

14. Spacetime as a Quantum Error Correction Code?

1. AdS/CFT
2. Entanglement Wedge Reconstruction
3. QEC Interpretation

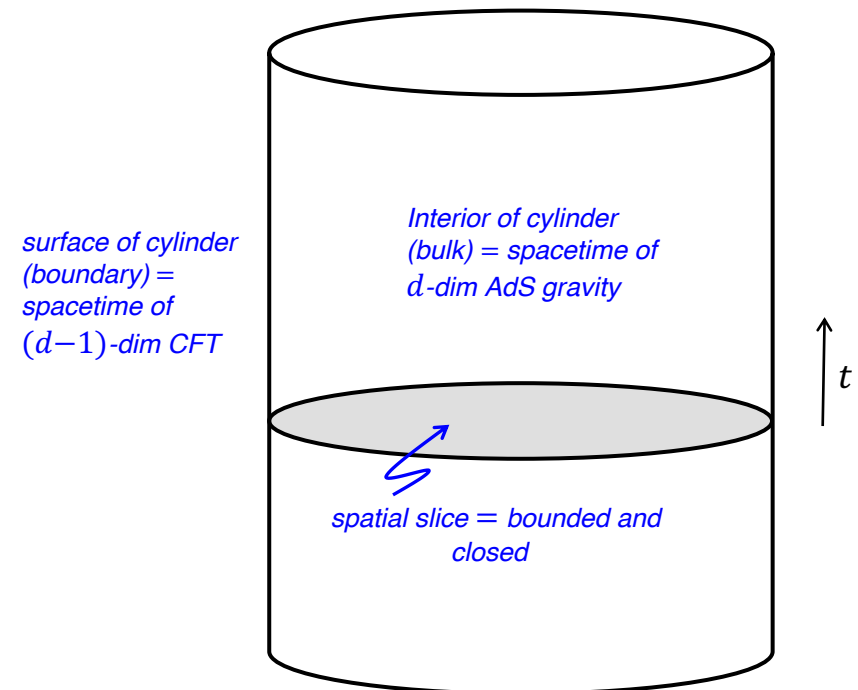
1. The AdS/CFT Correspondence *↔ An approach to quantum gravity*

- A "duality" map between quantities in two types of theories:



- Holographic view:

- Our universe is described by the bulk theory, an as yet not completely known quantum theory of gravity (QG).
- So our universe is encoded in a theory of one less dimension, like a hologram.
- And that theory (a CFT) is much more simple than a theory of QG.



14. Spacetime as a Quantum Error Correction Code?

1. AdS/CFT
2. Entanglement Wedge Reconstruction
3. QEC Interpretation

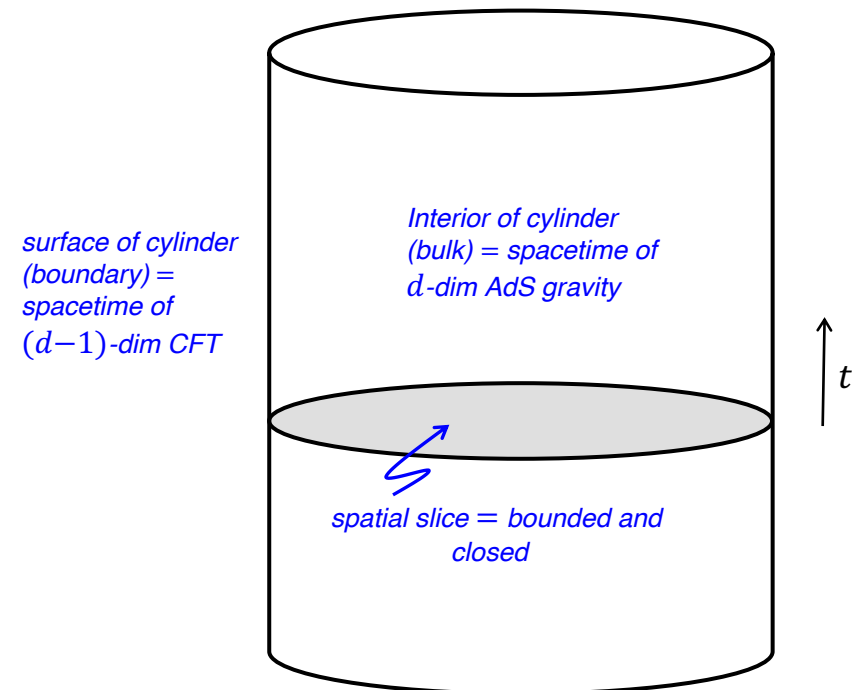
1. The AdS/CFT Correspondence *↔ An approach to quantum gravity*

- A "duality" map between quantities in two types of theories:



- *Redundant aspect*: boundary degrees of freedom in $(d-1)$ -dim are mapped to bulk degrees of freedom in d -dim.
- Suggests a quantum error correction code (QEC): A way to protect quantum information *redundantly* against degradation.

Fodder for mystery-mongering...

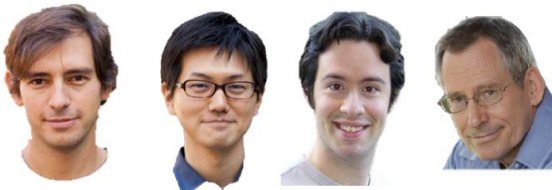


Is spacetime a quantum error-correcting code?



Fernando Pastawski, Beni Yoshida,
Daniel Harlow, John Preskill
= *HaPPY*

J. of High Energy Physics 06 (2015) 149



John Preskill
STOC in Montreal
20 June 2017

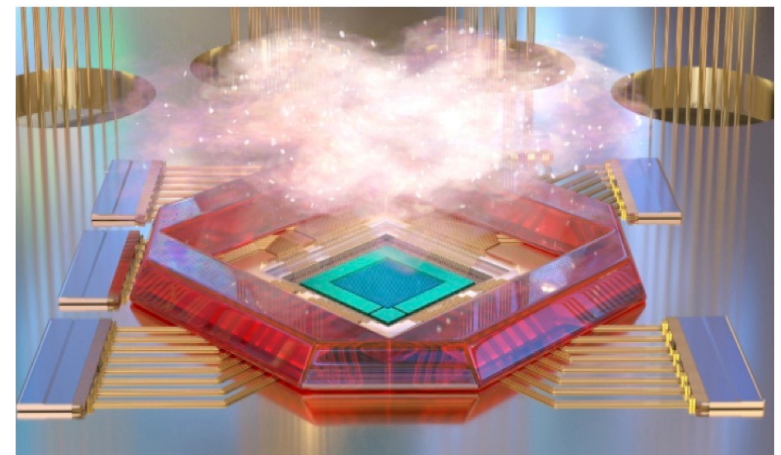


How Space and Time Could Be a Quantum Error-Correcting Code

By NATALIE WOLCHOVER

January 3, 2019

The same codes needed to thwart errors in quantum computers may also give the fabric of space-time its intrinsic robustness.



Towards De-Mystification



↖ "Entanglement Wedge Reconstruction"

↖ Like the 3-qutrit erasure protection code!

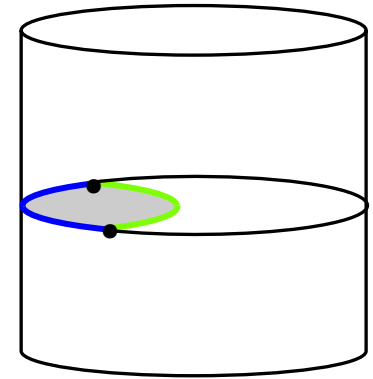
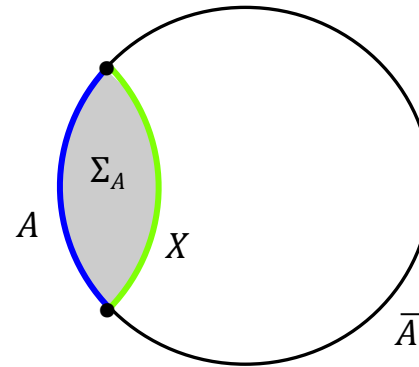
↖ But what does this have to do with spacetime?

- One proposed form of the AdS/CFT map relates boundary quantities in a boundary region A to bulk quantities in a bulk region called the "*entanglement wedge of A* ".
- And: One way to make this "EWR" map precise is equivalent to a quantum erasure protection map!
- And: This suggests a *QEC interpretation* of AdS/CFT, according to which a certain collection of bulk states is dual to a subspace of boundary states that forms the codespace for a quantum erasure protection code.

2. Entanglement Wedge Reconstruction

- Set-up:

- Take a spatial slice of the AdS/CFT "cylinder".
- Divide the boundary of the slice into regions A and $\bar{A}$.
- Connect the boundary of A with a minimal area surface X through the bulk.



Def. 1 (*Homology surface*). Let A be a region of a boundary spatial slice of an asymptotically AdS spacetime, and let X be a minimal area bulk surface with the same boundary as A . The **homology surface Σ_A of A** is defined as the bulk surface bounded by $A \cup X$.

The homology surface of A defines the entanglement wedge of A ...

Def. 2 (Entanglement wedge). Let A be a subregion of a boundary spatial slice of an asymptotically AdS spacetime, and let Σ_A be its homology surface. The **entanglement wedge of A** is defined by

$$W[A] \equiv D_{\text{bulk}}[\Sigma_A]$$

where $D_{\text{bulk}}[\Sigma_A]$ is the *bulk domain of dependence* of Σ_A .

↖ All bulk points that can be connected to Σ_A by a causal curve.

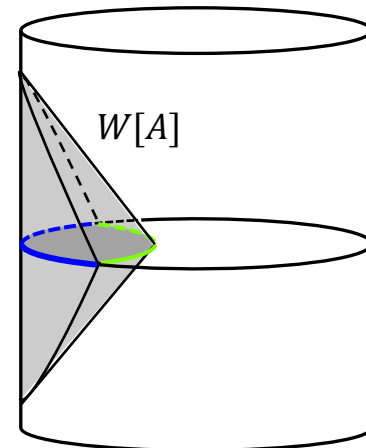
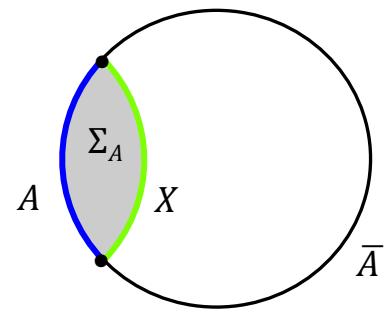
Claim 1 (Entanglement Wedge Reconstruction).

Any bulk observable located in $W[A]$ can be represented by a boundary observable located in A .

EWR_{precise}

Suppose $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$ has a subspace $\mathcal{H}_C = \mathcal{H}_{W[A]} \otimes \mathcal{H}_{W[\bar{A}]}$ that encodes bulk degrees of freedom located in $W[A]$ and $W[\bar{A}]$. Then for any operator $\mathcal{O}$ on $\mathcal{H}_{W[A]}$, there is an operator $\mathcal{O}_A$ on $\mathcal{H}_A$ such that, for any $|\tilde{\psi}\rangle \in \mathcal{H}_C$,

$$\mathcal{O}_A |\tilde{\psi}\rangle = \mathcal{O} |\tilde{\psi}\rangle \quad \text{and} \quad \mathcal{O}_A^\dagger |\tilde{\psi}\rangle = \mathcal{O}^\dagger |\tilde{\psi}\rangle$$



↖ One way to make EWR precise...

Claim 2. EWR_{precise} is equivalent to the existence of a quantum erasure protection map.

More precisely: Let $\mathcal{H}^{(n)} = \mathcal{H}_A^{(m)} \otimes \mathcal{H}_{\bar{A}}^{(n-m)}$ be an n -qudit Hilbert space and let $\mathcal{H}_C = \mathcal{H}_a \otimes \mathcal{H}_{\bar{a}}$ be a subspace of $\mathcal{H}^{(n)}$ with basis $|\tilde{i}\rangle$. Then the following are equivalent:

- (i) For any operator $\mathcal{O}$ on $\mathcal{H}_a$, there is an operator $\mathcal{O}_A$ on $\mathcal{H}_A$ such that, for any $|\tilde{\psi}\rangle \in \mathcal{H}_C$,

$$\mathcal{O}_A |\tilde{\psi}\rangle = \mathcal{O} |\tilde{\psi}\rangle \text{ and } \mathcal{O}_A^\dagger |\tilde{\psi}\rangle = \mathcal{O}^\dagger |\tilde{\psi}\rangle \quad \leftarrow EWR_{\text{precise}}!$$

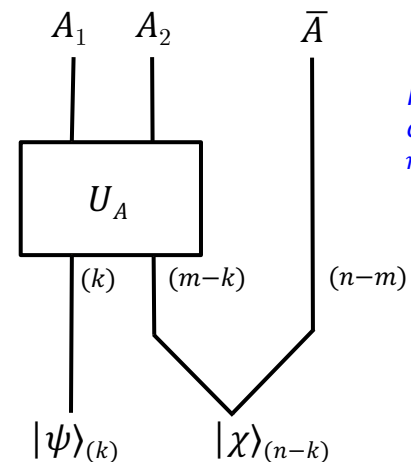
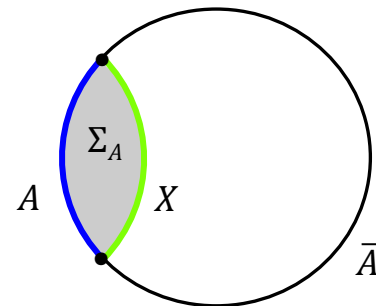
- (ii) Let $\mathcal{H}_A^{(m)} = \mathcal{H}_{A_1}^{(k)} \otimes \mathcal{H}_{A_2}^{(m-k)}$. Then there exists a transformation U_A on $\mathcal{H}_A$ and a state $|\chi\rangle_{A_2 \bar{A}_2} \in \mathcal{H}_{A_2} \otimes \mathcal{H}_{\bar{A}_2}$ such that

$$|\tilde{i}\rangle = (U_A \otimes I_{\bar{A}}) (|i\rangle_{A_1} |\chi\rangle_{A_2 \bar{A}_2}) \quad \leftarrow \text{QEC map!}$$

n -qudit basis for codeword

k -qudit basis for logical qudit

$(n-k)$ -qudit state that adds redundant qudits to the logical qudit



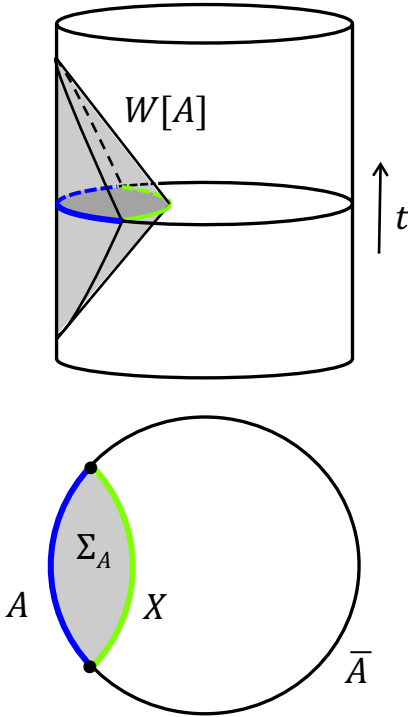
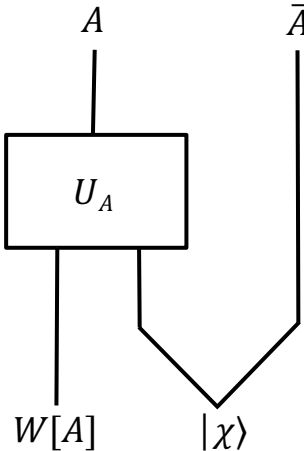
For 3-qudit code, $n = 3$, $m = 2$, $k = 1$.

- Claim (ii) says: Take a logical k -qudit state $|\psi\rangle$, attach an ancillary $(n-k)$ -qudit state $|\chi\rangle$, and act on result with U_A . Result is an n -qudit codeword $|\tilde{\psi}\rangle$ that protects $|\psi\rangle$ against the erasure of $(n-m)$ qudits.
- Claim (i) says: If $a = W[A]$, then the codespace $\mathcal{H}_C$ is the Hilbert space of boundary systems dual to bulk systems in $W[A]$ and $W[\bar{A}]$.

3. The QEC Interpretation of AdS/CFT

QEC	AdS/CFT
Physical qudit Hilbert space: $\mathcal{H}^{(n)} = \mathcal{H}_A^{(m)} \otimes \mathcal{H}_{\bar{A}}^{(n-m)}$	CFT Hilbert space for boundary of spatial slice: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$
Code subspace $\mathcal{H}_C \subset \mathcal{H}^{(n)}$ (<i>encoded logical qudits</i>)	Subspace $\mathcal{H}_C \subset \mathcal{H}$ of boundary states that are dual to bulk states in $W[A]$ and $W[\bar{A}]$.
<u>QEC map</u> : Basis states in $\mathcal{H}_C$ are related to basis states in $\mathcal{H}^{(n)}$ by: $ \tilde{i}\rangle = (U_A \otimes I_{\bar{A}})(i\rangle_{A_1} \chi\rangle_{A_2 \bar{A}_2})$	<u>EWR</u> : Bulk operators in $W[A]$ are related to boundary operators in A by: $\mathcal{O}_A \tilde{\psi}\rangle = \mathcal{O} \tilde{\psi}\rangle, \mathcal{O}_A^\dagger \tilde{\psi}\rangle = \mathcal{O}^\dagger \tilde{\psi}\rangle$

- The AdS/CFT correspondence as a quantum erasure protection code:
 - In particular: Bulk qudits in $W[A]$ protect information encoded in boundary qudits in $A \cup \bar{A}$ against the erasure of qudits in $\bar{A}$.
 - More generally (?): The bulk theory protects boundary information against erasure.

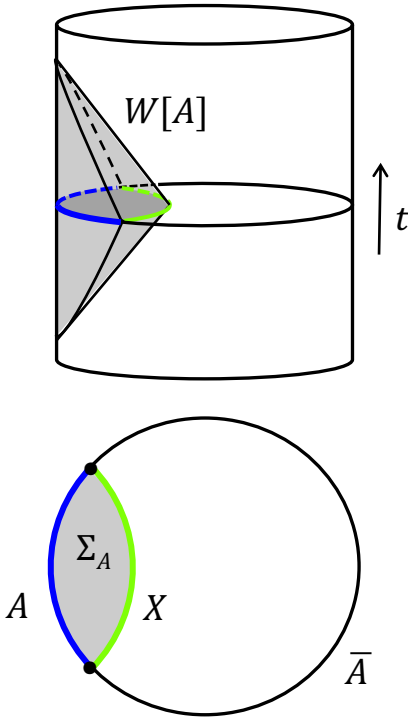
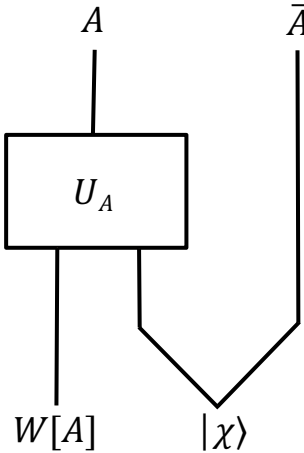


3. The QEC Interpretation of AdS/CFT

QEC	AdS/CFT
Physical qudit Hilbert space: $\mathcal{H}^{(n)} = \mathcal{H}_A^{(m)} \otimes \mathcal{H}_{\bar{A}}^{(n-m)}$	CFT Hilbert space for boundary of spatial slice: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$
Code subspace $\mathcal{H}_C \subset \mathcal{H}^{(n)}$ <i>(encoded logical qudits)</i>	Subspace $\mathcal{H}_C \subset \mathcal{H}$ of boundary states that are dual to bulk states in $W[A]$ and $W[\bar{A}]$.
<u>QEC map</u> : Basis states in $\mathcal{H}_C$ are related to basis states in $\mathcal{H}^{(n)}$ by: $ \tilde{i}\rangle = (U_A \otimes I_{\bar{A}})(i\rangle_{A_1} \chi\rangle_{A_2 \bar{A}_2})$	<u>EWR</u> : Bulk operators in $W[A]$ are related to boundary operators in A by: $\mathcal{O}_A \tilde{\psi}\rangle = \mathcal{O} \tilde{\psi}\rangle, \mathcal{O}_A^\dagger \tilde{\psi}\rangle = \mathcal{O}^\dagger \tilde{\psi}\rangle$

- Note: Two distinct redundancies:
 - *QEC redundancy*: Encoded logical qudit states are n -qudit states, logical qudit states are k -qudit states, where $n > k$.
 - *AdS/CFT redundancy*: $\dim(\text{bulk}) > \dim(\text{boundary})$.

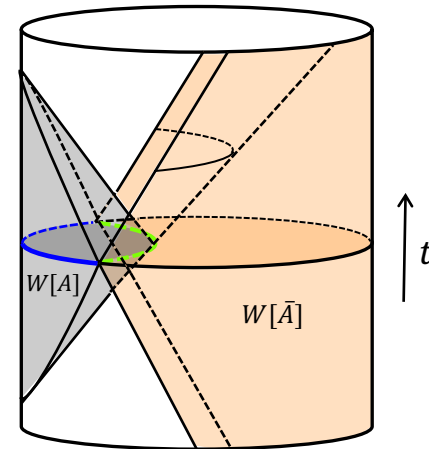
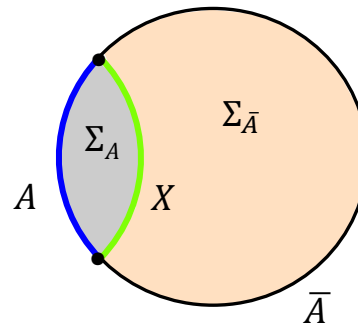
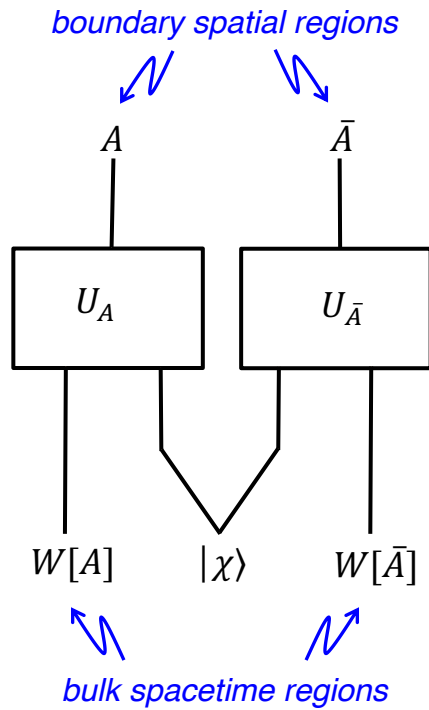
Not the same!



So: The bulk theory is a way of protecting information encoded in the boundary theory against erasure.

- Does this justify the claim that spacetime is a QEC?

- A QEC isn't the sort of thing associated with a spacetime.
- A QEC can be *realized* by physical systems, which can possess spatiotemporal properties.



The QEC map relates physical systems in bulk spacetime regions to physical systems in boundary spatial regions.

So spacetime (either bulk or boundary) really isn't a QEC...