Assignment #4

1. How can the Shannon entropy $S_{\text{Shan}}(X)$ be understood as the expected information gain associated with the measurement of a random variable? How can it be understood as a measure of the maximum amount a message can be compressed?

2. Why does a copying operation cost no entropy?

3. Why can't we mathematically represent the quantum spin property of "*Hardness*" as a function on a classical phase space?

4. (a) Suppose two electrons A, B are in the entangled state $|\Psi^+\rangle = \sqrt{½}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$, where $|0\rangle$ and $|1\rangle$ are eigenstates of *Hardness* that represent the values *hard* and *soft*, respectively. According to the Eigenvalue-Eigenvector Rule, do the electrons have a definite value of *Hardness*?
   (b) Suppose we measure the *Hardness* of electron A and get *hard*. According to the Projection Postulate, what is the post-measurement state of the two electrons? In this post-measurement state, according to the Eigenvalue-Eigenvector Rule, does electron B have a value of *Hardness*? If so, what is it?
   (c) Suppose the two electrons were separated by a huge distance prior to the measurement of electron A's *Hardness*. In what sense does the Eigenvalue-Eigenvector result above suggest what Einstein referred to as "spooky action at a distance"?