

06. Maxwell's Demon and Information

1. The Demon and the 2nd Law
2. Automated Demons
3. Computational Demons

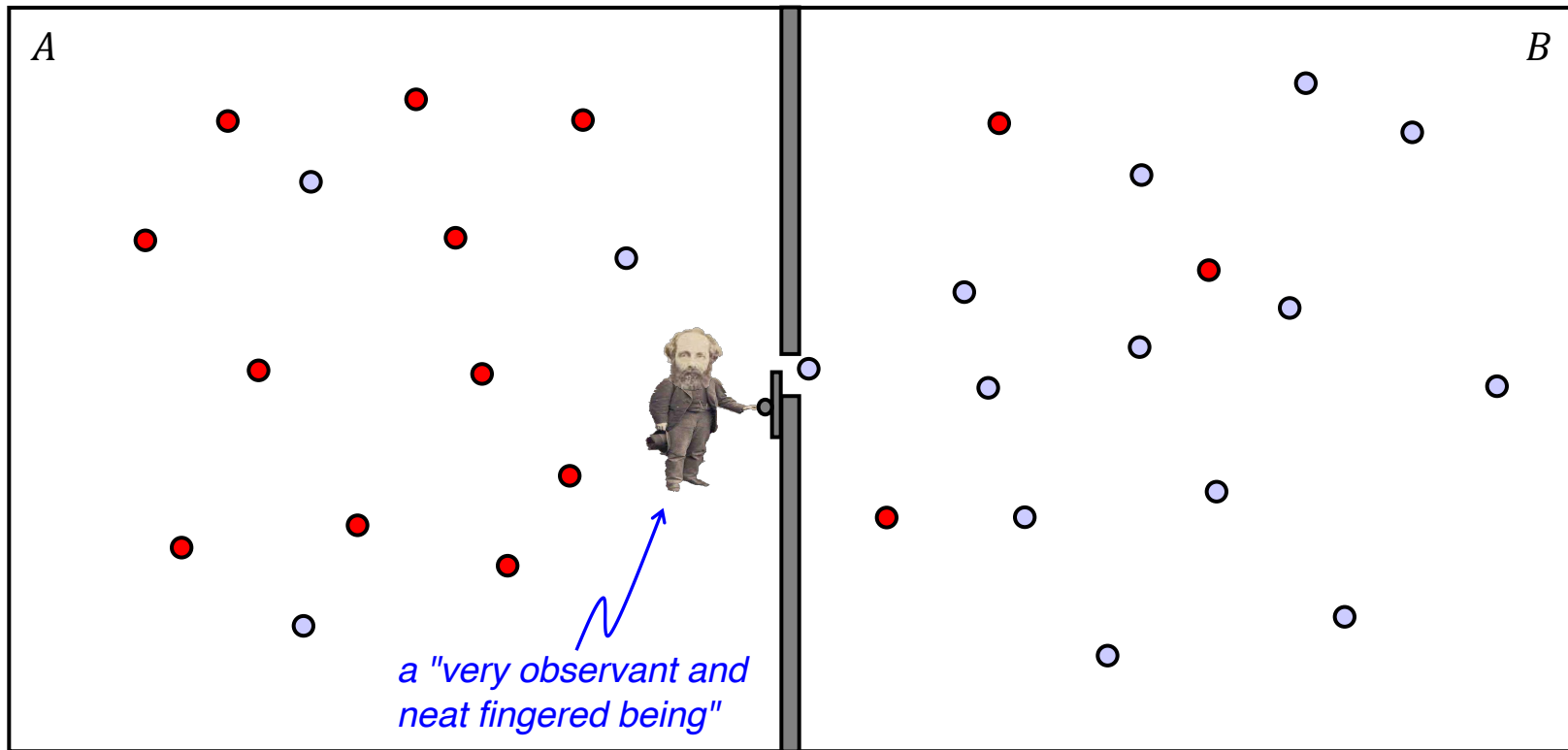


James Clerk Maxwell
(1831-1879)

1. The Demon and the 2nd Law

- Consider 2nd Law in the form: "If two things are in contact, the hotter cannot take heat from the colder without external agency."
- Maxwell's counterexample:

1867 letter to P. Tait



The neat-fingered being only lets hot molecules through to A and cold molecules through to B.

Upshot:

"The hot system has got hotter and the cold colder and yet no work has been done, only the intelligence of a very observant and neat fingered being has been employed."



Moral #1:



"The 2nd Law of Thermodynamics has only statistical certainty."

- In other words: It's very probable, but *not* completely certain, that "If two things are in contact, the hotter cannot take heat from the colder without external agency."

Moral #2: Attempts to derive the 2nd Law from (deterministic) mechanics will fail.

"...it is rare sport to see those learned Germans contending for the priority of the discovery that the 2nd law of [thermodynamics] is the Hamiltonische Princip... [It] soars along in a region unvexed by statistical considerations while the German Icari flap their waxen wings in *nephelo coccygia* amid those cloudy forms which the ignorance and finitude of human science have invested with the incommunicable attributes of the invisible Queen of heaven."



Moral #3: The distinction between dissipated energy (heat that we cannot make use of) and energy available for work depends on our state of knowledge.



[If we supposed]... our senses sharpened to such a degree that we could trace the motions of molecules as easily as we now trace those of large bodies... the distinction between work and heat would vanish...
[The truth of the 2nd Law depends]... on the fact that the bodies we deal with consist of millions of molecules and that we can never get hold of a single molecule."

- In other words: If we were neat-fingered beings capable of knowing the positions and velocities of molecules, then the 2nd Law would not apply (i.e., the 2nd Law doesn't hold for microscopic systems).

Vexing Unanswered Questions

(1) *Why* is the 2nd Law only statistical?

- Are the probabilities really *epistemic*? Do they really reflect our lack of knowledge of the micro-physics (Moral #3)?
 - *But then why do the vast majority of observable macroscopic systems obey the 2nd Law?*
- Are the probabilities *ontic*? Do they reflect an intrinsic probabilistic nature of micro-physical objects?
- Subsequent development of statistical mechanics and attempts to derive 2nd Law within it.

(2) Should the Demon itself be subject to thermodynamics?

- Must be: Otherwise why would we care if a non-thermodynamic demon was capable of violating the 2nd Law of thermodynamics?
- But: If so, then shouldn't we "naturalize" the Demon?

- Perhaps a comprehensive thermodynamical analysis of Demon-plus-system will indicate that the 2nd Law is not violated.
- Subsequent 20th-century history of the Demon:
 - *Fluctuation phenomena as naturalized demons.*
 - *Computational devices as naturalized demons.*

(3) Is Demonology Necessary?

- To investigate the conceptual significance of the 2nd Law, look to securing the foundations of statistical mechanics, as opposed to demon-bashing.
(Earman & Norton 1998)

2. Automated Demons

- Early 20th century thermal fluctuation phenomena:
 - *Brownian motion.*
 - *Density fluctuations in fluids near critical states.*
- Idea: Exploit such phenomena to violate 2nd Law.



Later idea: Exploit such phenomena to predict the stock markets...



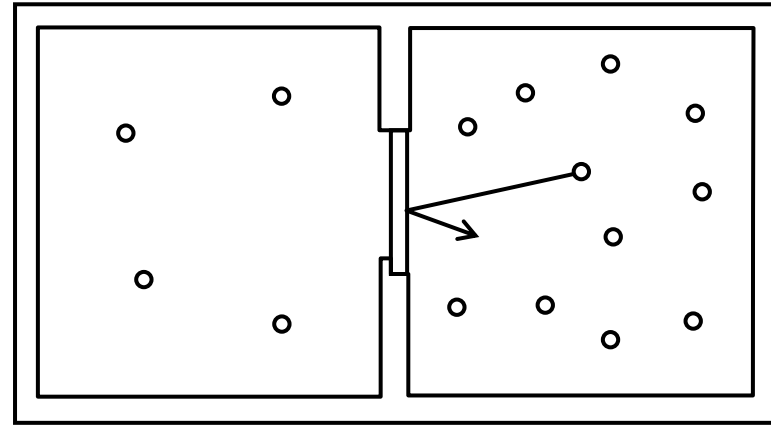
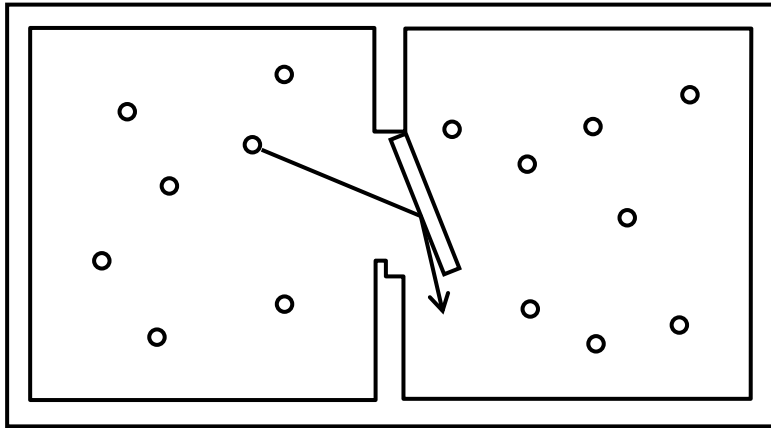
Key characteristic:
*Completely random
processes!*



Marian
Smoluchowski
(1872-1917)

Smoluchowski and Automated Demons

Smoluchowski, M. (1914) 'Gultigkeitsgrenzen des zweiten Hauptsatzes der Warmetheorie'



- Gas in separate chambers initially at equal pressures and temperatures.
- One-way spring-loaded trapdoor allows *randomly fluctuating* molecules to pass from one side to the other, but not *vice-versa*.
- Result: Build-up of pressure on one side that can be exploited to perform work.
Violation of 2nd Law!

Is this an example of a decrease in TD entropy of a thermally-isolated system?

Yes!

Can this decrease in TD entropy be used to perform work?

No!

Sensitivity of spring and trapdoor to single molecules (!) makes them susceptible to thermal fluctuations; hence they can't function as promised.

Is this automated valve what Maxwell was concerned with?

- *Wasn't he concerned with an "intelligent being" who knows when to open and close the valve?*
- *What if we replace the automated valve with a "naturalized" intelligent demon?*



*A "computational demon"
that functions by acquiring
and/or erasing information...*

Def. *(Computational Demons, Types A and B).*

- A **Type A computational demon** is a device that functions (in part) by storing data in a memory register (i.e., "acquiring information").
- A **Type B computational demon** is a device that functions (in part) by resetting a memory register (i.e., "erasing information").

3. Computational Demons

Szilard and Type A Computational Demons

(1929) "On the Decrease of Entropy in a Thermodynamical System by the Intervention of Intelligent Beings"

Claim. Attempts to violate the 2nd Law with a Type A computational demon fail since there is a TD entropy cost in the acquisition of information needed to run the device.

"...measurements themselves are necessarily accompanied by a production of entropy."

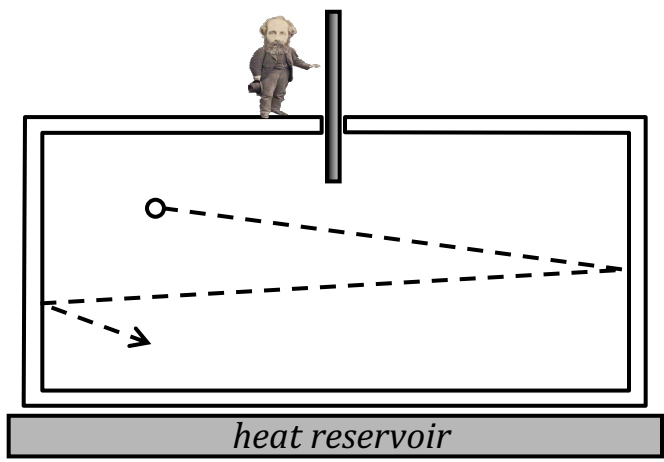


Leo Szilard
(1898-1964)

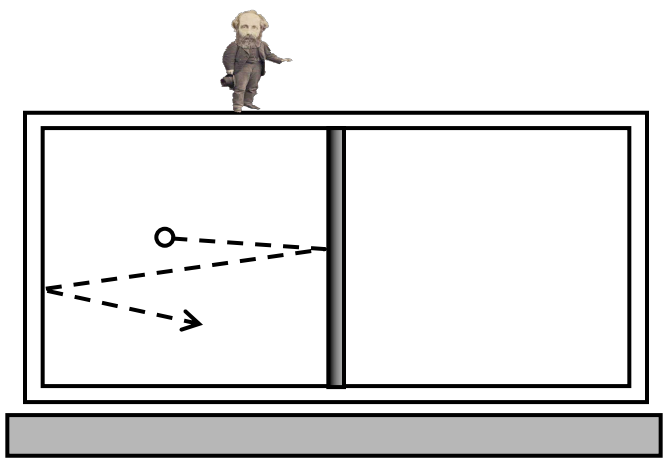
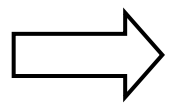
More precisely:

Szilard's Principle: Gaining *information* that allows us to discern between n states weighted with probabilities p_i , $i = 1...n$, is associated with a minimum increase in TD entropy of $-k \sum_i p_i \ln p_i$.

Example #1: Szilard's One-Molecule Engine

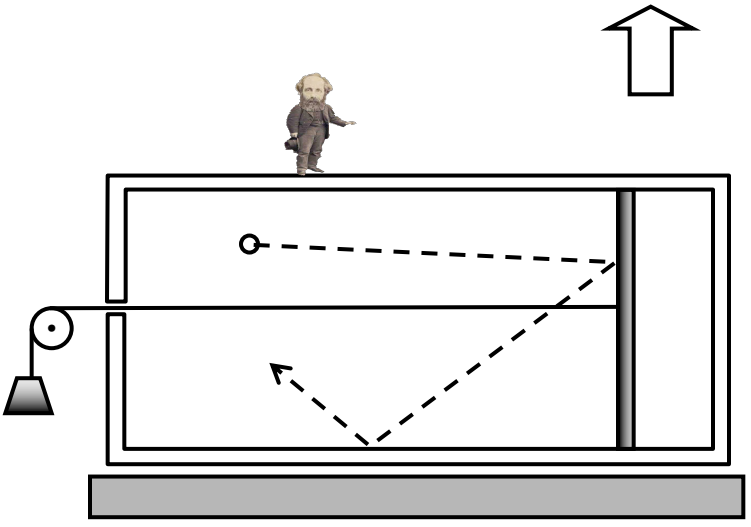
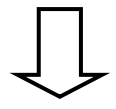


(a) Single molecule exhibiting thermal fluctuations. System at *const. temp.*

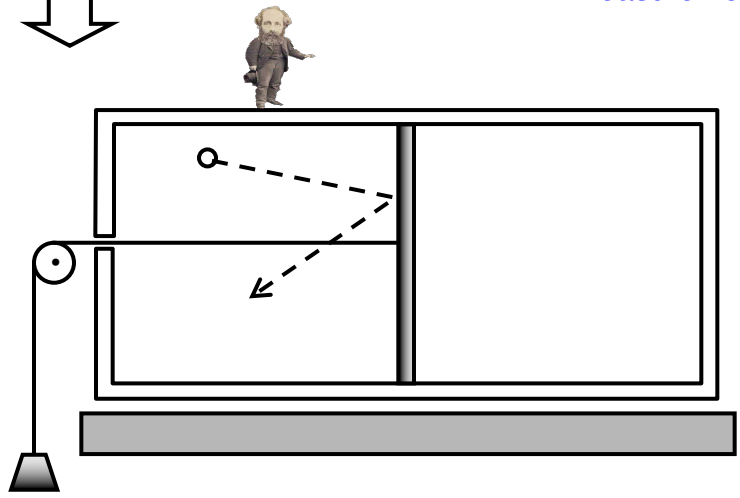
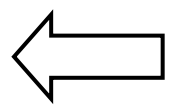


(b) Demon inserts partition/piston and determines which side molecule is on.

Conducts a measurement!



(d) Weight is detached. Partition/piston removed. Cycle returns to (a).



(c) Weight is attached to side with molecule. Gas expands reversibly and isothermally by absorbing heat from reservoir. Work performed on weight.

Violation of 2nd Law: Heat converted to work with no exhaust!

- Stage (c): $\Delta U = 0 = -W + Q$  gas absorbs heat (+) and performs work (-)

So: $Q = W = \int_{V_i}^{V_f} P dV$  work done by gas

$$= \int_{V_i}^{V_f} \frac{kT}{V} dV$$
  for an ideal gas: $PV = NkT$, $N = 1$

$$= kT \ln \frac{V_f}{V_i} = kT \ln 2$$
  heat absorbed by gas from reservoir (assume $V_f = 2V_i$)

So: Change in S_{TD} in reservoir is:

$$\Delta S_{TD} = \int_{\sigma_i}^{\sigma_f} \frac{\delta Q_R}{T} = \frac{-kT \ln 2}{T} - 0 = -k \ln 2$$
  heat emitted by reservoir  A decrease in S_{TD} !

Szilard's Solution: There must be an increase in S_{TD} of $k \ln 2$ in the demon which balances the decrease in S_{TD} of the reservoir.

Fine print:

- In Stage (b) the demon gains "information" that lets it discern between two states, each with a probability of $\frac{1}{2}$.
- So Szilard's Principle entails an increase of S_{TD} of at least $-k\{\frac{1}{2} \ln \frac{1}{2} + \frac{1}{2} \ln \frac{1}{2}\} = k \ln 2$.

Example #2: Szilard's Molecule-Sorting Engine

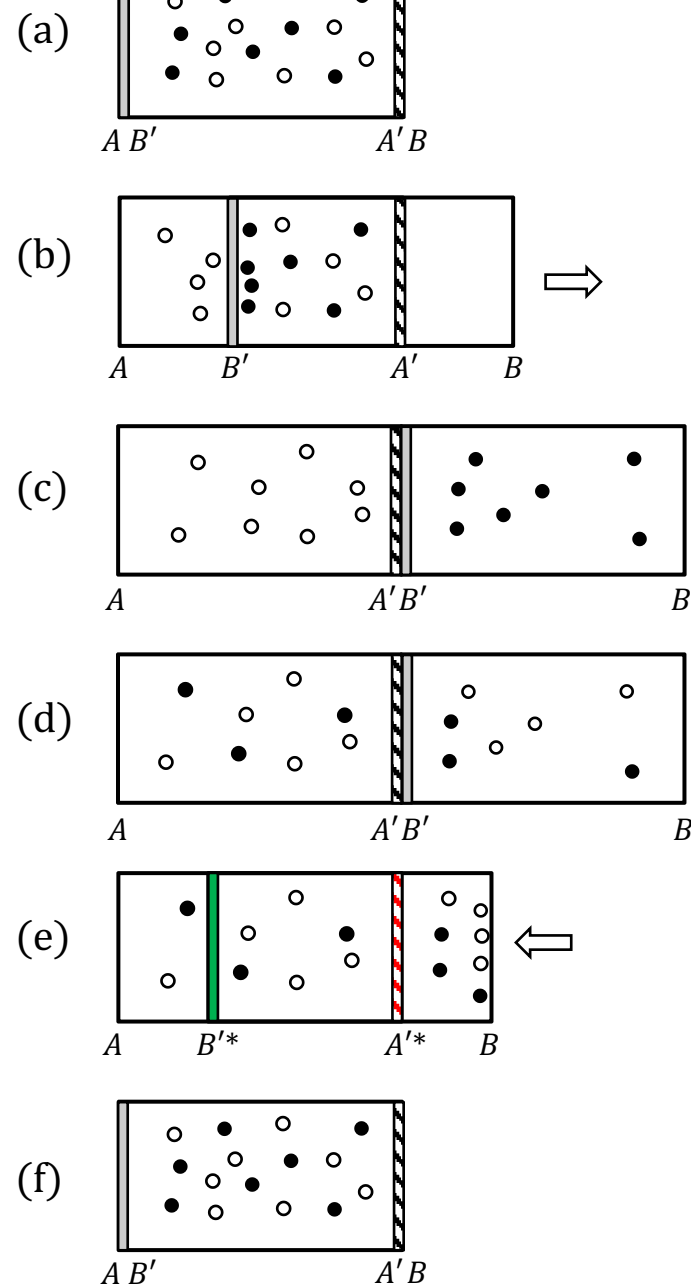
Type 1 = black
Type 2 = white

Stage (a): Measurement at Equilibrium

- N -molecule gas in cylinder with vol V consisting of two molecule types that transition into each other over long time intervals.
- $\{p_1, p_2\}$ = equilibrium distribution over single-particle states so that $N_1 = p_1 N$, $N_2 = p_2 N$ are the # of Type 1 and Type 2 molecules at equilibrium.
- *Molecules are measured to determine their type.*

Stage (b): Reversible Separation

- Section BB' is moved to right reversibly until B' coincides with A' .
- Semi-permeable membrane B' moves through Type 2s and pushes Type 1s to right.
- Semi-permeable membrane A' allows Type 1s to pass through and blocks Type 2s.
- No work done, no heat exchanged.



Example #2: Szilard's Molecule-Sorting Engine

Type 1 = black
Type 2 = white

Stage (c): Separate Nonequilibrium

- Gases in left and right are not in equilibrium:

- Left: $(N_1)_L^{(c)} = 0$ $(N_2)_L^{(c)} = p_2 N$

- Right: $(N_1)_R^{(c)} = p_1 N$ $(N_2)_R^{(c)} = 0$ $\leftarrow$ Equilibrium characterized by $N_1/N_2 = p_1/p_2$

Stage (d): Separate Equilibrium

- Molecules transition between types to reestablish equilibrium in each side.

- # in left: $xN + yN = p_2 N$, with $x/y = p_1/p_2$

Thus: $(p_1/p_2)y + y = p_2$, or $y = p_2^2$; so $x = p_1 p_2$

- # in right: $zN + tN = p_1 N$, with $z/t = p_1/p_2$

Thus: $(p_1/p_2)t + t = p_1$, or $t = p_1 p_2$; so $z = p_1^2$

So:

- Left: $(N_1)_L^{(d)} = p_1 p_2 N$ $(N_2)_L^{(d)} = p_2^2 N$

- Right: $(N_1)_R^{(d)} = p_1^2 N$ $(N_2)_R^{(d)} = p_1 p_2 N$

S_{Boltz} decrease
of $Nk(p_1 \ln p_1 + p_2 \ln p_2)$

Claim. From Stage (c) to Stage (d),

(i) S_{Boltz} of gas increases by $Nk\{p_1 \ln p_1 + p_2 \ln p_2\}$

(ii) S_{Boltz} of environment decreases by $Nk\{p_1 \ln p_1 + p_2 \ln p_2\}$

- Assumption: $\Delta S_{\text{gas}} + \Delta S_{\text{Environ}} = 0$ $\leftarrow$ Equilibration is reversible

Claim 1(i). From Stage (c) to Stage (d), S_{Boltz} of gas increases by $Nk\{p_1\ln p_1 + p_2\ln p_2\}$.

Proof

- For an N -molecule ideal gas in vol V , $S_{\text{Boltz}} = Nk\ln [fV/N]$.
- The change in S_{Boltz} between Stages (c) and (d) is:

$$\Delta S_{\text{Boltz}} = \Delta S_L + \Delta S_R = (S_L^{(d)} - S_L^{(c)}) + (S_R^{(d)} - S_R^{(c)})$$

Where:

$$\begin{aligned} S_L^{(d)} &= (N_1)_L^{(d)} k\ln [f_1 V / (N_1)_L^{(d)}] + (N_2)_L^{(d)} k\ln [f_2 V / (N_2)_L^{(d)}] \\ &= p_1 p_2 N k\ln [f_1 V / p_1 p_2 N] + p_2^2 N k\ln [f_2 V / p_2^2 N] \\ &= p_2 N k\{p_1\ln [f_1 V / p_2 N] - p_1\ln p_1 + p_2\ln [f_2 V / p_2 N] - p_2\ln p_2\} \end{aligned}$$

$$\begin{aligned} S_R^{(d)} &= (N_1)_R^{(d)} k\ln [f_1 V / (N_1)_R^{(d)}] + (N_2)_R^{(d)} k\ln [f_2 V / (N_2)_R^{(d)}] \\ &= p_1^2 N k\ln [f_1 V / p_1^2 N] + p_1 p_2 N k\ln [f_2 V / p_1 p_2 N] \\ &= p_1 N k\{p_1\ln [f_1 V / p_1 N] - p_1\ln p_1 + p_2\ln [f_2 V / p_1 N] - p_2\ln p_2\} \end{aligned}$$

So:

$$\begin{aligned} \Delta S_{\text{Boltz}} &= p_2 N k\{p_1\ln [f_1 / f_2] - p_1\ln p_1 - p_2\ln p_2\} + p_1 N k\{-p_2\ln [f_1 / f_2] - p_1\ln p_1 - p_2\ln p_2\} \\ &= -Nk\{p_1\ln p_1 + p_2\ln p_2\} \end{aligned}$$

 Positive since $0 < p_1, p_2 < 1$

Stage (c): Separate Nonequilibrium

$$\begin{aligned} (N_1)_L^{(c)} &= 0 & (N_2)_L^{(c)} &= p_2 N \\ (N_1)_R^{(c)} &= p_1 N & (N_2)_R^{(c)} &= 0 \end{aligned}$$

Stage (d): Separate Equilibrium

$$\begin{aligned} (N_1)_L^{(d)} &= p_1 p_2 N & (N_2)_L^{(d)} &= p_2^2 N \\ (N_1)_R^{(d)} &= p_1^2 N & (N_2)_R^{(d)} &= p_1 p_2 N \end{aligned}$$

$$\begin{aligned} S_L^{(c)} &= (N_1)_L^{(c)} k\ln [f_1 V / (N_1)_L^{(c)}] \\ &\quad + (N_2)_L^{(c)} k\ln [f_2 V / (N_2)_L^{(c)}] \\ &= p_2 N k\ln [f_2 V / p_2 N] \end{aligned}$$

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Example #2: Szilard's Molecule-Sorting Engine

Type 1 = black
Type 2 = white

S_{TD} increase of
 $Nk(p_1 \ln p_1 + p_2 \ln p_2)$

Stages (e)-(f): Return to initial state

- Membranes B' , A' are replaced with B'^* , A'^* :
- B'^* moves through original Type 2s (everything on left).
- A'^* lets original Type 1s pass (everything on right)
- A'^* , B'^* are replaced with A' , B' at end of process.

Szilard's Argument

- All transitions between (a) and (f) are reversible, so 2nd Law requires $\Delta S_{TD} = 0$.
- There is a decrease in S_{Boltz} of $Nk\{p_1 \ln p_1 + p_2 \ln p_2\}$ during Stage (d).
- So: There must be an increase in S_{TD} prior to Stage (d) of at least $Nk\{p_1 \ln p_1 + p_2 \ln p_2\}$.
- And: This increase is due to the measurement at Stage (a).

Fine print:

- In Stage (a) the demon gains "information" that lets it discern between the two states, with probabilities p_1 , p_2 , of N molecules.
- So Szilard's Principle entails an increase of S_{TD} of at least $Nk\{p_1 \ln p_1 + p_2 \ln p_2\}$.

But Ooops: Assumes $S_{TD} = S_{Boltz}$!

Landauer and Type B Computational Demons

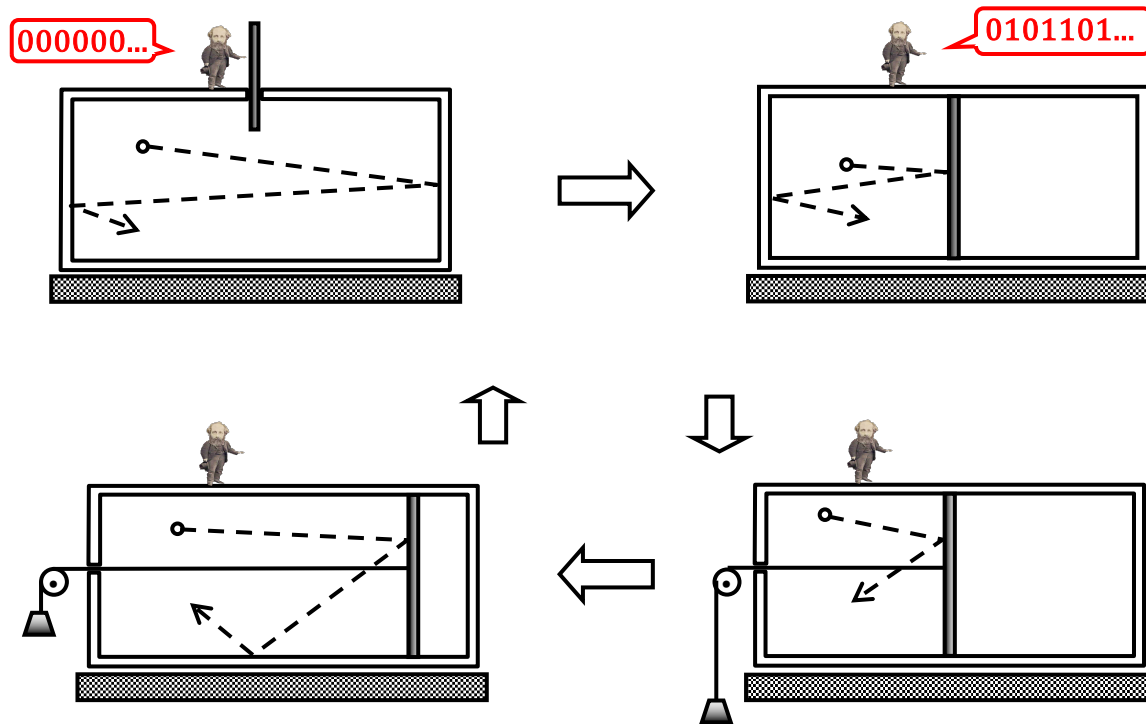
(1961) "Irreversibility and Heat Generation in the Computing Process", *IBM J of R&D* 5, 183-91.



Rolf Landauer
(1927-1999)

Landauer's Principle: Erasing *information* that allows us to discern between n states weighted with probabilities p_i , $i = 1...n$, is associated with a minimum increase in TD entropy of $-k\sum_i p_i \ln p_i$.

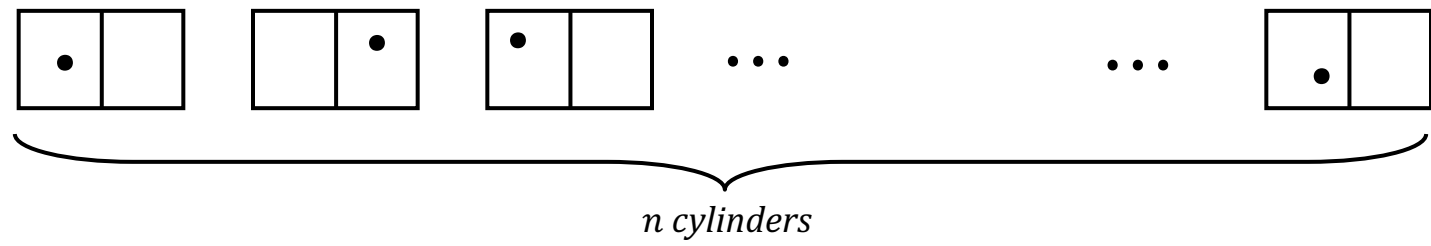
- The demon works in a *cycle*.
- At a given time, it must acquire information.
- At a later time, it must erase this information to return to its initial state.



*The 2nd Law is saved...
(from Type B computational demons)...*

General Idea: Logical states of a computer must be represented by *physical states* of its hardware.

- Ex: An n -bit memory register as an array of n two-chambered cylinders, each filled with a one-molecule gas.



Let molecule in left correspond to "0"; molecule in right correspond to "1".

- Each cylinder has 2 possible states.
 - *So entire register has 2^n possible states.*
- Now: Set register to zero (erase all bits).
 - Before erasure: Register can be in any of 2^n states.
 - After erasure: Register is in exactly one state.

"The operation has therefore compressed many logical states into one, much as a piston might compress a gas."



Charles Bennett
(1943-present)

- So: Erasure = compression of many physical states (high S_{Boltz}) into exactly one (low S_{Boltz}).

"Hence one cannot clear a memory register without generating heat and adding to the entropy of the environment. Clearing a memory is a thermodynamically irreversible operation."

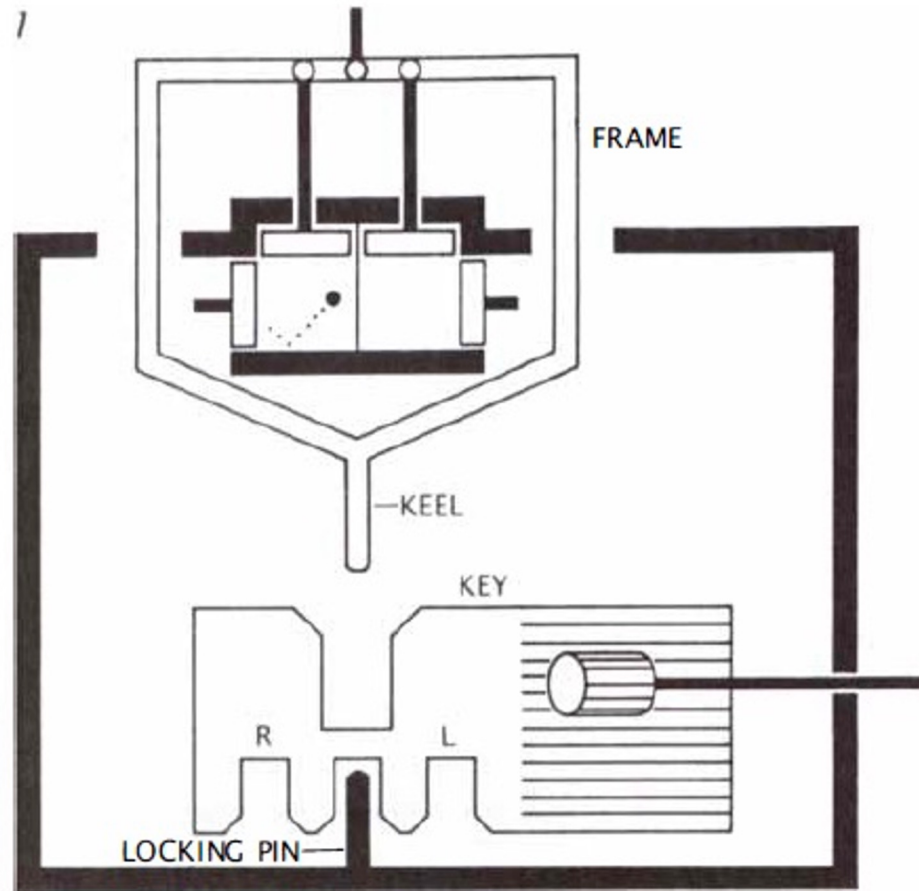


Concern: Are we talking about S_{Boltz} (measure of microstates), or S_{TD} (irreversibility, heat exchanges)?

Shakey reasoning...

Shakey Reasoning, Part II

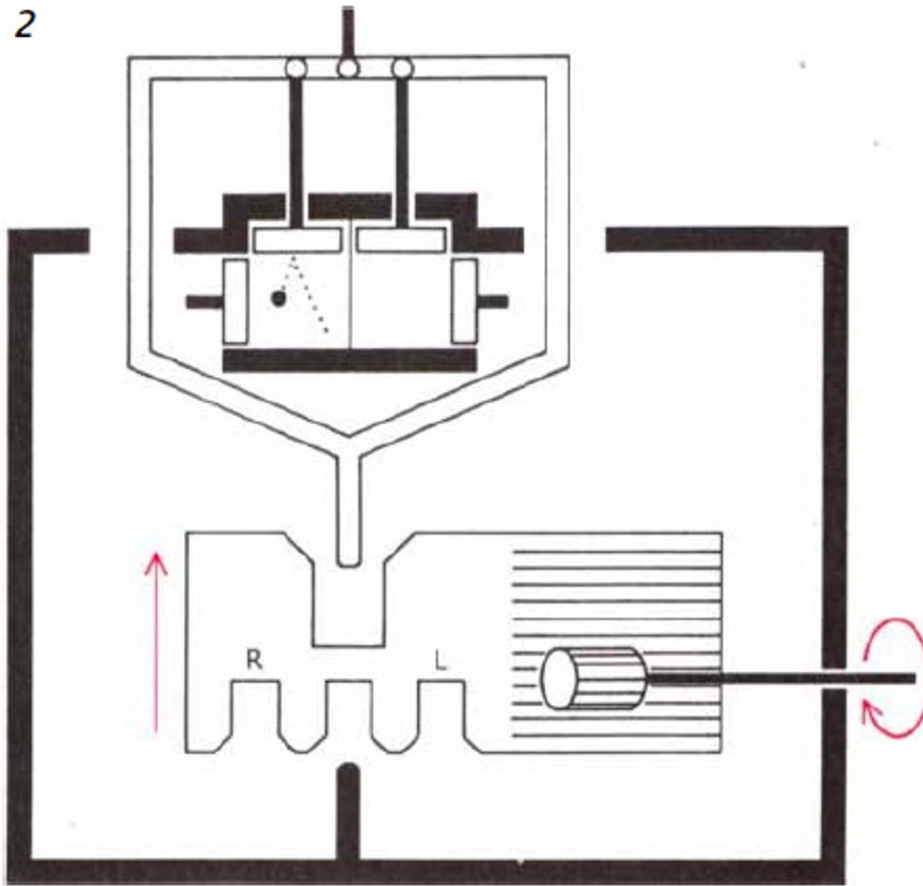
Claim. Info acquisition can be achieved without entropy cost.



↖ Against Szilard's Principle

"A slightly modified Szilard engine sits near the top of the apparatus (1) within a boat-shaped frame; a second pair of pistons has replaced part of the cylinder wall. Below the frame is a key, whose position on a locking pin indicates the state of the machine's memory."

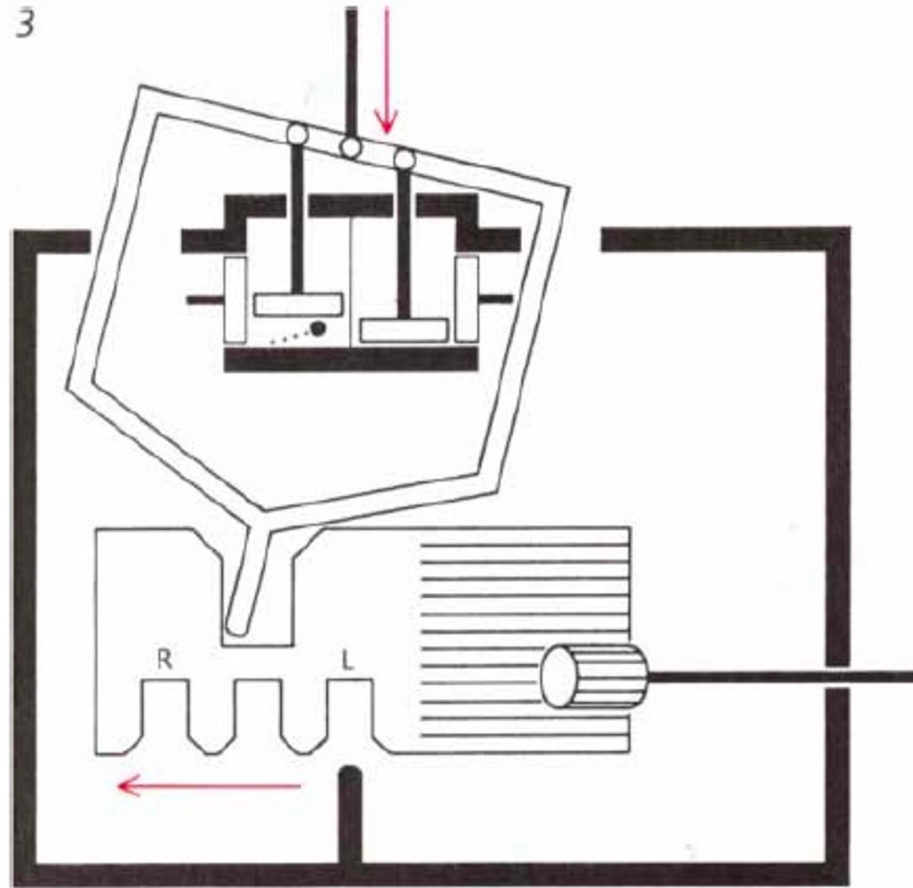
Claim. Info acquisition can be achieved without entropy cost.



"To begin the measurement (2) the key is moved up so that it disengages from the locking pin and engages a 'keel' at the bottom of the frame."

Shakey Reasoning, Part II

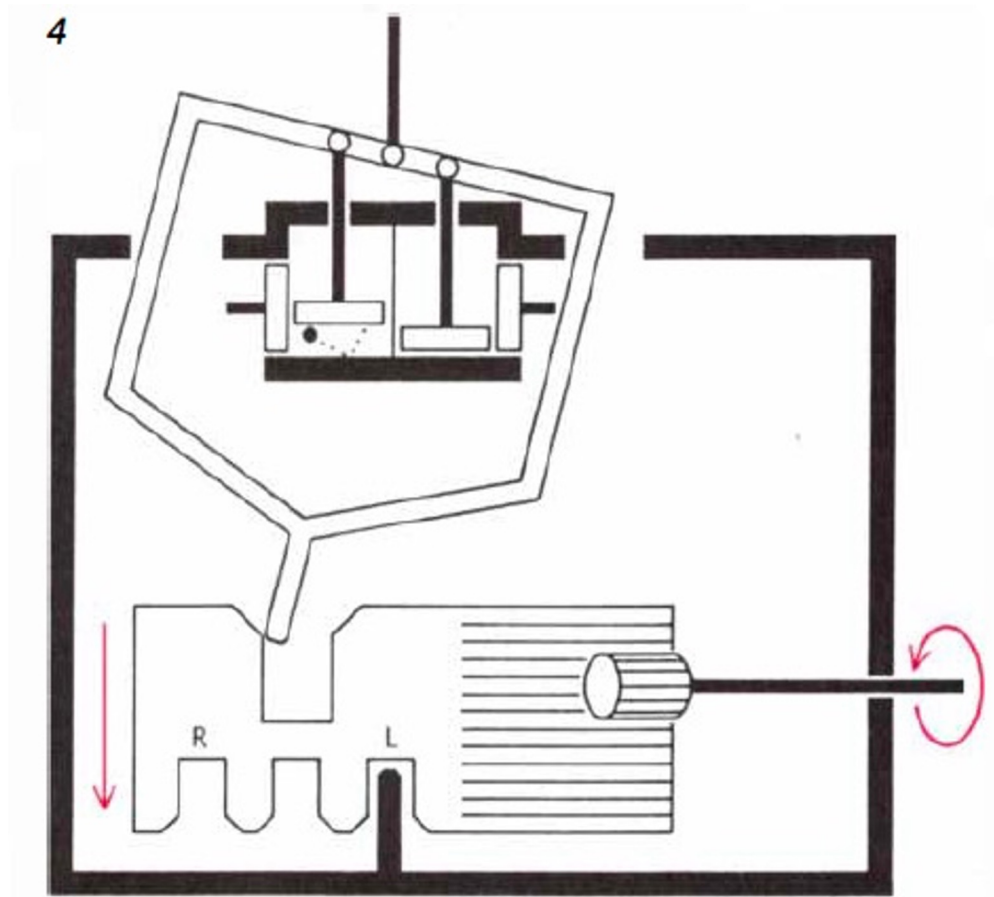
Claim. Info aquisition can be achieved without entropy cost.



"Then the frame is pressed down (3). The piston in the half of the cylinder containing no molecule is able to descend completely, but the piston in the other half cannot, because of the pressure of the molecule. As a result the frame tilts and the keel pushes the key to one side."

Shakey Reasoning, Part II

Claim. Info aquisition can be achieved without entropy cost.

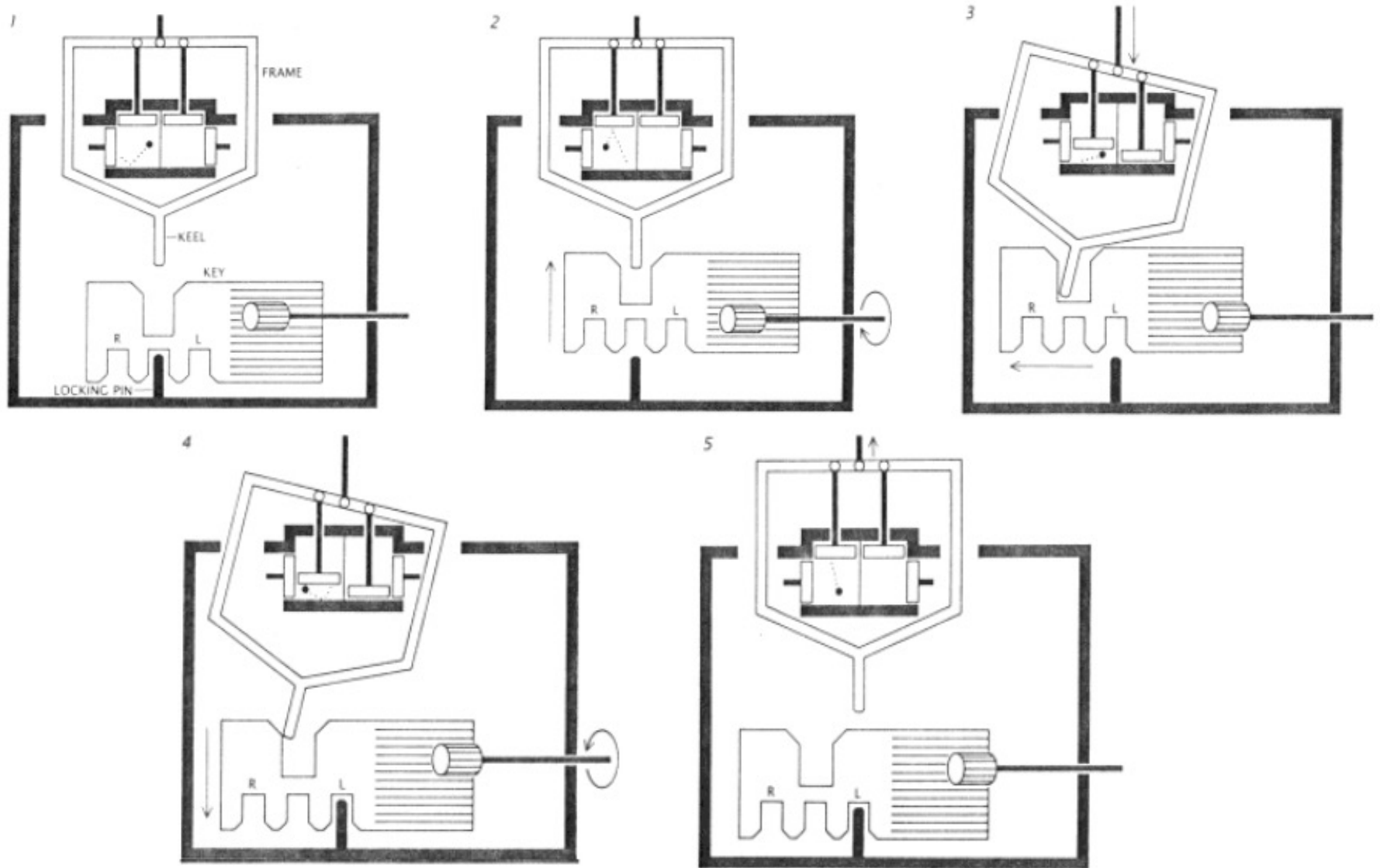


"The key, in its new position, is moved down to engage the locking pin (4), and the frame, is allowed to move back up (5)..."

5

The diagram shows a mechanical assembly within a hexagonal frame. At the top, three vertical rods are connected to a horizontal bar. A red arrow points upwards from the central rod. Below this, there are two vertical rods passing through a series of horizontal bars. The central part of the assembly consists of two vertical rods passing through a series of horizontal bars. The entire assembly is mounted on a base. To the right of the assembly, there is a control knob with a cylindrical body and a horizontal shaft. The knob is connected to a horizontal shaft that passes through a series of horizontal bars. The shaft is labeled 'R' and 'L' at its ends. The shaft is connected to a series of horizontal bars. The shaft is labeled 'R' and 'L' at its ends. The shaft is connected to a series of horizontal bars.

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- Measurement without entropy cost?

No: Any mechanical device will be subject to thermal fluctuations thus obliterating its measuring function (Earman and Norton 1999).

Shakey Reasoning, Part III

- Maybe the info acquisition of a mechanical locking pin device isn't the type that should be associated with a computational device.
- Maybe for *computational measurement*, there is no TD entropy cost.

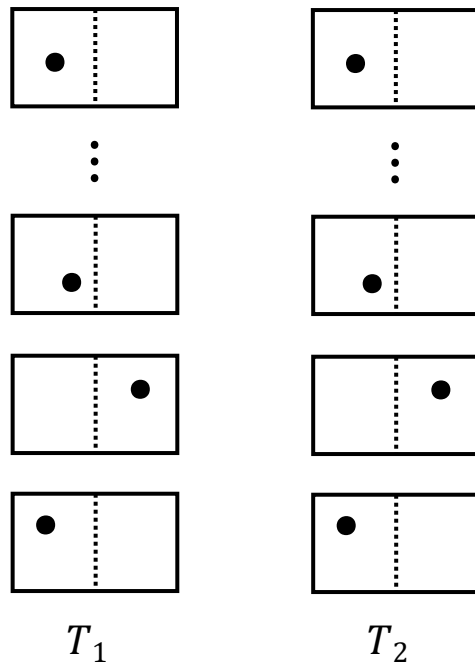
Why?

$$\left[\begin{array}{c} \text{computational} \\ \text{measurement} \end{array} \right] = \left[\begin{array}{c} \text{correlation between the state} \\ \text{of a measured system and the} \\ \text{state of the memory register of} \\ \text{a measurement device.} \end{array} \right] = \left[\begin{array}{c} \text{copying} \\ \text{operation} \end{array} \right]$$

- Now show: *Copying operations cost no entropy.*

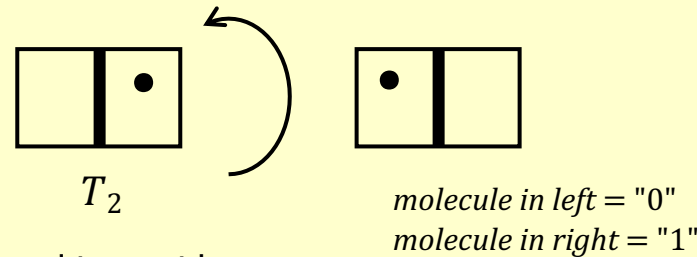
Two physical memory registers T_1, T_2 initially in same state.

- Task: Reset T_2 to zero state.



To reset T_2 using T_1 :

1. If first box of T_1 is "0", do nothing to first box of T_2 .
2. If first box of T_1 is "1", insert partition into first box of T_2 (trapping molecule on right) and then flip box:

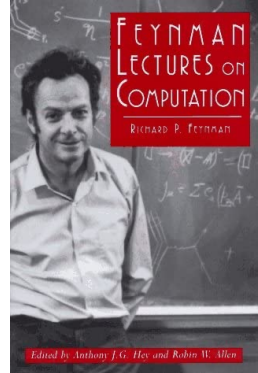


3. Proceed to next box.

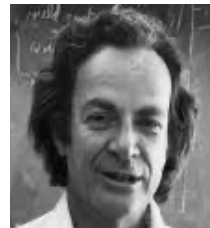
- Claim: Reset procedure of T_2 using T_1 is *reversible*: no entropy cost.

Why?

- T_1 tells us the state T_2 is in, so resetting T_2 using T_1 does not involve a decrease in the number of its possible states; hence no decrease in entropy.
- If we did not have T_1 (if T_2 was in an *unknown* state), then resetting T_2 to zero would involve a decrease in the number of its possible states; hence there would be a decrease in its entropy.



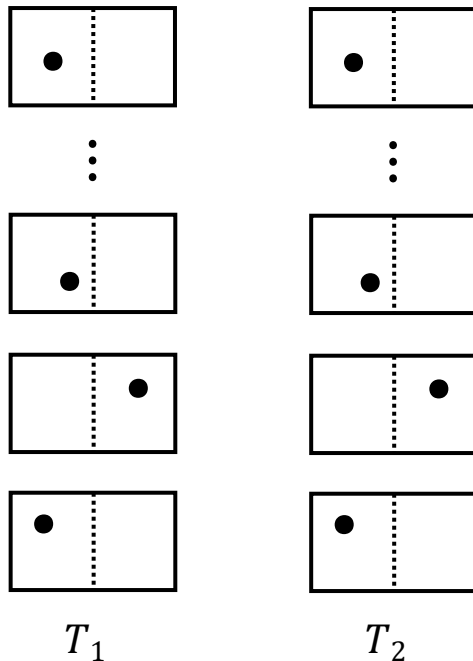
(1996) *Lectures on Computation*



Richard Feynman
(1918-1988)

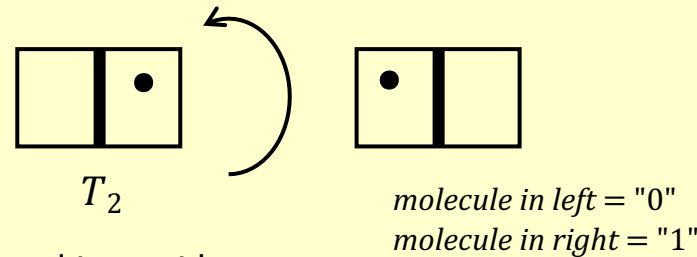
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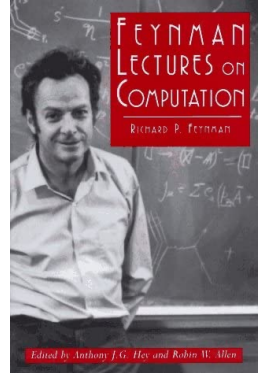
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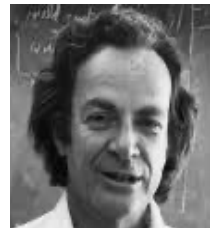
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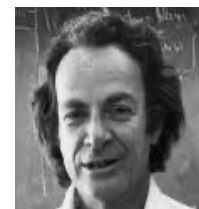
"...it might seem odd to be able to insert pistons and turn boxes without expending energy. In the real world, of course, you can't--but we are dealing with abstractions here and, as I have said, we are not interested in the kinetic energy or weight of the 'boxes'. Given our assumptions, it is possible to do so, although the downside is that we would have to take an eternity to do it!" (Feynman 1996, pg. 144.)



(1996) *Lectures on Computation*

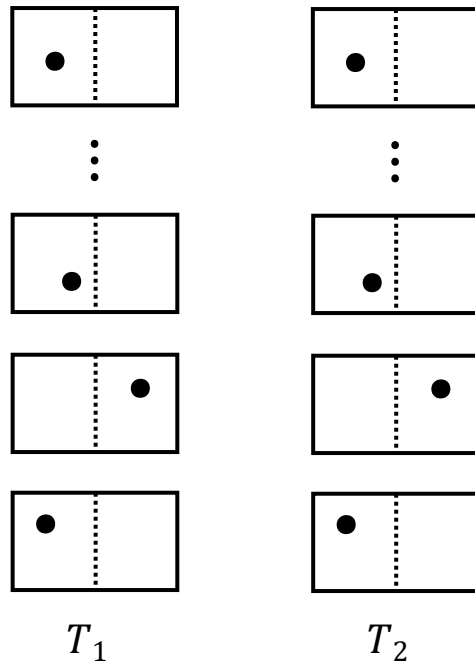


Richard Feynman
(1918-1988)



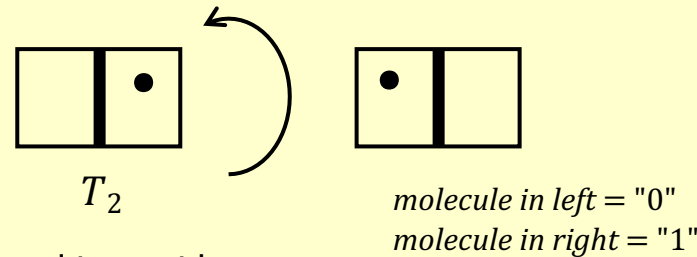
Two physical memory registers T_1, T_2 initially in same state.

- Task: Reset T_2 to zero state.



To reset T_2 using T_1 :

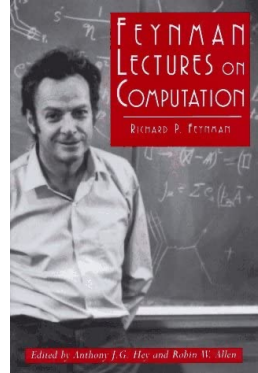
1. If first box of T_1 is "0", do nothing to first box of T_2 .
2. If first box of T_1 is "1", insert partition into first box of T_2 (trapping molecule on right) and then flip box:



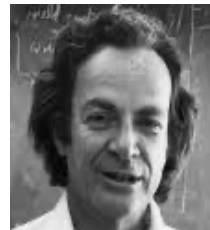
3. Proceed to next box.

- Claim: Reset procedure of T_2 using T_1 is *reversible*: no entropy cost.
- In reverse operation (no entropy cost), T_1 is *copied* onto initially blank T_2 .
 - And: This is a *generic copying operation*.

Conclusion: Copying operations cost no entropy.



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Richard Feynman
(1918-1988)

Issues

1. Is measurement the "reverse" of a reset operation?

Two types of resetting operation

- T_1 is known.

 Feynman/Bennett/Landauer:
This is reverse of copying/measurement.

- T_1 is unknown.

 But: Isn't this measurement? A measurement involves acquiring information, in addition to, perhaps, copying it.

2. What exactly is a "computational" process?

- Just a process that involves erasure?
- Or a process that involves both measurement and erasure?

3. What type of information is associated with a computation?

- Shannon information?
- Fisher information?

4. Do arguments for Landauer's Principle conflate S_{Boltz} with S_{TD} ?

- Claim: Erasure involves a decrease in S_{Boltz} of the computational device, which requires a compensating increase in S_{TD} in its environment.
- A decrease in S_{Boltz} might be associated with a decrease in S_{Shan} , hence a decrease in Shannon information; but no link to the 2nd Law here...