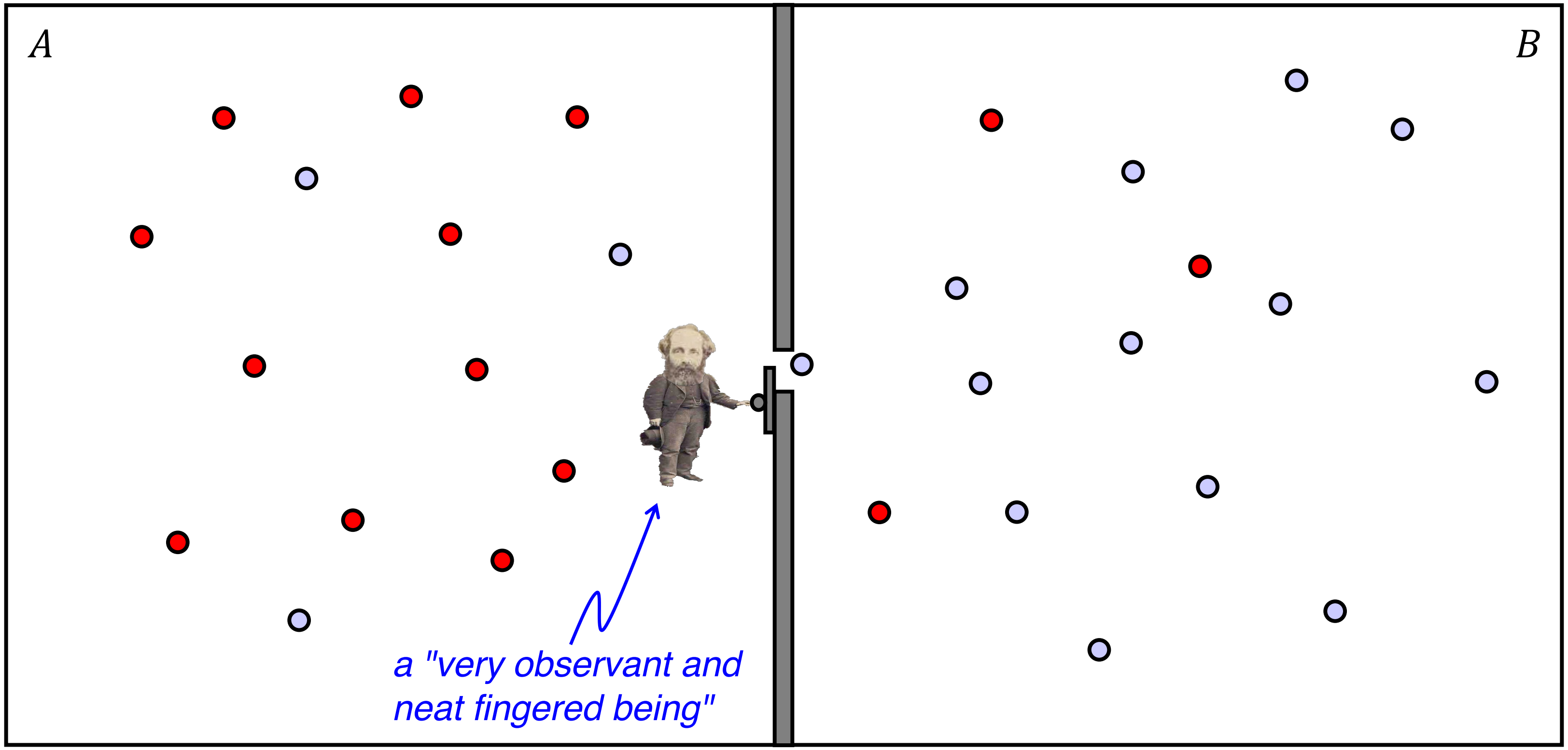
03. Maxwell's Demon

James Clerk Maxwell
(1831-1879)

1. Maxwell's (1867) letter to Tait

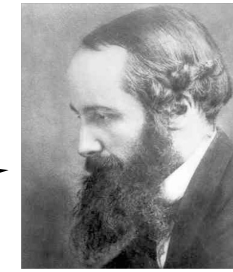
- Consider 2nd Law in the form: "If two things are in contact, the hotter cannot take heat from the colder without external agency."
- Maxwell's counterexample:



The neat-fingered being only lets hot molecules through to A and cold molecules through to B.

Upshot:

"The hot system has got hotter and the cold colder and yet no work has been done, only the intelligence of a very observant and neat fingered being has been employed."



Moral #1:

"The 2nd Law of Thermodynamics has only statistical certainty."

- *In other words*: It's very probable, but *not* completely certain, that "If two things are in contact, the hotter cannot take heat from the colder without external agency."

<u>*Moral* #2</u>: Attempts to derive the 2nd Law from (deterministic) mechanics will fail.

"...it is rare sport to see those learned Germans contending for the priority of the discovery that the 2nd law of [thermodynamics] is the Hamiltonische Princip... [It] soars along in a region unvexed by statistical considerations while the German Icari flap their waxen wings in *nephelo coccygia* amid those cloudy forms which the ignorance and finitude of human science have invested with the incommunicable attributes of the invisible Queen of heaven."

<u>*Moral #3*</u>: The distinction between dissipated energy (heat that we cannot make use of) and energy available for work depends on our state of knowledge.

[If we supposed]... our senses sharpened to such a degree that we could trace the motions of molecules as easily as we now trace those of large bodies... the distinction between work and heat would vanish...

[The truth of the 2nd Law depends]... on the fact that the bodies we deal with consist of millions of molecules and that we can never get hold of a single molecule."

- <u>*In other words*</u>: *If* we were neat-fingered beings capable of knowing the positions and velocities of molecules, then the 2nd Law would not apply.

2. Vexing Unanswered Questions

(1) *Why* is the 2nd Law only statistical?

- Are the probabilities really *epistemic*? Do they really reflect our lack of knowledge of the micro-physics (Moral #3)?
  - *But then why do the vast majority of observable macroscopic systems obey the 2nd Law?*

- Are the probabilities *ontic*? Do they reflect an instrinsic probabilitistic nature of micro-physical objects?

- Subsequent development of statistical mechanics and attempts to derive 2nd Law within it.

(2) Should the Demon itself be subject to thermodynamics?

- *Must be*: Otherwise why would we care if a non-thermodynamic demon was capable of violating the 2nd Law of thermodynamics?
- *But*: If so, then shouldn't we "naturalize" the Demon?

  - Perhaps a comprehesive thermodynamical analysis of Demon-plus-system will indicate that the 2nd Law is not violated.
  - Subsequent 20th-century history of the Demon:
    - *Fluctuation phenomena as naturalized demons*.
    - *Information-theoretic analyses of entropy*.

(3) Is Demonology Necessary?

- To investigate the conceptual significance of the 2nd Law, look to securing the foundations of statistical mechanics, as opposed to demon-bashing (Earman & Norton 1998).

3. Naturalized Demons

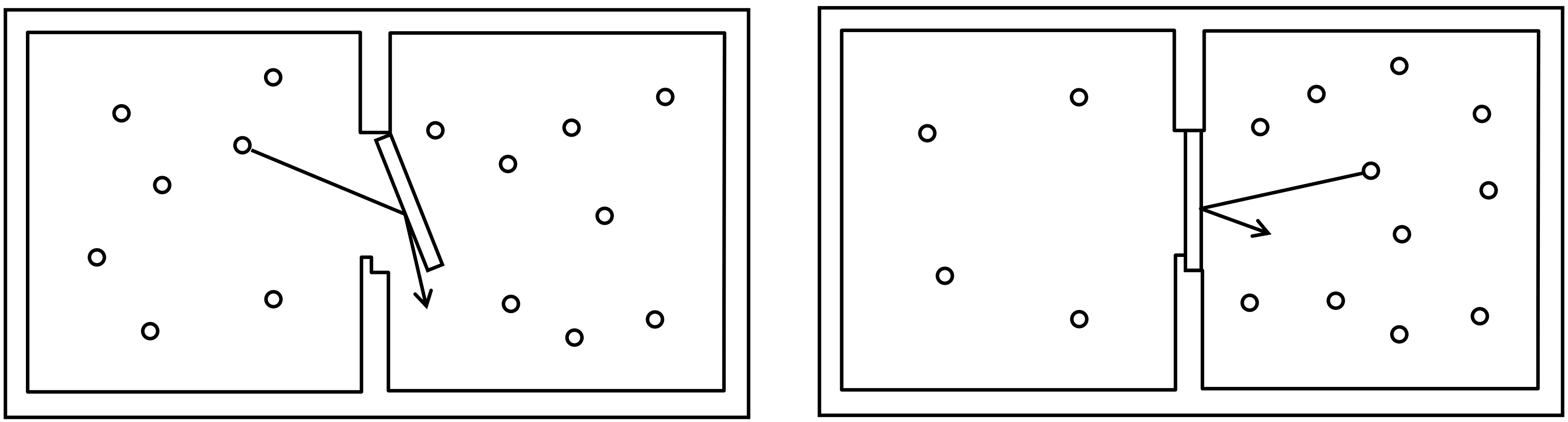
- Early 20th century thermal fluctuation phenomena:
  - *Brownian motion*.
  - *Density fluctuations in fluids near critical states*.

Key characteristic: *Completely random processes!*

Marian Smoluchowski (1872-1917)

- *Idea*: Exploit such phenomena to construct devices that violate 2nd Law.

Smoluchowski's (1914) trapdoor device



- Gas in separate chambers initially at equal pressures and temperatures.
- Spring-loaded trapdoor allows *randomly fluctuating* molecules to pass from one side to the other, but not *vice-versa*.
- *Expected Result*: Build-up of pressure on one side that can be exploited to perform work. Violation of 2nd Law!

Questions:

1. Is this an example of a decrease in entropy of a thermally-isolated system?
   - *Yes!*
2. Can this decrease in entropy be used to perform work?
   - *No! Spring must be sufficiently weak, and trapdoor sufficiently light*.
   - *But then trapdoor itself will be subject to thermal fluctuations that will prevent its intended operation*.

- Smoluchowski's response to (1):
  - Weaken the 2nd Law: *In the long run, on average, a thermally isolated system's entropy will increase.*

- *New (old) question*: What if the trapdoor is replaced with an intelligent being who knows when to open/close it?

Szilard on Entropy and Information

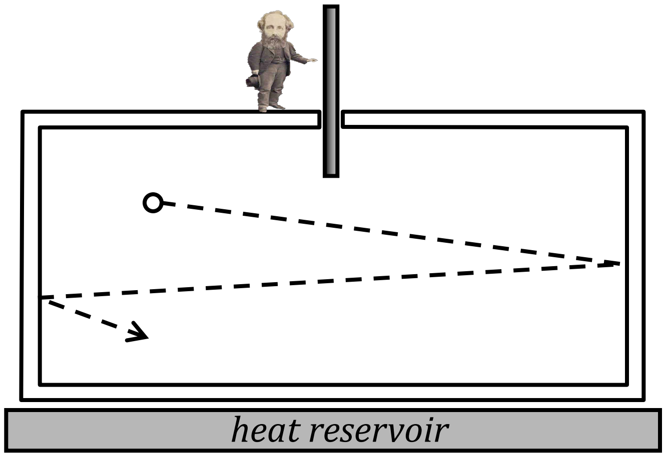
(1929) "On the Decrease of Entropy in a Thermodynamical System by the Intervention of Intelligent Beings"

Claim. Any device that employs fluctuations in an attempt to violate the 2nd Law will fail since there is an inevitable hidden entropy cost in the aquisition of information needed to run the device.

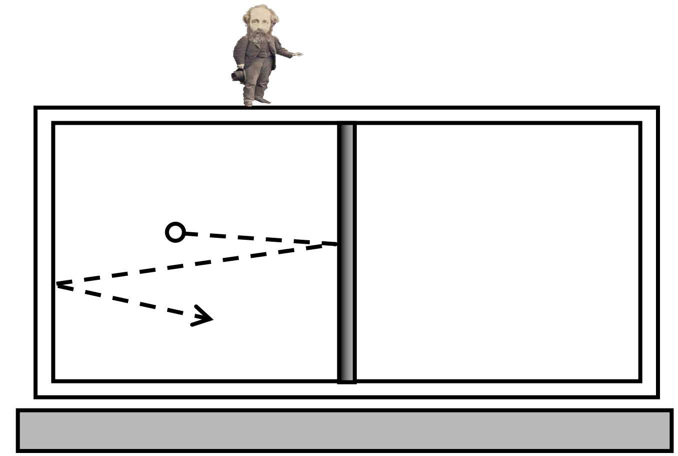
"...measurements themselves are necessarily accompanied by a production of entropy."

Leo Szilard
(1898-1964)

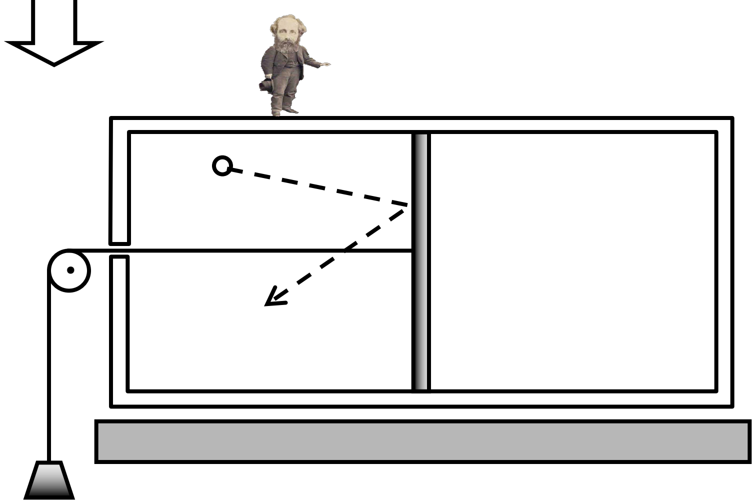
Szilard's One-Molecule Engine



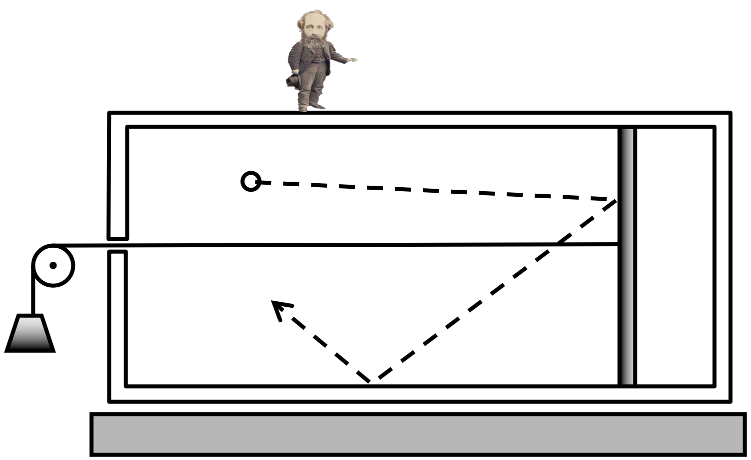
(a) Single molecule exhibiting thermal fluctuations. System at *const. temp*.



(b) Being inserts partition/piston *and determines which side molecule is on*.



(c) Being attaches weight to side with molecule. Gas expands reversibly and isothermally by absorbing heat from reservoir. Work performed on weight.



(d) Weight is detached. Partition/piston removed. Cycle returns to (a).

Result: ***Violation of 2nd Law!*** Heat converted to work with no exhaust.

- Stage (c): $\Delta U = 0 = -W + Q$ ← gas absorbs heat (+) and performs work (-)

So:

$$Q = W = \int_{V_i}^{V_f} P dV$$ ← work done by gas

$$= \int_{V_i}^{V_f} \frac{kT}{V} dV$$ ← for an ideal gas

assume $V_f = 2V_i$

$$= kT \ln \frac{V_f}{V_i} = kT \ln 2$$ ← heat absorbed by gas from reservoir

- So: Change in TD entropy in reservoir is:

heat emitted by reservior

$$\Delta S_{\text{TD}} = \int_{\sigma_i}^{\sigma_f} \frac{\delta Q_R}{T} = \frac{-kT\ln 2}{T} - 0 = -k\ln 2$$ ← A decrease in entropy!

Szilard's Solution: There must be an entropy increase of $k\ln 2$ in the being which balances the entropy decrease in the reservoir.

This entropy increase is associated with measurement

- *Assume*: Only two possible measurement outcomes (simplest case).
- Let $\bar{S}_1, \bar{S}_2$ be the entropies associated with outcomes 1 and 2, respectively.

Claim. A lower bound on $\bar{S}_1, \bar{S}_2$ is given by:

$$e^{-\bar{S}_1/k} + e^{-\bar{S}_2/k} \leq 1$$

"...*if* the amount of entropy produced by the 'measurement' is to compensate the entropy decrease..., the relation must always hold good."

Why?

- Let w_1, w_2 be the probabilities of getting outcomes 1 and 2, respectively.

- Then (it turns out), lower bounds for $\bar{S}_1$ and $\bar{S}_2$ are given by:

$$\bar{S}_1 \geq -k\ln w_1 \qquad \bar{S}_2 \geq -k\ln w_2$$

or

$$w_1 \geq e^{-\bar{S}_1/k} \qquad w_2 \geq e^{-\bar{S}_2/k}$$

Boltzmann entropy
$S_{\text{Boltz}} = -k\sum_i w_i \ln w_i$

- *And*: The claim then follows from $w_1 + w_2 = 1$.

Def. The average entropy cost of measurement per cycle is

$$\bar{S} = w_1\bar{S}_1 + w_2\bar{S}_2$$

- Now show that for any values of $\bar{S}_1, \bar{S}_2$ that satisfy lower-bound constraint, the resulting value for $\bar{S}$ is no less than the entropy decrease that violates the 2nd Law.

Ex: Szilard choses $\bar{S}_1 = \bar{S}_2 = k\ln 2$

- *These satisfy lower-bound constraint*:

$$e^{-\bar{S}_1/k} + e^{-\bar{S}_2/k} = 2e^{-\ln 2} \leq 1$$

- *And*: $\bar{S} = w_1\bar{S}_1 + w_2\bar{S}_2 = k\ln 2$

- *Thus*: *On average*, the entropy increase due to measurement is no less than the entropy decrease from the conversion of heat to work.

The 2nd Law is saved!