16. Einstein and General Relativistic Spacetimes

Problem: Special relativity does not account for gravitational force.

- To include gravity...

Geometricize it! Make it a feature of spacetime geometry.

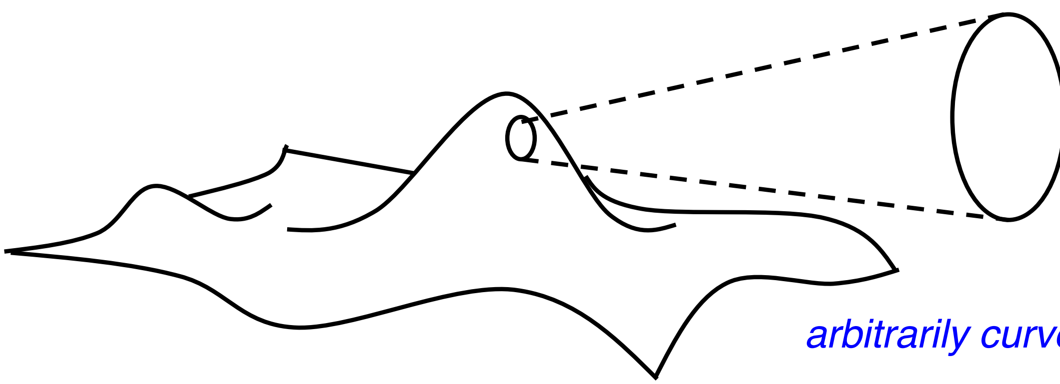
Two requirements

(1) New theory ("general relativity") must reduce to special relativity in sufficiently flat regions of spacetime:

- Replace $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu$ with $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$.

flat Minkowski metric *non-flat metric*

- Require $g_{\mu\nu}$ to reduce to $\eta_{\mu\nu}$ in small regions of spacetime.



16. Einstein and General Relativistic Spacetimes

Problem: Special relativity does not account for gravitational force.

- To include gravity...

Geometricize it! Make it a feature of spacetime geometry.

Two requirements

(2) Curvature of spacetime must be related to matter density:

- The Einstein equations (1916):

$$G_{\mu\nu}(g_{\mu\nu}) = \kappa T_{\mu\nu}$$

Einstein tensor = encodes curvature of spacetime as a function of $g_{\mu\nu}$

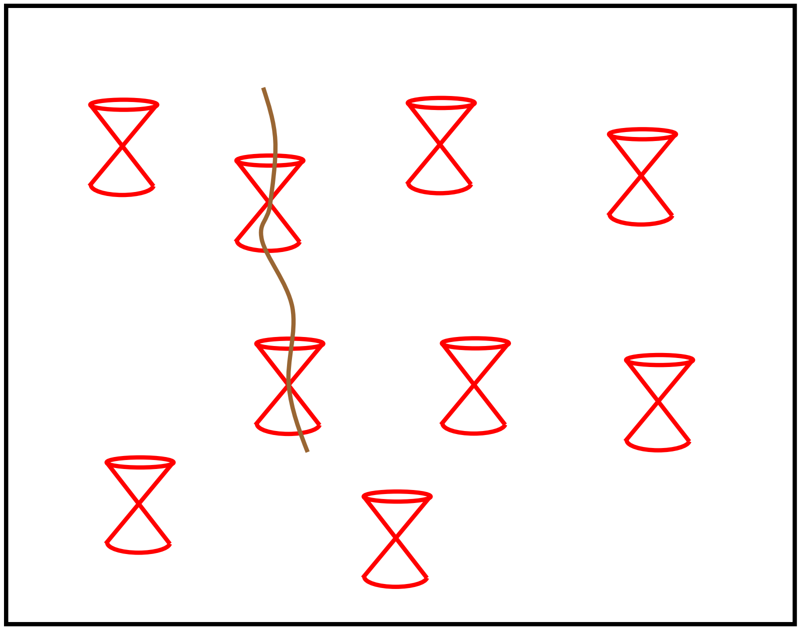
Stress-energy tensor = encodes matter density

- *Consequence*: The Minkowski metric is the solution for zero curvature $G_{\mu\nu}(\eta_{\mu\nu}) = 0$ (*i.e.*, spatiotemporal flatness).

A general relativistic spacetime is a 4-dim collection of points such that between any two (infinitesimally close) points, there is a definite spacetime interval given by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where $g_{\mu\nu}$ is a *Lorentzian* metric that satisfies the Einstein equations.

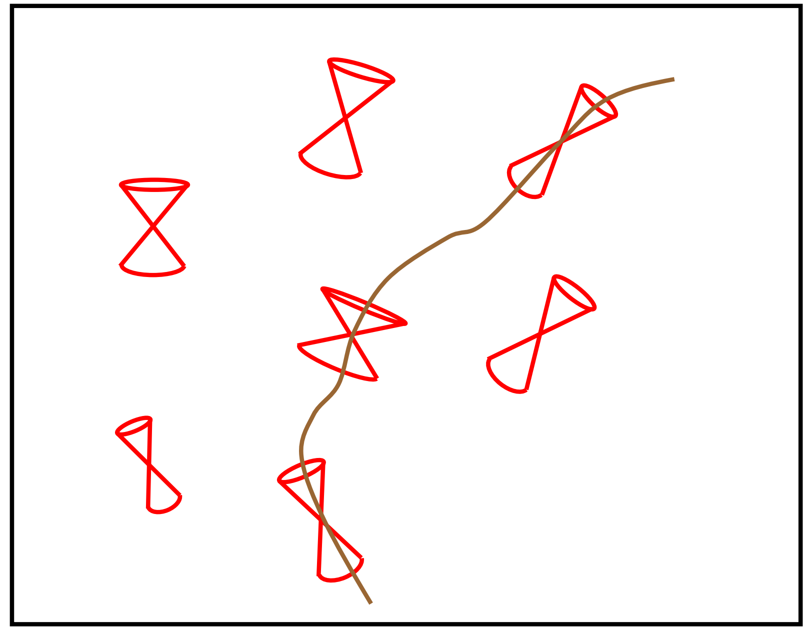
"reduces to the Minkowski metric at any point"

Minkowski Spacetime



Invariant light-cone structure at each point: light-cones all have same size and orientation.

Arbitrary General Relativistic Spacetime



Variable light-cone structure at each point: light-cones can twist and turn due to curvature.

- *Idea*: The light-cone structure constrains the motion of physical objects (traveling on timelike worldlines).
- *And*: In an arbitrary general relativistic spacetime, the matter density determines the light-cone structure.

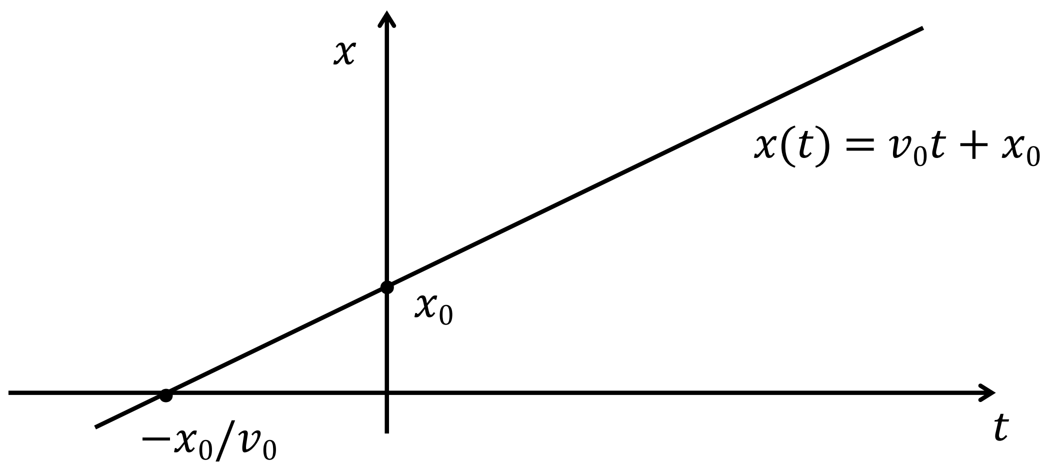
1. The Geometrization of Gravity in GR

Two key observations:

(i) *Geometry*

Equation for a straight line:

$$\frac{d^2x}{dt^2} = 0, \quad \text{or} \quad x(t) = v_0 t + x_0, \quad \text{where } v_0, x_0 = \text{constants}$$



- In inertial frames, Newton's 2nd Law is $F = ma = m\dfrac{d^2x}{dt^2}$.
- In the absence of external forces ($F = 0$) an object's position $x(t)$ as a function of time is the equation of a straight line! (Newton's 1st Law.)

(ii) *Physics*

- Consider when the external force an object experiences is due to gravity:

$$F = \frac{GMm_g}{r^2}$$

The *Newtonian gravitational force* on an object of mass m_g due to another object of mass M a distance r away.

$$= -m_g \partial \Phi$$

$\Phi = -GM/r$ is the *Newtonian gravitational potential field* (describes the particular gravitational field produced by mass M).

- Newton's 2nd Law becomes: $-m_g \partial \Phi = m_i a$

gravitational mass

⇓

measure of degree to which an object experiences the Newtonian gravitational force

inertial mass

⇓

measure of inertia of an object--tendency of object to obey Newton's 1st Law

- Is m_g the same as m_i?
  - *Conceptually and mathematically, no!*
  - *Physically, yes! All known experiments indicate that $m_g = m_i$.*

Consequence of $m_g = m_i$:

<u>Universality of Gravitational Force</u>

In any given gravitational field (described by some Φ), *all* objects fall with the *same* acceleration $a = -\partial\Phi$.

- This is *regardless* of the object's internal properties (it's mass, charge, *etc*.).

The gravitational force is universal: it affects all objects in the same way.

<u>Constrast with the electromagnetic force</u>:

$$\vec{F}_{EM} = q(\vec{E} + \vec{v}\times\vec{B})$$

Electromagnetic force experienced by an object with electric charge q moving at speed v in the presence of electric E and magnetic B fields.

- Newton's 2nd Law becomes:

$$q\left(\vec{E} + \vec{v}\times\vec{B}\right) = m_i\vec{a} \qquad or \qquad \vec{a} = \frac{q}{m_i}\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

- *<u>Not universal!</u>* How much an object accelerates in given E- and B-fields depends on its charge and its inertial mass in the ratio q/m_i.

Different objects will experience different electromagnetically-induced accelerations.

Since gravity is universal, let's incorporate it into the structure of spacetime!

- *Let's "geometrize" it.*

- The motion of an object in a gravitational field is given by

$$\frac{d^2x}{dt^2} = a = -\partial\Phi$$

A curved line in a flat space.

- We can rewrite it as:

$$\left(\frac{d^2x}{dt^2} + \partial\Phi\right) = 0$$

A straight line in a curved space!

- We can view these *particular* "$\partial\Phi$"-straight lines in the curved space as the paths of objects that are undergoing gravitationally-induced acceleration.

- The "extra" term $\partial\Phi$ can be encoded into a "non-flat" metric.

In *flat* Galilean and Minkowski spacetimes, there is a distinction between:

straight trajectories;
no forces present

$$\frac{d^2x}{dt^2} = 0$$

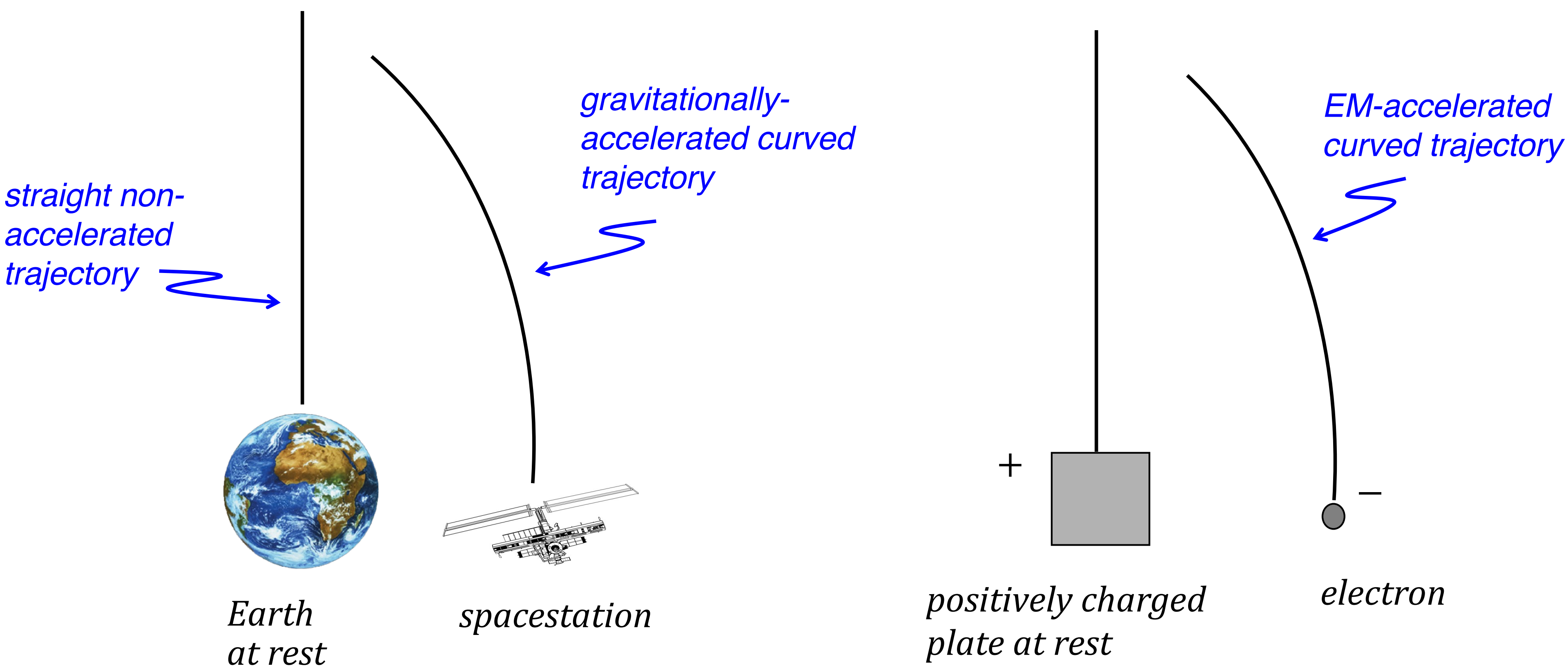
$$ma = 0$$

vs.

curved trajectories;
forces present

$$\frac{d^2x}{dt^2} \neq 0$$

$$ma = F$$



In *curved* general relativistic spacetimes:

- *No* distinction between straights and grav.-accelerated trajectories.
- *Still* a distinction between straights/grav.-accelerated trajectories, and all *other* force-induced accelerated trajectories.

straight trajectories;
no forces present

$$\left(\frac{d^2x}{dt^2} + \partial\Phi\right) = 0$$

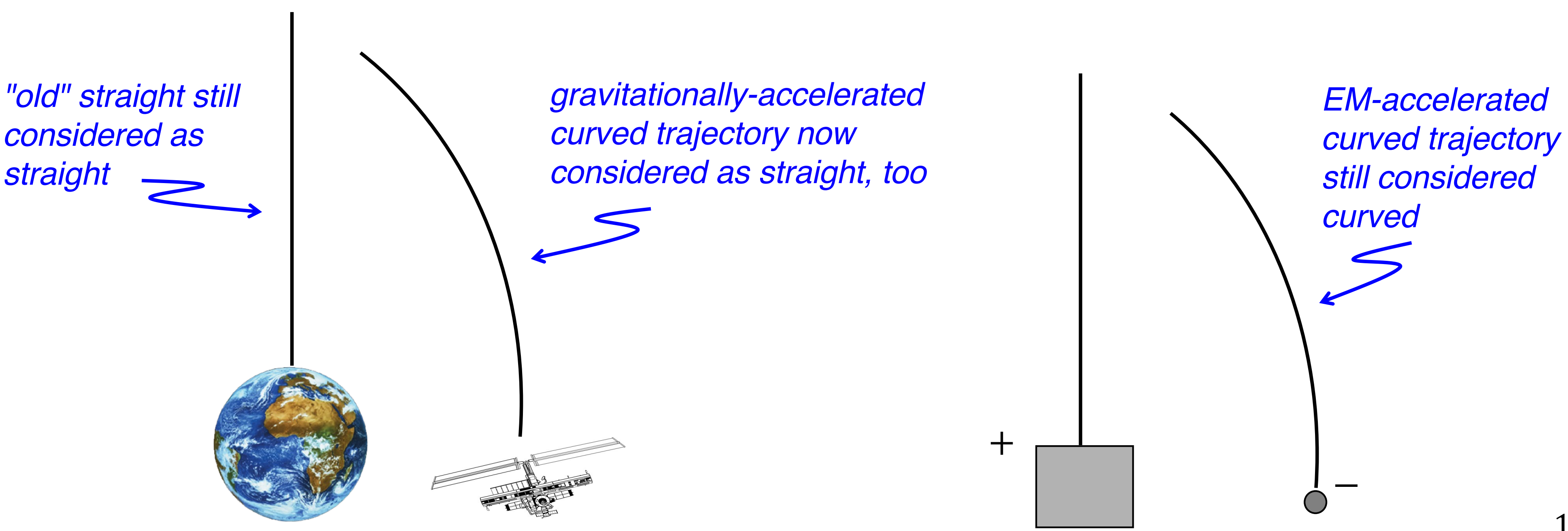
$$ma = 0$$

vs.

curved trajectories;
forces present

$$\left(\frac{d^2x}{dt^2} + \partial\Phi\right) \neq 0$$

$$ma = F$$



Consequences of Geometrizing Gravity

1. Inertial reference frames (defined by the families of straight trajectories in spacetime) now include objects at rest, in constant motion, *or* gravitationally accelerating.

2. *Gravitationally-induced* acceleration is thus relative (in exactly the same way that position and velocity are relative): Whether or not you are gravitationally accelerating depends on your frame of reference.

3. All *other* types of acceleration are still absolute: Whether or not you are non-gravitationally-accelerating is independent of your frame of reference (such accelerations always come "packaged" with attendant forces).

Interpretation of Geometrizing Gravity

(a) Under a *substantivalist interpretation*: The gravitational field is no longer a physical field that exists *in* spacetime; rather it is now part of the curvature of spacetime itself.

- *We've demoted the status of the gravitational field from physics to geometry.*

(b) Under a *relationist interpretation*: The metric field is physically real and just is what was previously called the gravitational field.

- *We've promoted the status of the metric field from geometry to physics.*

- Both interpretations agree that the *structure* of spacetime is no longer flat, as in Special Relativity and Newtonian dynamics.
- They disagree over how spacetime structure manifests itself.
  - *A substantivalist says it's the structure of a real spacetime.*
  - *A relationalist says it's the structure of a real physical field (the metric field).*

2. The Conventionality of Geometry in GR

- Is it a matter of convention whether or not to geometrize the gravitational force?

(A) *Flat geometry & grav. force*

$$\frac{d^2x}{dt^2}m_i = -m_g\partial\Phi$$

(B) *Curved geometry & no grav. force*

$$\left(\frac{d^2x}{dt^2}m_i + m_g\partial\Phi\right) = 0$$

- Can the grav force be thought of as an undetectable deformation force?
  - *Present in the "simple" flat geometry, but absent in the complicated curved geometry.*

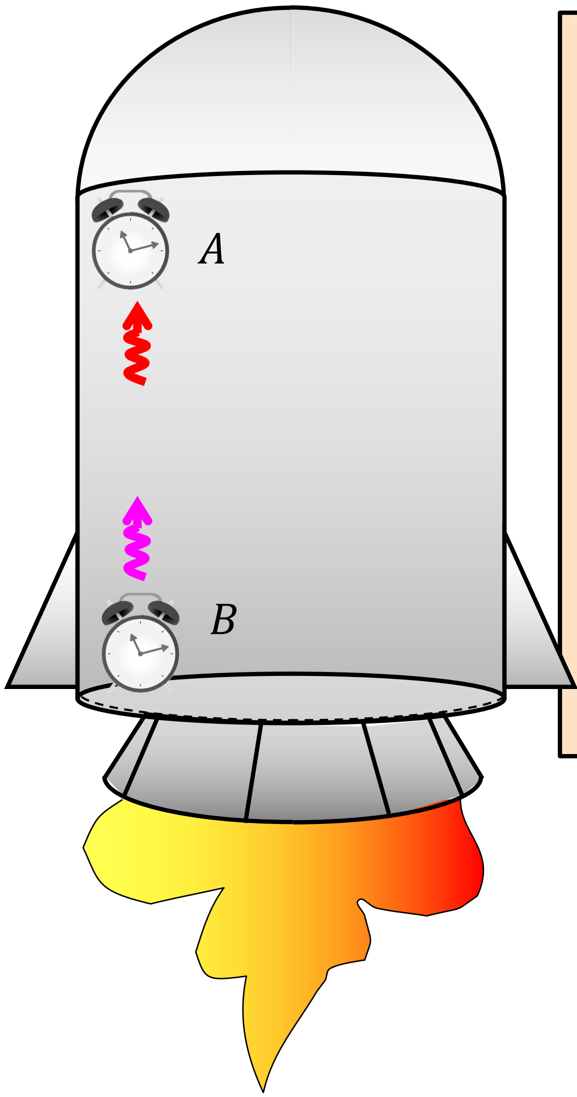
- <u>*Suppose*</u>: There is a *unique* split between *inertial structure* and *gravity* in GR.
  - *Suppose the contents of the parenthesis in* (B) *can always be written uniquely as two distinct terms).*

- <u>*Then*</u>: Since all observations indicate $m_g = m_i$, there would be no observational difference between (A) and (B).

- <u>*Realist Response*</u>: The curved geometry description is, arguably, much simplier.

One consequence of $m_g = m_i$: The gravitational redshifting of clocks.

Observations of clocks in a uniformly accelerating frame...



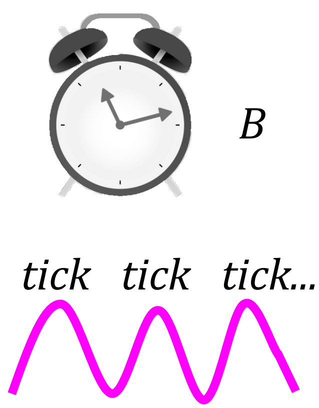
Claim: Clock B ticks slower than Clock A.

Why? To compare clocks, send light signals:

(i) Correlate *frequency* of a light signal with Clock B.

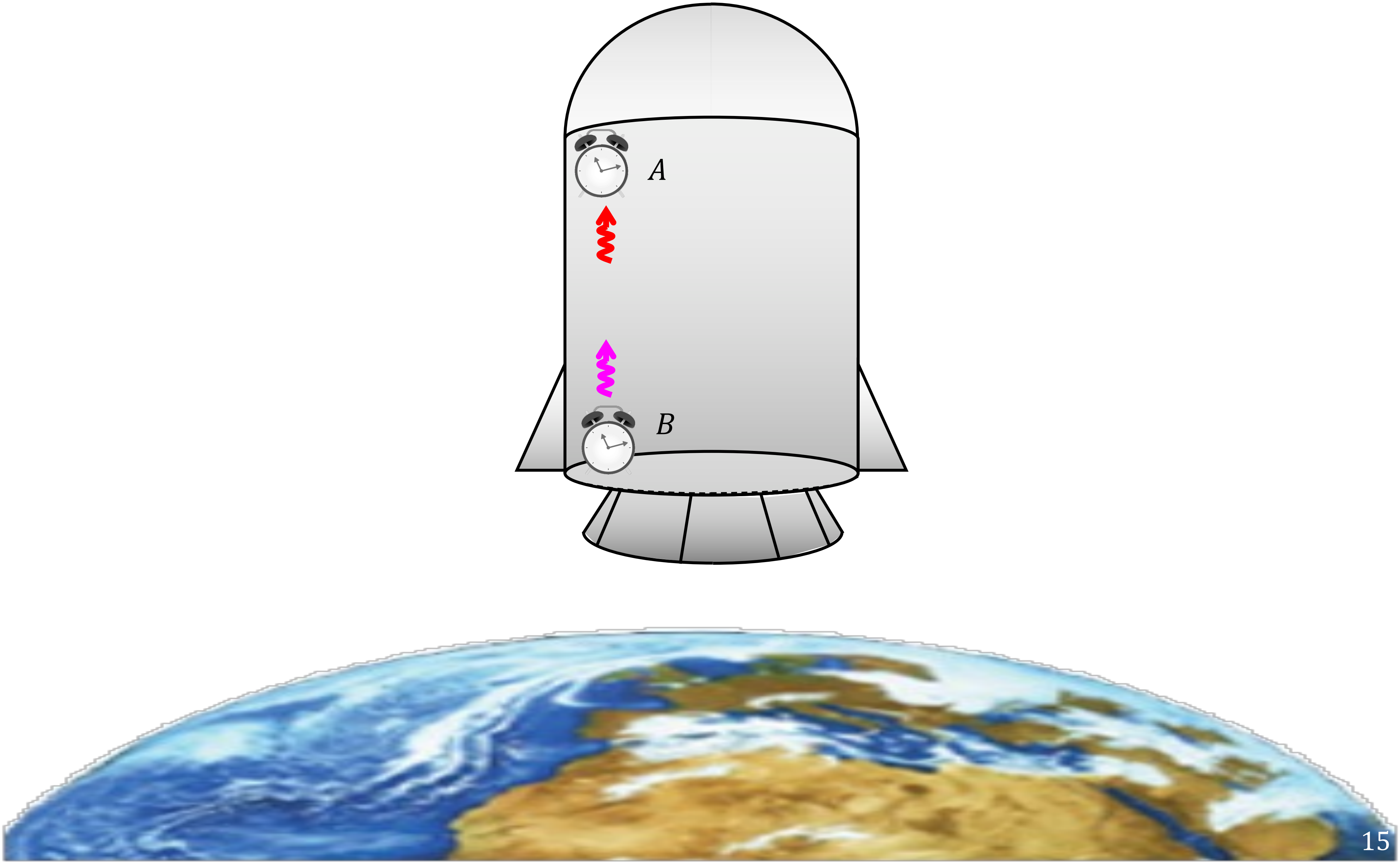
(ii) Send correlated light signal from B to A.

(iii) Since A is accelerating away from light signal, it will receive signal at *lower* frequency (*i.e.*, *shifted to the red*); hence A will measure B as ticking slower.



One consequence of $m_g = m_i$: The gravitational redshifting of clocks.

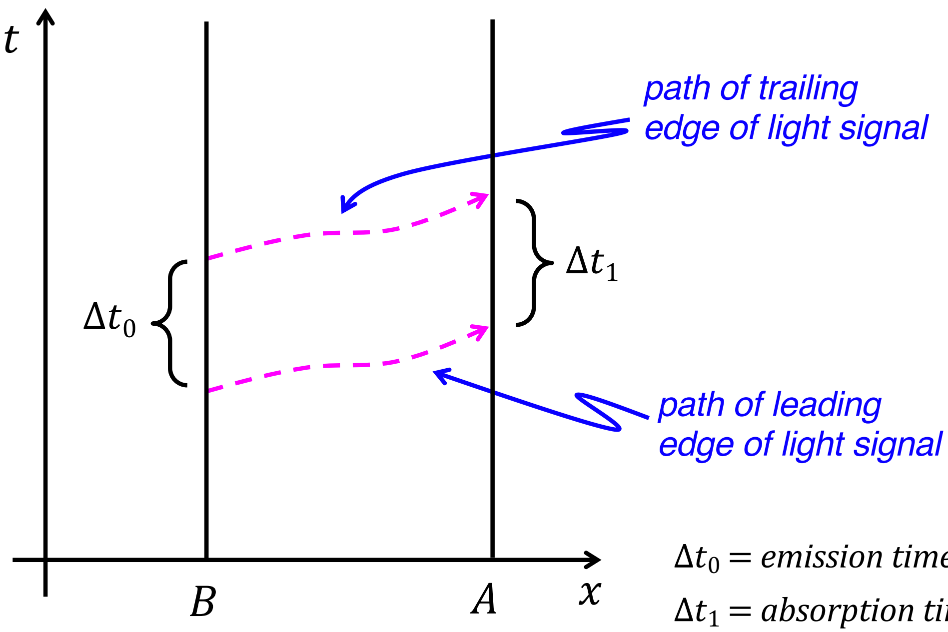
...should be indistinguishable from observations of clocks in a (homogeneous) gravitational field:



Prediction: Gravity slows clocks (gravitational "red-shift").

Experimental Evidence: 1956 Pound-Rebka experiment in tower at Jefferson Lab on Harvard campus.

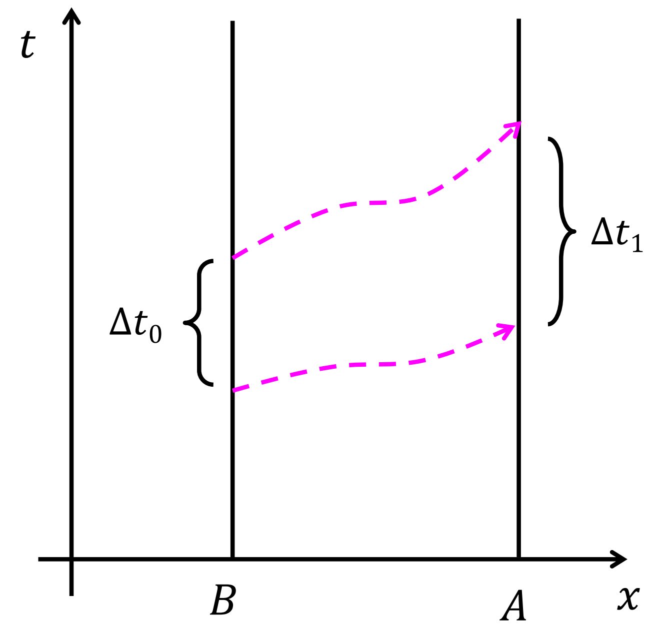
Two ways to explain the gravitational red-shifting of clocks



Flat Geometry

Δt_0 = *emission time for light signal*

Δt_1 = *absorption time for light signal*



Curved Geometry

- Experiments indicate $\Delta t_1 > \Delta t_0$.
- If spacetime is flat, we should have $\Delta t_1 = \Delta t_0$.
  - *The experimental result must be explained by claiming that gravity affects clocks in a way different from its effects on other objects (so it doesn't affect all things in the same way).*
- If spacetime is curved, then the result is explained by the fact that the paths taken by leading and trailing edges of light signal are not "parallel".
  - *The experimental result is explained without reference to a force acting on clocks in a way different from how it acts on other things.*
  - *Rather, we can say that gravity, as spacetime curvature, affects all objects in the same way.*

<u>BUT</u>: There isn't a unique split between inertial structure and gravity in GR.

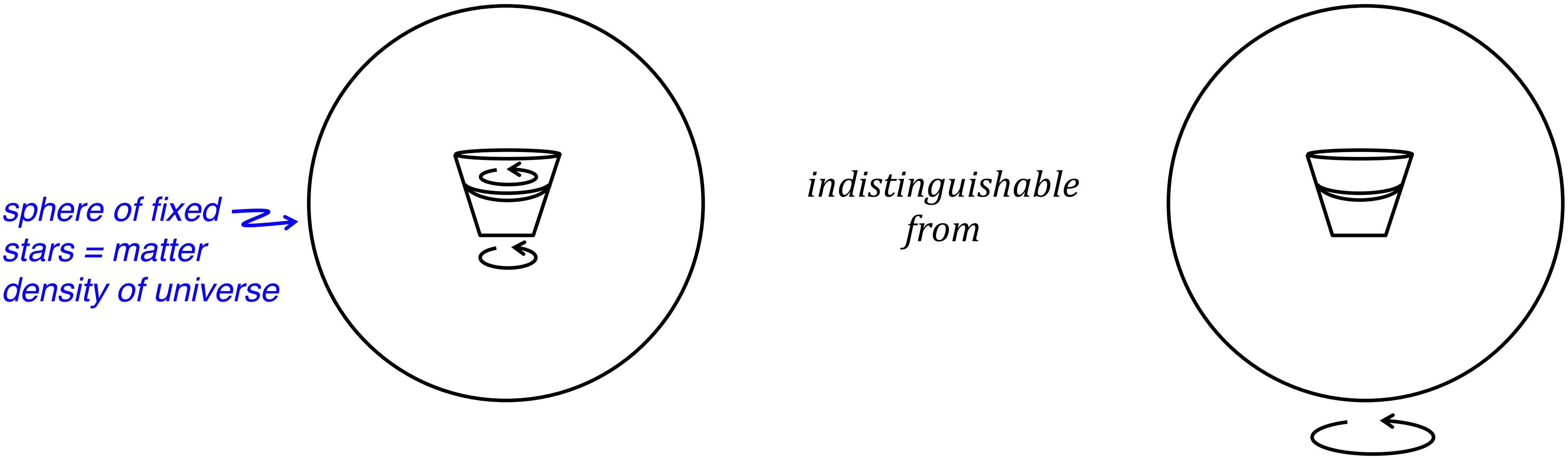
- Under a standard condition (that the connection be symmetric), the term schematically represented by $(d^2x/dt^2 + \partial\Phi)$ cannot be split into an inertial part d^2x/dt^2 and a gravitational part $\partial\Phi$.
- *<u>So</u>*: Under this standard condition, geometry is *not* conventional in GR!

<u>Suppose we relax this standard condition.</u>

- We get a theory ("teleparallel gravity") that looks like GR and in which a "split" between (what looks like) inertial structure and (what looks like) gravitation can be achieved.
- *<u>But</u>*: The verdict is still out on whether this is an equivalent way of formulating GR, or whether it counts as an entirely different theory!

3. Mach's Principle and GR

Recall Mach: Water is rotating with respect to the "fixed stars" in Newton's Bucket.



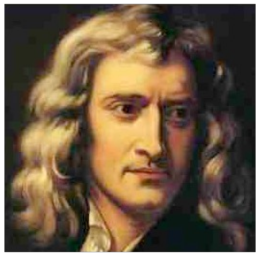
Water rotating; fixed stars at rest. *Water at rest; fixed stars rotating.*

> *Mach's Principle*: The *matter density* in the universe is the cause of inertial forces on objects undergoing non-inertial motion

- *Details? How* does the matter density of the universe cause inertial forces?
  - *Mach provides no explanation.*
  - *Einstein thinks general relativity supplies the explanation!*

In GR: The structure of spacetime...
...*determines* the inertial frames of reference (i.e., the families of straights).
...*is determined by* the matter density.

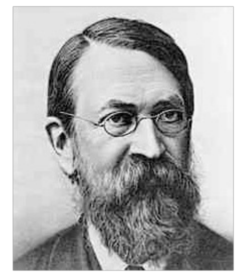
- *Newton's substantivalist (Einstein's interpretation):*



The structure of spacetime is the cause of inertial forces on accelerating objects.

- *Mach's relationalist:*

The matter density in the universe is the cause of inertial forces on accelerating objects.



- *In GR*: The matter density in the universe determines the structure of spacetime, which then determines the inertial frames of reference.
- Is "determining the inertial frames of reference" the same as "being the cause of inertial forces"?

Does GR agree with Newton's substantivalist or Mach's relationalist?

Three Questions of Interpretation

<u>(1) *Does the GR account support substantivalism or relationalism?*</u>

Depends on how you interpret the "structure of spacetime":

(a) <u>*A substantivalist may say*</u>: "The structure of spacetime is given by properties of real spacetime points."

- *Take all physical fields out of the universe and real spacetime would be left.*

(b) <u>*A relationalist may say*</u>: "The structure of spacetime is given by properties of the metric field, which is a real physical field."

- *Take all physical fields out of the universe and nothing would be left.*

$$G_{\mu\nu}(g_{\mu\nu}) = \kappa T_{\mu\nu}$$

metric field *matter fields*

<u>*Should the metric field also be considered a matter field?*</u>

Substantivalist: No!

Relationalist: Yes!

(2) *Does the GR account support Mach's Principle?*

Depends on how you interpret what matter is!

- In GR, there are *"vacuum" solutions* to the Einstein equations.
  - *Non-flat solutions in which the matter density is zero:* $T_{\mu\nu} = 0$
  - *"Gravitational waves" with no sources.*

$$G_{\mu\nu} = \kappa T_{\mu\nu} = 0$$

Doesn't necessarily
mean zero curvature!

(a) *A substanativalist may say*: "In GR, there can be inertial forces (as experienced by gravitational waves) in a universe *devoid of matter!*

- *So Mach's Principle does not hold in general."*

(b) *A relationalist may say*: "In vacuum solutions, the inertial forces are still determined by a matter field; namely, the metric field!

- *Moreover, such 'vacuum' solutions don't really describe universes devoid of matter; what they describe are universes in which the only matter field is the metric field!*
- *So Mach's Principle does hold in general!"*

(3) *Do vacuum solutions support substantivalism or relationalism?*

(a) ***A substantivalist may say***: "This supports my view: Gravitational waves are propagations of spacetime itself.

(b) ***A relationalist may respond***: "This supports my view: Gravitational waves are propagations in the metric field."

"One hundred years after Albert Einstein predicted the existence of gravitational waves, scientists have finally spotted these elusive ripples in space-time. In a highly anticipated announcement, physicists with the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) revealed on 11 February that their twin detectors have heard the gravitational 'ringing' produced by the collision of two black holes about 400 megaparsects (1.3 billion light-years) from Earth."
Casteivecchi & Witze (2016) 'Eintein's Gravitational Waves Found at Last', *Nature News*.

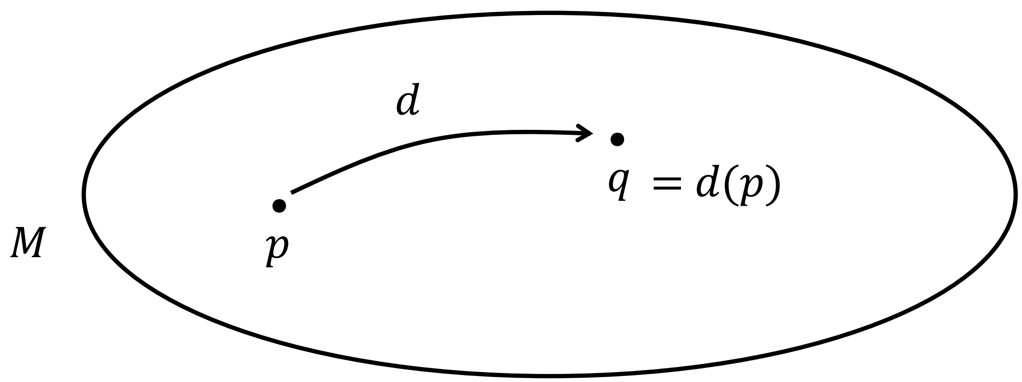
4. Leibniz Shifts in GR: The Hole Argument

- *Manifold Substantivalism*: The 4-dim collection of points (a *manifold*) of a general relativistic spacetime represents real substantival spacetime points.
- *Claim (Hole Argument)*: If we adopt a manifold substantivalist interpretation of GR, then we have to conclude that GR is *indeterministic*!

Symmetries and Equations of Motion

Symmetries = transformations that leave equations of motion unchanged.

- Newton's equations: Symmetries = Galilean transformations
- Maxwell's equations: Symmetries = Lorentz transformations
- Einstein equations: Symmetries = "diffeomorphisms"

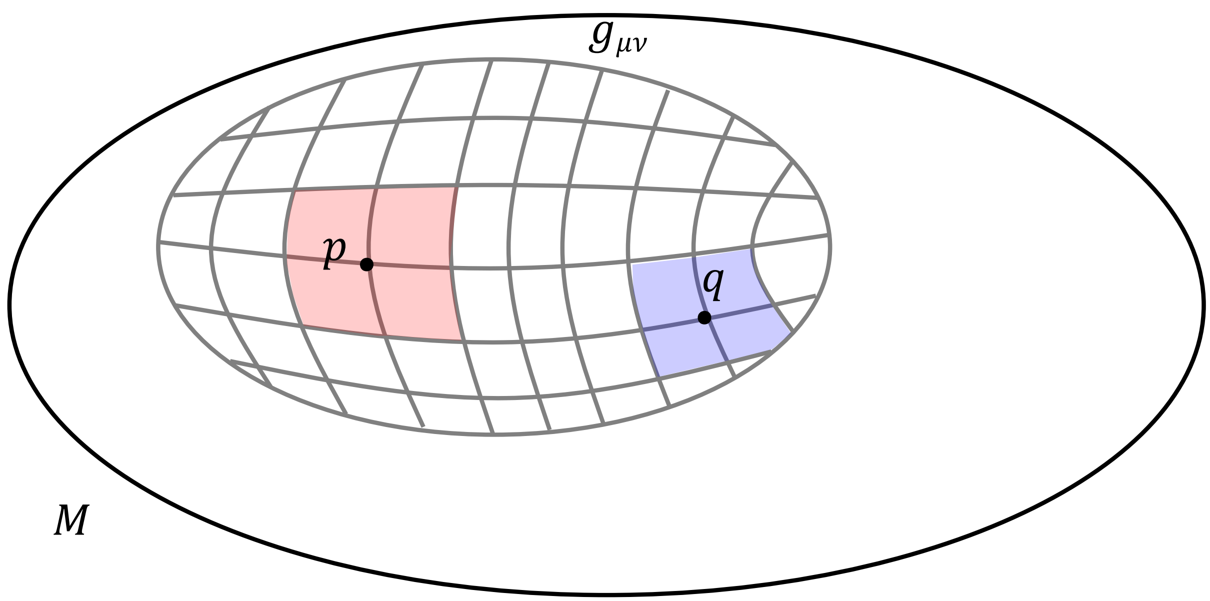


diffeomorphism = transformation between points on a manifold
= *arbitrary* coordinate transformation

<u>What this means:</u>

- Let $d{*}g_{\mu\nu}$ be what you get when you act with d on $g_{\mu\nu}$.
- *<u>Then</u>*: If $G_{\mu\nu}(g_{\mu\nu}) = \kappa T_{\mu\nu}$, then $G_{\mu\nu}(d{*}g_{\mu\nu}) = \kappa T_{\mu\nu}$.
  - *If $g_{\mu\nu}$ is a solution to the Einstein equations with matter distribution $T_{\mu\nu}$, then so is $d{*}g_{\mu\nu}$.*

- $d{*}g_{\mu\nu}$ is obtained from $g_{\mu\nu}$ by "shifting" it to a different set of points.



What this means:

- Let $d{*}g_{\mu\nu}$ be what you get when you act with d on $g_{\mu\nu}$.
- *Then*: If $G_{\mu\nu}(g_{\mu\nu}) = \kappa T_{\mu\nu}$, then $G_{\mu\nu}(d{*}g_{\mu\nu}) = \kappa T_{\mu\nu}$.
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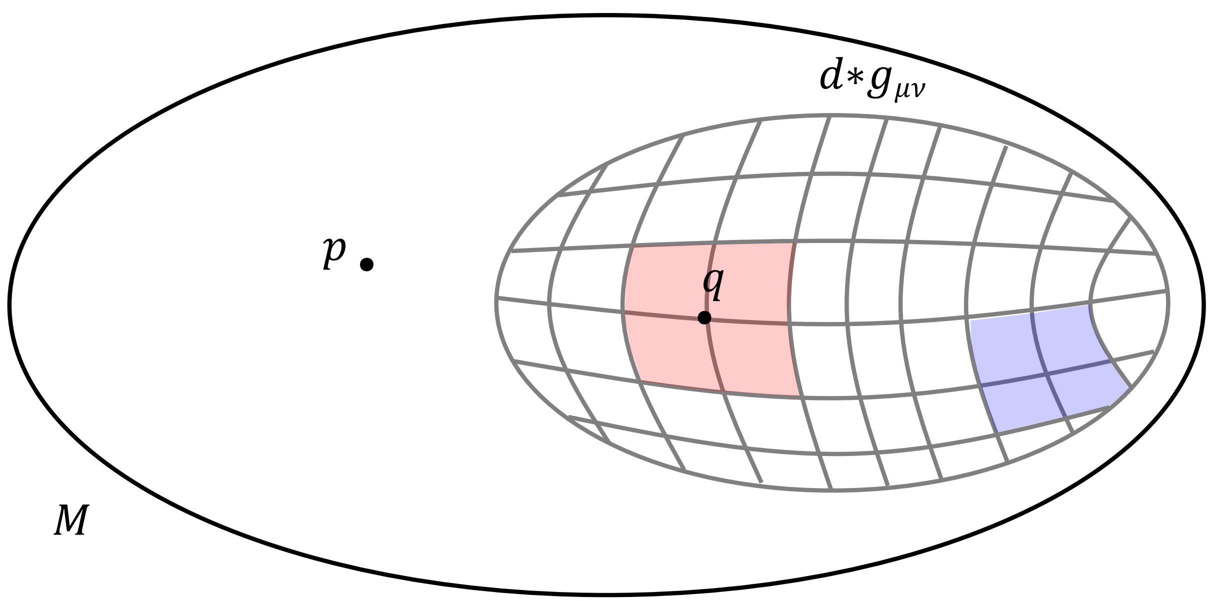
- $d{*}g_{\mu\nu}$ is obtained from $g_{\mu\nu}$ by "shifting" it to a different set of points.

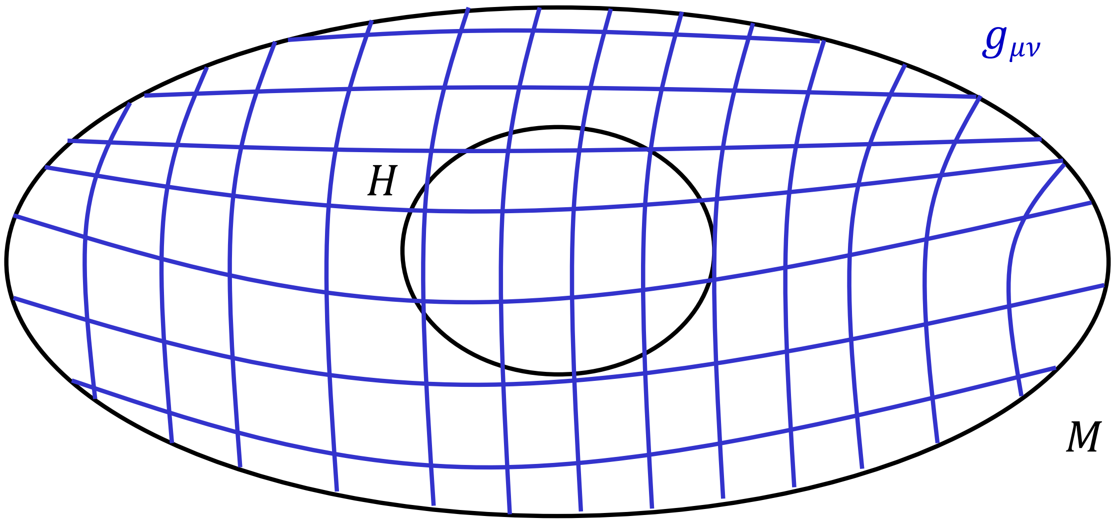


- *Recall*: Leibniz proposed static and kinematic shifts.
  - *The appropriate type of shift in GR is a shift-by-a-diffeomorphism!*

<u>Now: Construct a "hole" diffeomorphism h such that:</u>

(1) $h =$ identity outside a region H (the "hole") of M.

(2) $h \neq$ identity inside H.



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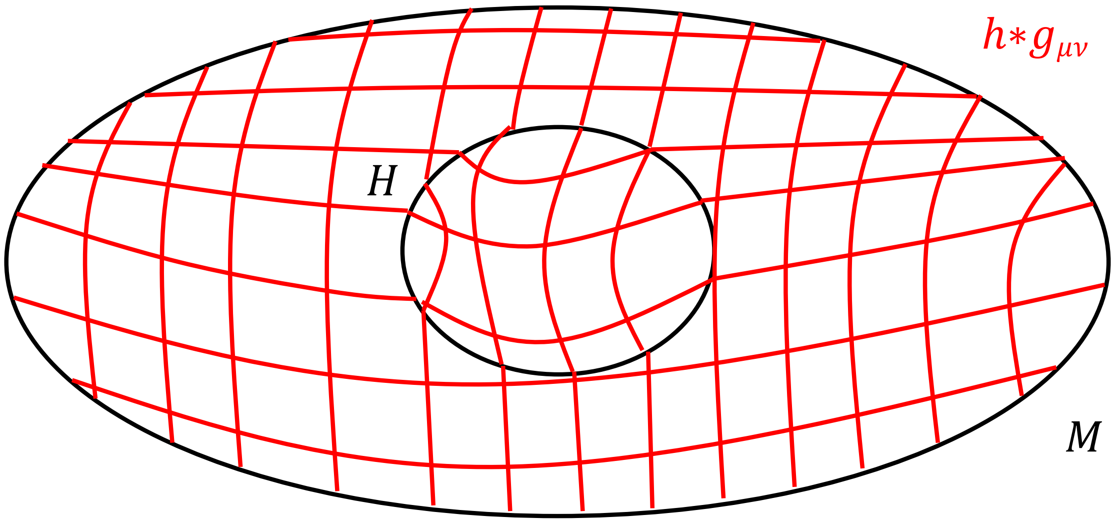
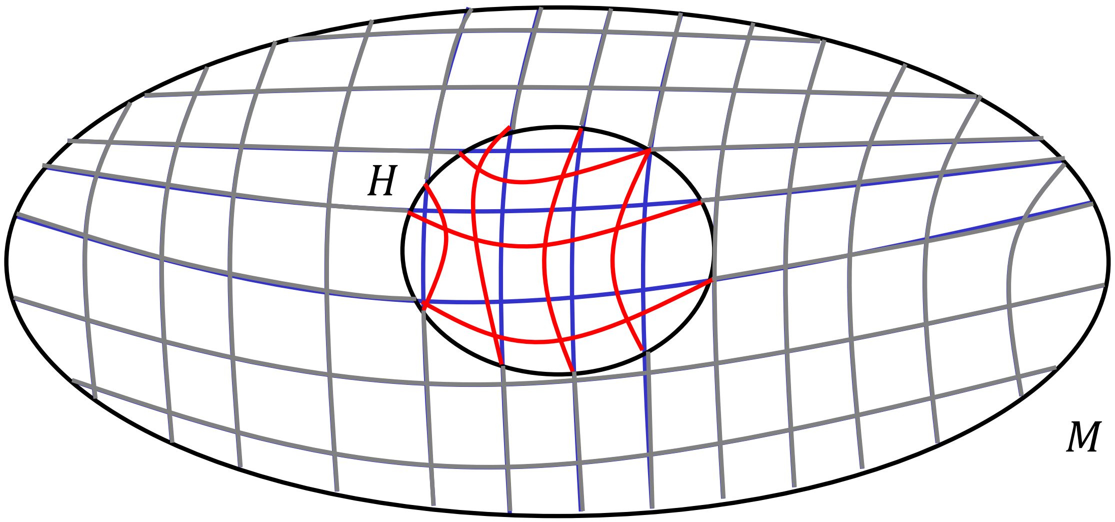


Now: *Construct a "hole" diffeomorphism h such that:*

(1) $h =$ identity outside a region H (the "hole") of M.

(2) $h \neq$ identity inside H.



- $g_{\mu\nu}$ and $h{*}g_{\mu\nu}$ disagree inside H.
- $g_{\mu\nu}$ and $h{*}g_{\mu\nu}$ agree outside H.
- h shifts the metric $g_{\mu\nu}$ only in the hole.

- Manifold substantivalists must claim that $g_{\mu\nu}$ and $h{*}g_{\mu\nu}$ describe different states of affairs.
- *But*: $g_{\mu\nu}$ and $h{*}g_{\mu\nu}$ are *physically indistinguishable* (both are solutions for the same matter distribution).
- *So*: Manifold substantivalists must conclude that the Einstein equations are *indeterministic*.

A complete specification of the matter distribution outside the hole fails to uniquely determine the metric inside the hole.

Some Options:

1. Adopt a relationist interpretation of GR.
   - *Since $g_{\mu\nu}$ and $h{*}g_{\mu\nu}$ describe the same spacetime relations between objects, and differ only on what points they are spread over, a relationist will claim they are not distinct: they represent the same state of affairs.*

2. Modify your spacetime substantivalism.
   - *Claim that spacetime points (or regions) are real, but this doesn't necessarily mean $g_{\mu\nu}$ and $h{*}g_{\mu\nu}$ describe distinct states of affairs.*
   - *Maybe spacetime points obtain their "identities" in strange ways.*
   - *Maybe they obtain them only after a field has been "spread" over them, and not before.*

3. Modify your spacetime realism.
   - Claim that spacetime structure can be thought of as real without having to additionally claim that spacetime points are real (or that manifold regions are real).

Non-Trivial options: they influence how you might attempt to reconcile GR with quantum theory!