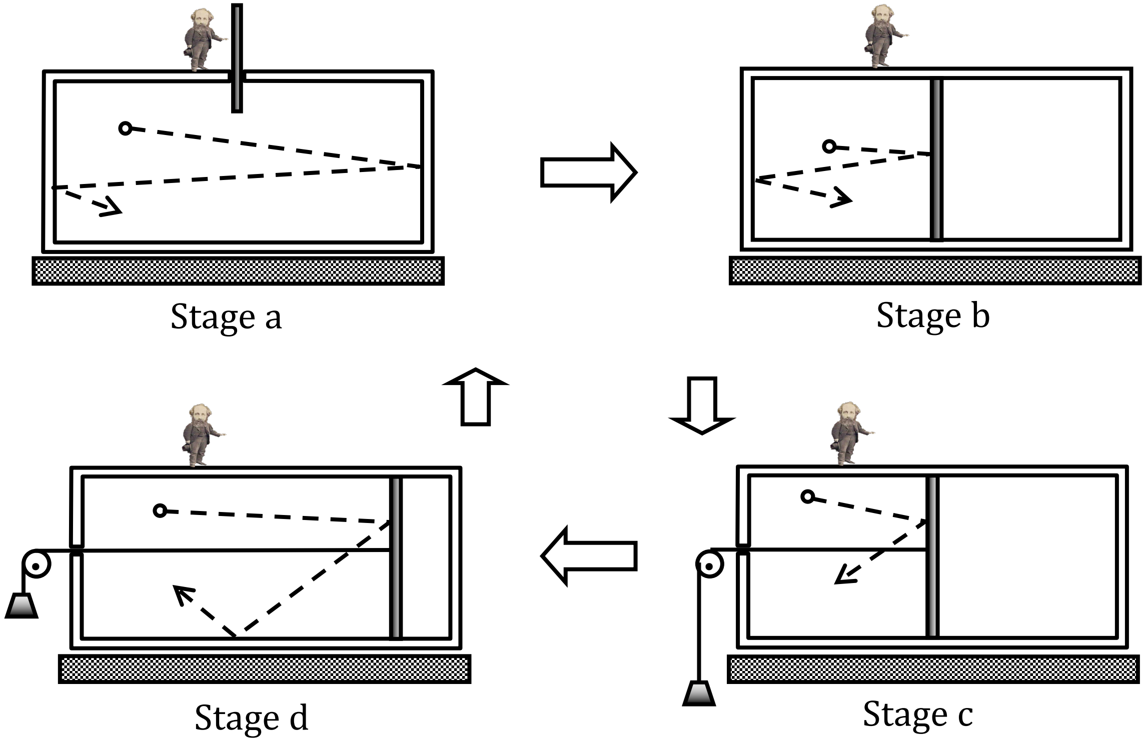
This is just $-I$

This transition is associated with a conversion of information I into "negentropy" (negative entropy)!



These remarks lead to an explanation of the problem of the Maxwell's Demon, which simply represents a device changing negentropy into information and back to negentropy...

- *But*: This takes option (S).
  - *So there's no need for references to "information" or "negentropy".*
  - *If demon and gas obey 2nd Law, then any increase in "negentropy" (decrease in entropy) associated with the demon will be compensated for by an increase in entropy somewhere else.*



Szilard's analysis:

- a→b: Molecule loses entropy $-S_m$.
- c→d: Molecule gains entropy $+S_m$; reservoir loses entropy $-S_h$.
- Overall decrease in entropy.

Szilard's solution (Szilard's Principle):

- a→b: Molecule loses entropy $-S_m$; Demon gains entropy $+S_h$ (info acquisition).
- c→d: Molecule gains entropy $+S_m$; reservoir loses entropy $-S_h$.
- No net gain or loss of entropy.

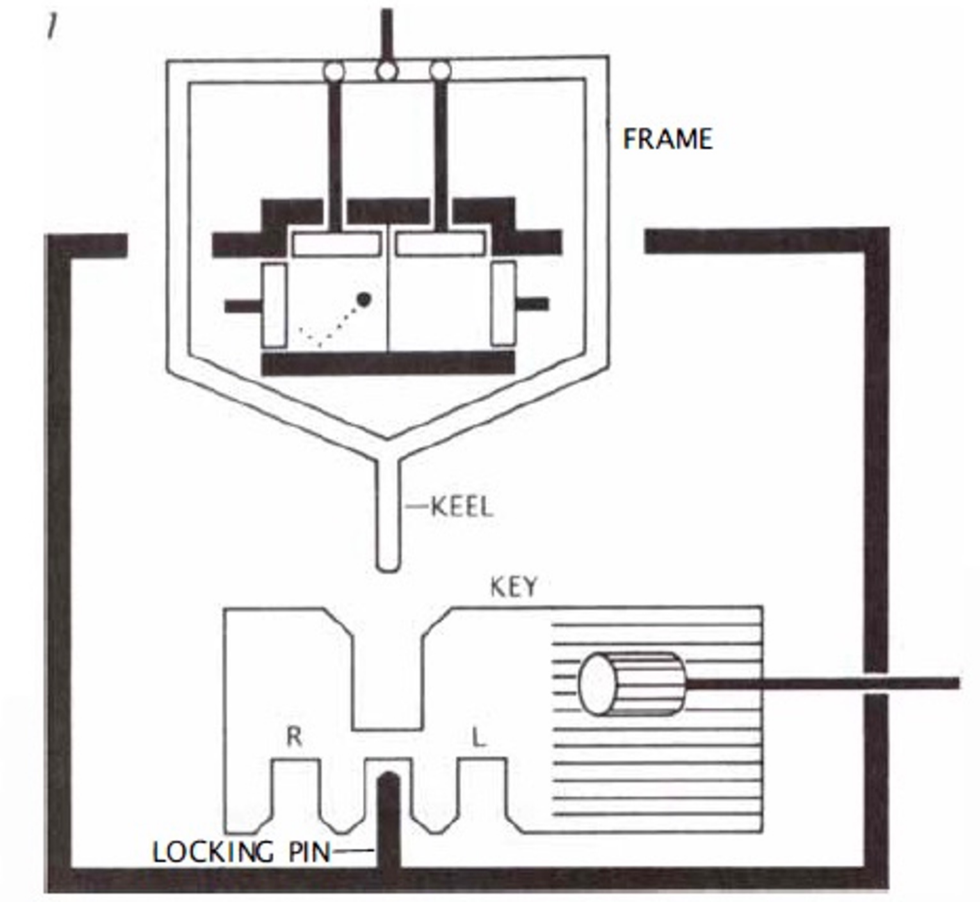
Brillioun's Interpretation:

- a→b: Molecule loses entropy $-S_m$; information ("negentropy") gained I is $+S_m$.
- c→d: Molecule gains entropy $+S_m$; reservoir loses entropy $-S_m$.
- No net gain or loss of entropy.
- "...a device changing negentropy into information (1→2) and back to negentropy (3→4)."

(b) Challenge to Szilard's Principle

Bennett, C. (1987) "Demons, Engines and the Second Law", *Scientific American 257 (5)*, 108.

Claim. Info aquisition can be achieved without entropy cost.



Charles Bennett
(1943-present)

"A slightly modified Szilard engine sits near the top of the apparatus (1) within a boat-shaped frame; a second pair of pistons has replaced part of the cylinder wall. Below the frame is a key, whose position on a locking pin indicates the state of the machine's memory."

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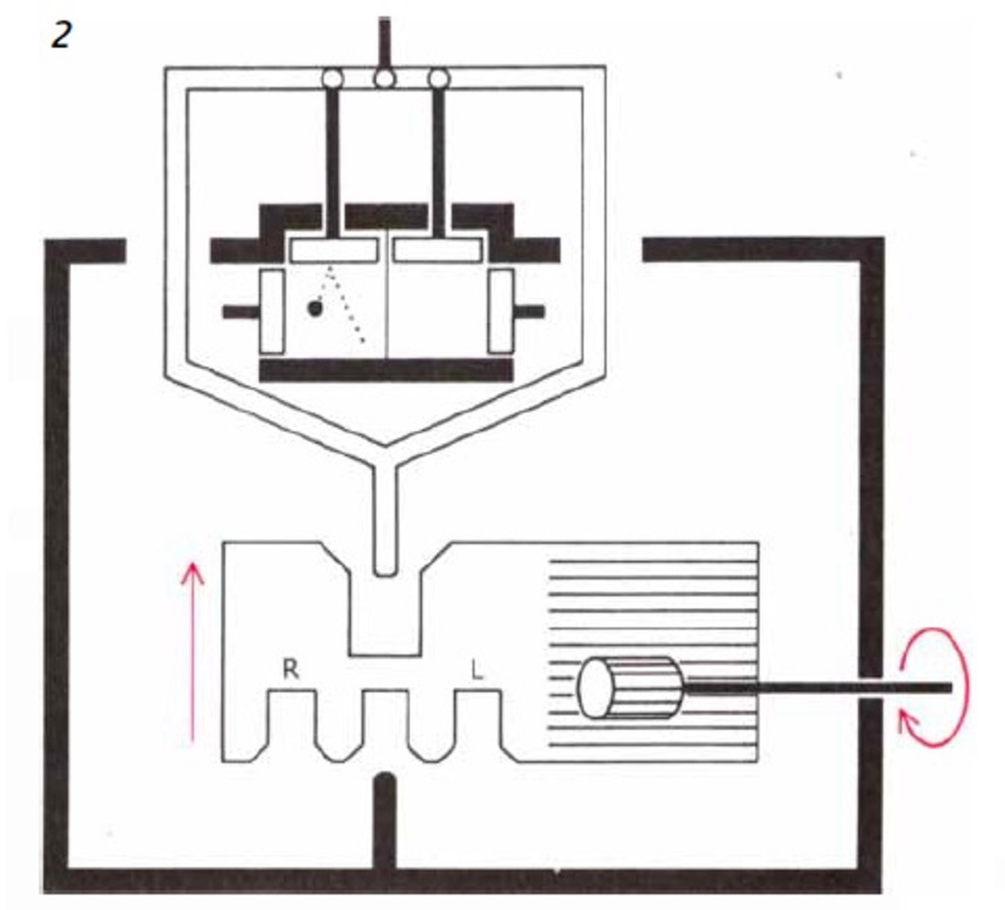

Charles Bennett
(1943-present)

"To begin the measurement (2) the key is moved up so that it disengages from the locking pin and engages a 'keel' at the bottom of the frame."

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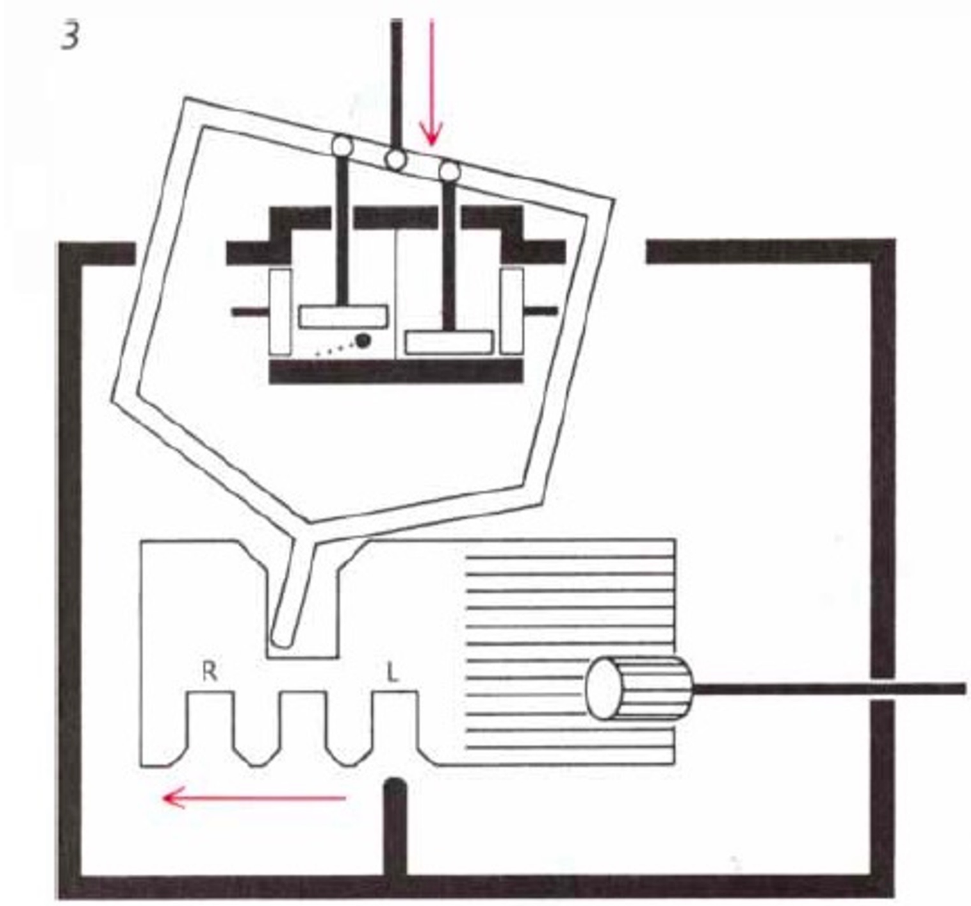

Charles Bennett
(1943-present)

"Then the frame is pressed down (3). The piston in the half of the cylinder containing no molecule is able to descend completely, but the piston in the other half cannot, because of the pressure of the molecule. As a result the frame tilts and the keel pushes the key to one side."

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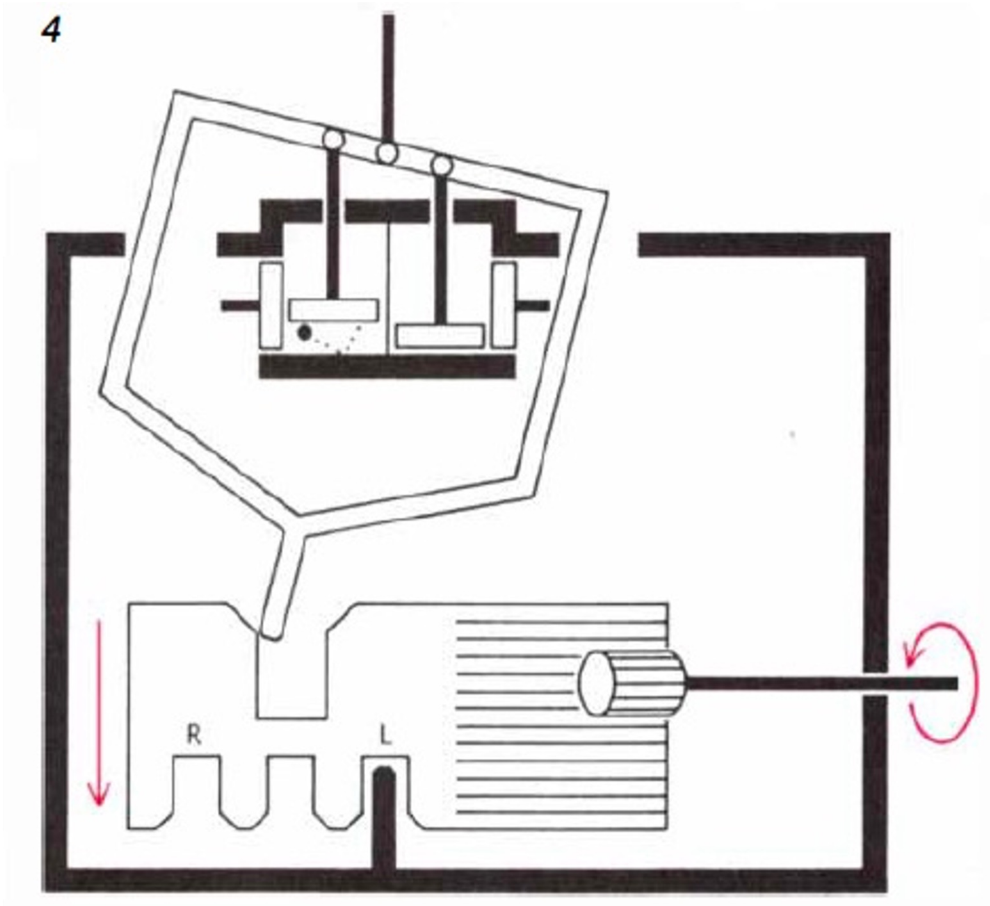

Charles Bennett
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"The key, in its new position, is moved down to engage the locking pin (4), and the frame, is allowed to move back up (5)..."

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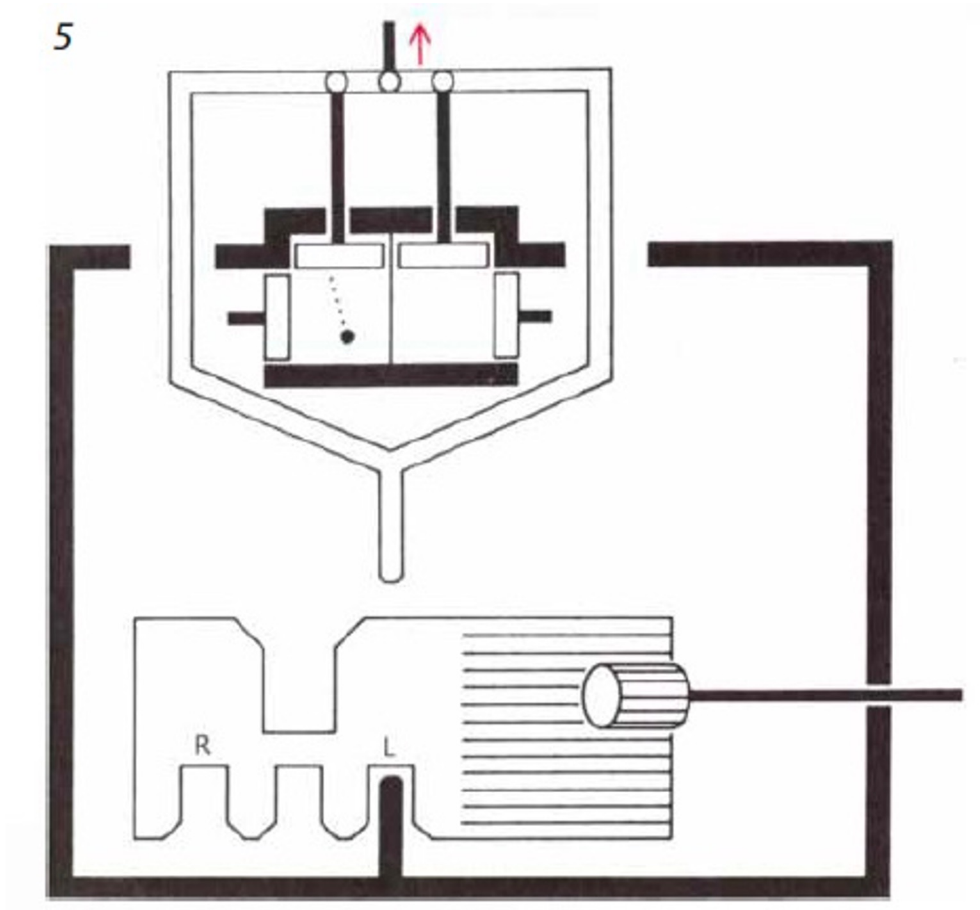
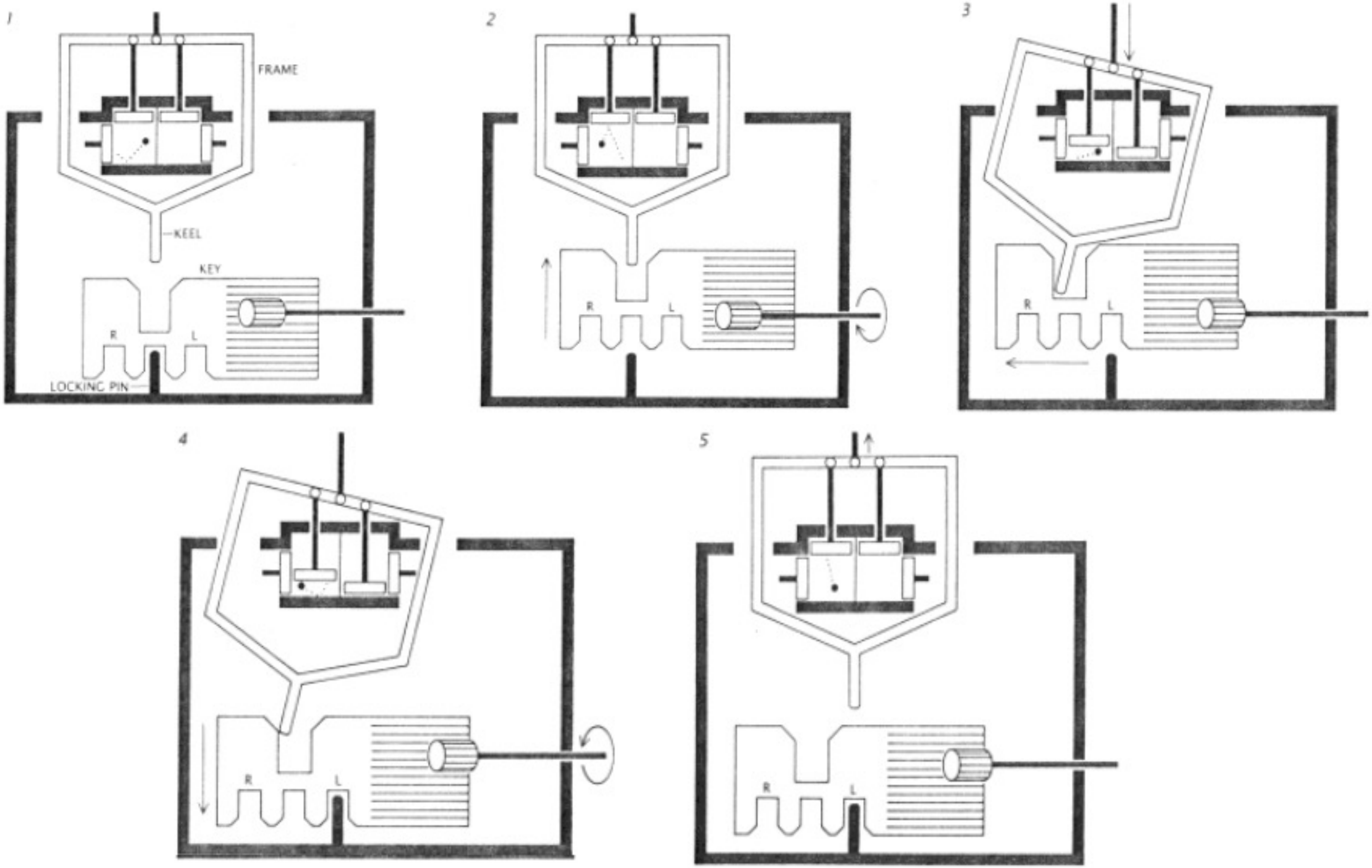


Charles Bennett
(1943-present)

"The key, in its new position, is moved down to engage the locking pin (4), and the frame, is allowed to move back up (5), undoing any work that was done in compressing the molecule when the frame was pressed down."



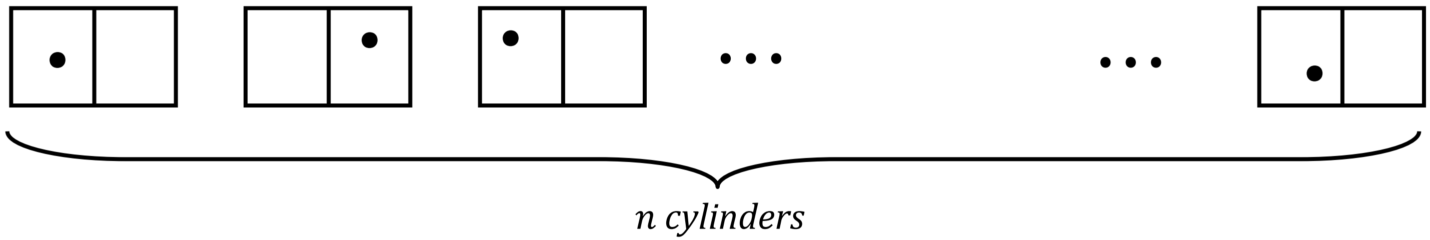
- Measurement without entropy cost?

  No: Any mechanical device will be subject to thermal fluctuations thus obliterating its measuring function (Earman and Norton 1999).

(c) Landauer's Principle

<u>*General Idea*</u>: *Logical states* of a computer must be represented by *physical states* of its hardware.

- <u>*Ex*</u>: An n-bit memory register as an array of n two-chambered cylinders, each filled with a one-molecule gas.



Let molecule in left correspond to "0"*; molecule in right correspond to* "1"*.*

- Each cylinder has 2 possible states.
  - *So entire register has* 2^n *possible states.*

- <u>*Now*</u>: Set register to zero (erase all bits).
  - <u>*Before erasure*</u>: Register can be in any of 2^n states.
  - <u>*After erasure*</u>: Register is in exactly one state.

Erasure involves compressing many logical states into one; just like a piston!

- *<u>So</u>*: Erasure = compression of many physical states (high entropy) into exactly one (low entropy).

> "Hence one cannot clear a memory register without generating heat and adding to the entropy of the environment. Clearing a memory is a thermodynamically irreverisble operation."

Charles Bennett

<u>What's the Moral for Maxwell's Demon?</u> (Earman & Norton 1999)

<u>Problem #1</u>

Not all physical processes admit descriptions in terms of information erasure (recall Smoluchowski's one-way valve).

<u>Problem #2</u>

Bennett claims Szilard's Principle *fails*, because we can *ignore* thermal fluctuations for measuring devices; while Landauer's Principle *succeeds*, because we *cannot ignore* thermal fluctuations for erasure devices (they are physical, thermal systems). Is this inconsistent?

Problem #3: Computerized demons don't need to erase information.

- Consider a 2-state memory device with states: "L" and "R".

Claim: A routine in which the system is found to be in state L, and then switched to state R, is not an erasure routine. (Bennett agrees.)

Why? It's logically reversible. It doesn't involve mapping many states to one.

Program for Szilard's One-Molecule Engine with No Erasure:

1. Begin in memory register state L.
2. If molecule is in left side, do nothing to register.
3. If molecule is in right side, switch to state R.
4. Check register:
   (i) If in state L, then do nothing. Commense expansion.
   (ii) If in state R, then commense expansion and reset register to state L.

- *Result*: No erasure of memory states needed to return to start of cycle.

- *So*: Landauer's Principle in particular, and information-theoretic analyses in general, provide no sound basis for the 2nd Law.

- *But*: What if we restrict attention to thermal systems that explicitly model computational processes?

> *In this particular context*: "The question at issue is at what stage of the information aquisition or information processing a *computerized* demon would fail as a perpetual motion machine, if we assume that the system is a canonical thermal system subject to the 2nd law." (Bub 2001*)

- *Relevent questions*:
  - Does *"computational measurement"* cost entropy?
  - Does *"computational memory erasure"* cost entropy?

* Bub, J. (2001) 'Maxwell's Demon and the Thermodynamics of Computation', *Studies in History & Philosophy of Modern Physics 32*, 569-579.

Computational Measurement

- *Claim*: No entropy cost for computational measurement.

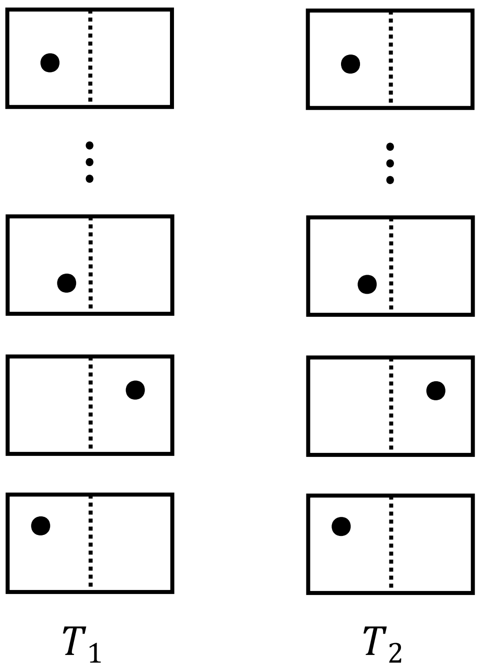
Why?

$$\begin{pmatrix}\textit{computational}\\ \textit{measurement}\end{pmatrix} = \begin{pmatrix}\textit{correlation between the state}\\ \textit{of a measured system and the}\\ \textit{state of the memory register of}\\ \textit{a measurement device.}\end{pmatrix} = \begin{pmatrix}\textit{copying}\\ \textit{operation}\end{pmatrix}$$

- *Now show*: *Copying operations cost no entropy.*

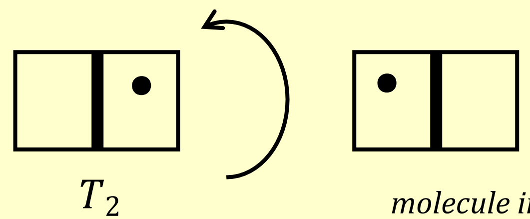
Two physical memory registers T_1, T_2 initially in same state.

- <u>*Task*</u>: Reset T_2 to zero state.



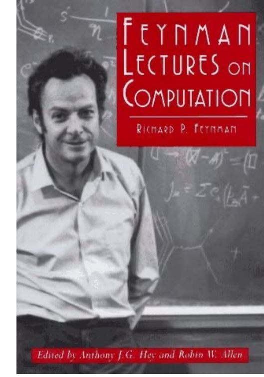
<u>To reset T_2 using T_1:</u>

1. If first box of T_1 is "0", do nothing to first box of T_2.
2. If first box of T_1 is "1", insert partition into first box of T_2 (trapping molecule on right) and then flip box:



molecule in left = "0"
molecule in right = "1"

3. Proceed to next box.



(1996) *Lectures on Computation*

Richard Feynman
(1918-1988)

- <u>*Claim*</u>: Reset procedure of T_2 using T_1 is *reversible*: no entropy cost.

  <u>*Why*</u>?

  - T_1 tells us the state T_2 is in, so resetting T_2 using T_1 does not involve a decrease in the number of its possible states; hence no decrease in entropy.
  - If we did not have T_1 (if T_2 was in an *unknown* state), then resetting T_2 to zero would involve a decrease in the number of its possible states; hence there would be a decrease in its entropy.

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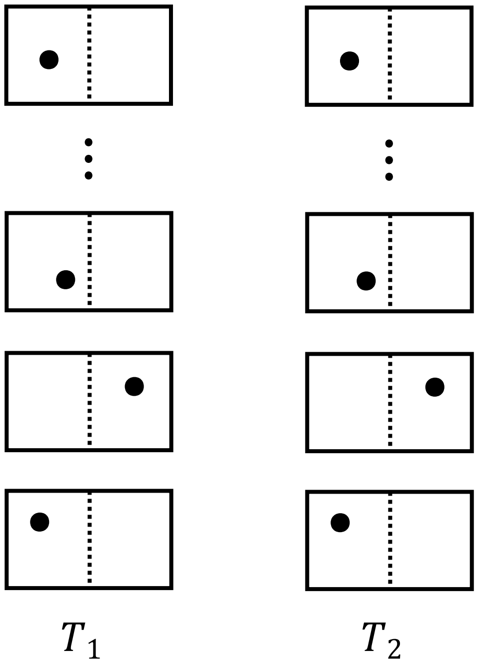

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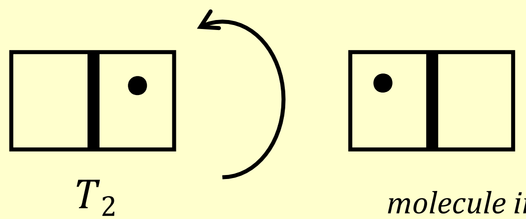

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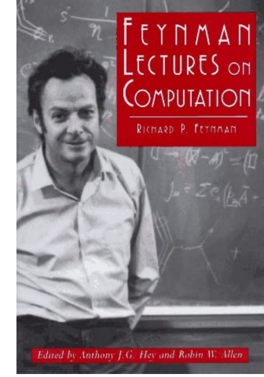
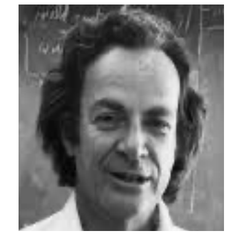


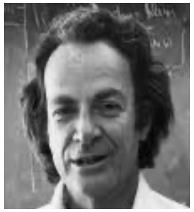
(1996) *Lectures on Computation*



Richard Feynman
(1918-1988)

- *Claim*: Reset procedure of T_2 using T_1 is *reversible*: no entropy cost.

> "...it might seem odd to be able to insert pistons and turn boxes without expending energy. In the real world, of course, you can't--but we are dealing with abstractions here and, as I have said, we are not interested in the kinetic energy or weight of the 'boxes'. Given our assumptions, it is possible to do so, although the downside is that we would have to take an eternity to do it!" (Feynman 1996, pg. 144.)



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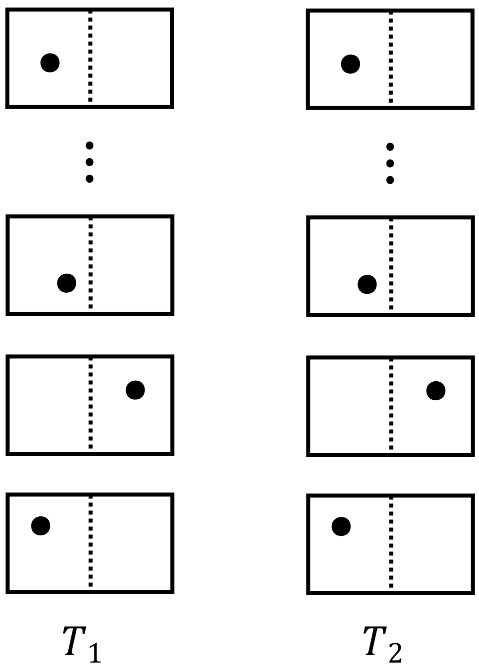

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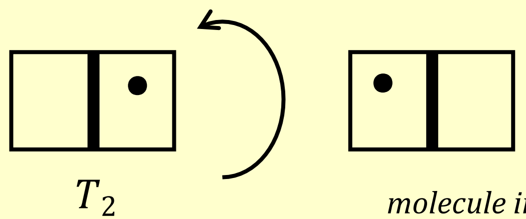

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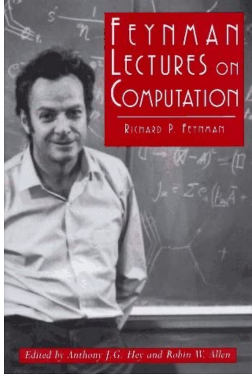

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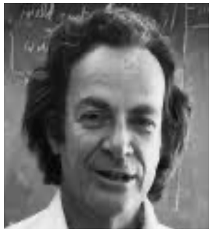

Richard Feynman (1918-1988)

- *Claim*: Reset procedure of T_2 using T_1 is *reversible*: no entropy cost.
- In reverse operation (no entropy cost), T_1 is *copied* onto initially blank T_2.
  - *And*: *This is a generic copying operation.*

Conclusion: Copying operations cost no entropy.

<u>*Computational Memory Erasure*</u>

- <u>*Recall*</u>: Earman & Norton's "computerized" demon example operates a Szilard One-Molecule Engine with no information erasure.

- <u>*Bub*</u>: This is not a computer, but rather an automatic mechanism.

"In most instances, a computer pushes information around in a manner that is independent of the exact data which are being handled, and is only a function of the physical circuit connections."

Rolf Landauer

"[Earman & Norton's] example only succeeds in evading the issue: without a state-independent reset operation, their demon is reduced to an automatically functioning switching device, and the question raised by Szilard is not addressed." (Bub 2001).

<u>Implication</u>: Any process that does not involve erasure is not a computational process.

Issues

1. *Is* measurement the "reverse" of a generic copying operation?

Two types of resetting operation

- T_1 is known.
- T_1 is unknown.

Bub/Feynman/Bennett/Landauer: This is copying, and reverse is measurement.

But: Isn't this measurement? A measurement involves aquiring information, in addition to, perhaps, copying it.

2. What exactly is a "computational" process?
   - *Just* a process that involves erasure?
   - Or a process that involves *both* measurement *and* erasure?
   - Or...?