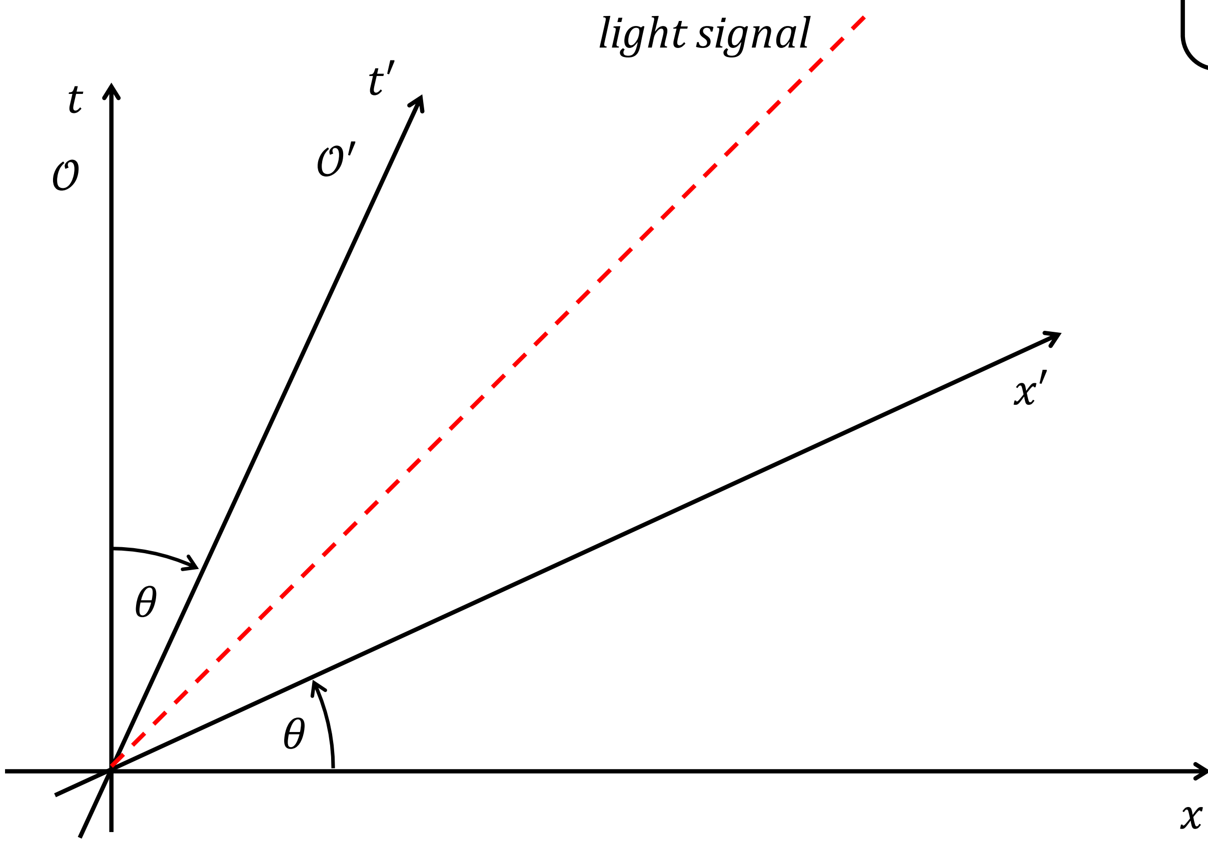
15b. Minkowski Spacetime

1. The Relativity of Simultaneity

Principle of Relativity and Light Postulate entail:

The speed of light c is the same in all inertial reference frames.



$$c = \frac{\Delta x}{\Delta t} = \frac{\Delta x'}{\Delta t'}$$

value of c for $\mathcal{O}$ = *value of c for* $\mathcal{O}'$

- $\mathcal{O}'$ is moving at constant velocity with respect to O.
- $\mathcal{O}$ and $\mathcal{O}'$ must measure same speed c for light signal.
- <u>*So*</u>: $\mathcal{O}$ and $\mathcal{O}'$ must disagree on spatial and temporal measurements!

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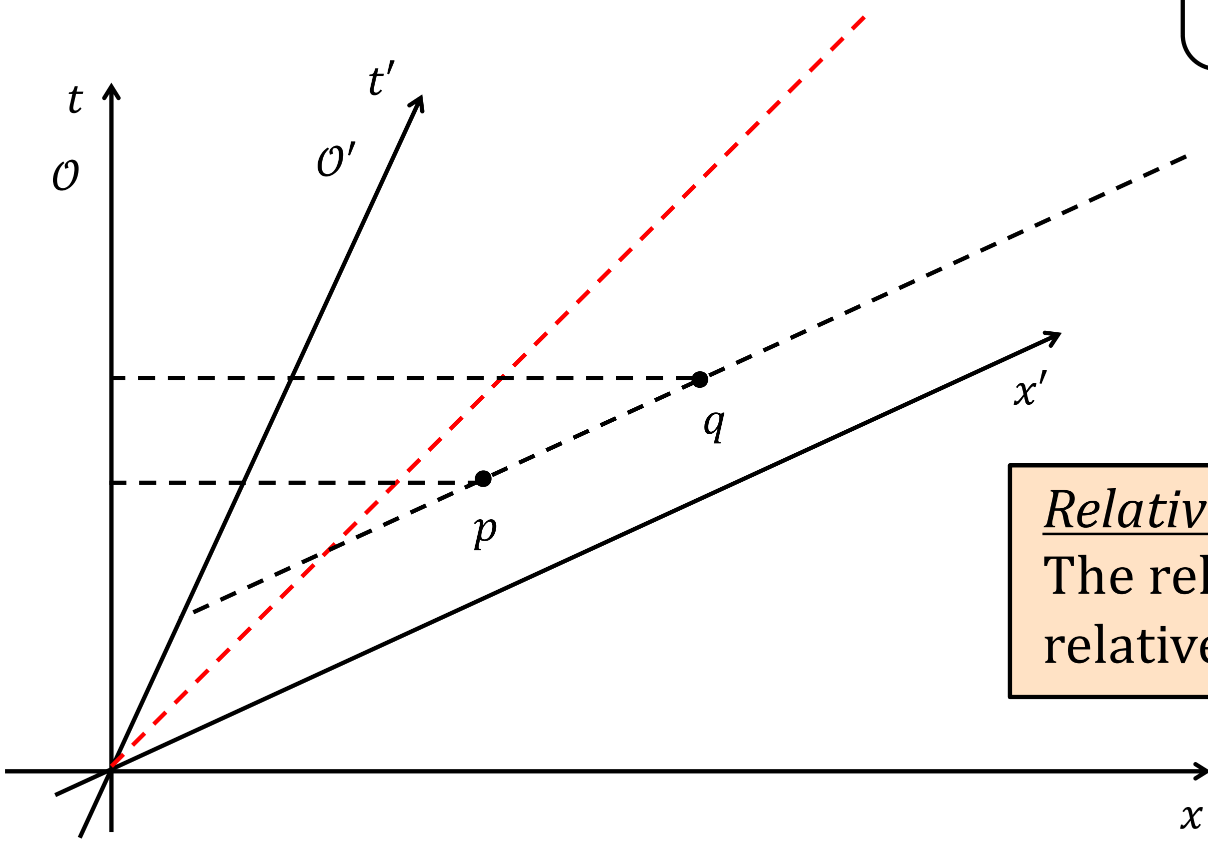


Relativity of Simultaneity
The relation of simultaneity is relative to inertial reference frames.

- $\mathcal{O}$ and $\mathcal{O}'$ make different judgements of simultaneity.
- p and q are simultaneous according to $\mathcal{O}'$.
- p happens before q according to $\mathcal{O}$.

2. Minkowski Spacetime

Hermann Minkowski (1864-1909)

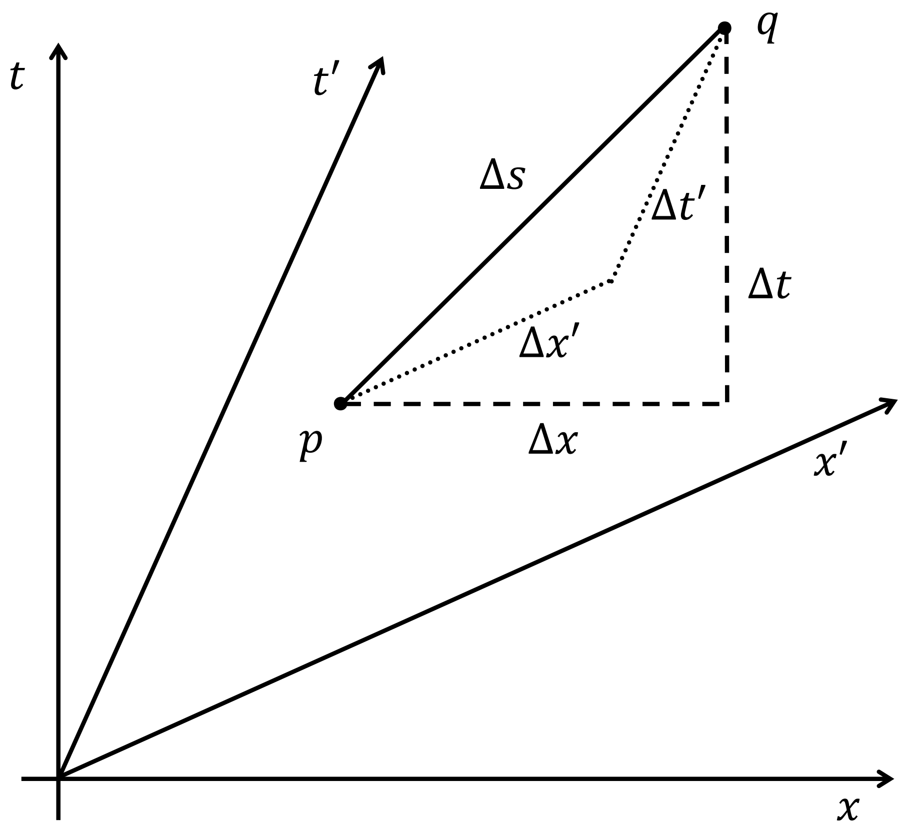
Spacetime of Special Relativity = Minkowski spacetime

Minkowski spacetime is a 4-dim collection of points such that between any two points p, q with coordinates (t, x, y, z) and $(t+\Delta t, x+\Delta x, y+\Delta y, z+\Delta z)$, there is a definite spacetime interval given by

$$\Delta s = \sqrt{-(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

- Similar to Euclidean *spatial* interval $\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$.
- *But*: Includes the time coordinate difference, too! And it's *negative*!

- *Idea*: All inertial frames agree on the *spatiotemporal* distance Δs between any points p and q.
- But they disagree on how Δs gets split into a *temporal* part and a *spatial* part: They disagree on measurements of time and measurements of space.



$$\Delta s = \sqrt{-(c\Delta t)^2 + (\Delta x)^2}$$
$$= \sqrt{-(c\Delta t')^2 + (\Delta x')^2}$$

- All inertial frames agree on the *spatiotemporal* distance Δs between any two points p and q.
- They disagree on the *temporal* distance between p and q (time dilation) and on the *spatial* distance (length contraction).
- They disagree on how they split Δs into temporal and spatial parts.

The Minkowski spacetime interval is encoded in the Minkowski metric $\eta_{\mu\nu}$

$$(\Delta s)^2 = \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu} \Delta x^\mu \Delta x^\nu$$

$$= \eta_{00}\Delta x^0 \Delta x^0 + \eta_{01}\Delta x^0 \Delta x^1 + \cdots + \eta_{33}\Delta x^3 \Delta x^3$$

$$= -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

$\Delta x^0 = c\Delta t$, $\Delta x^1 = \Delta x$,

$\Delta x^2 = \Delta y$, $\Delta x^3 = \Delta z$,

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Infinitesimally: $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$

<u>Three forms of $(\Delta s)^2$</u>

(a) Timelike. $(\Delta s)^2 < 0$, or: $\dfrac{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}{\Delta t} < c$

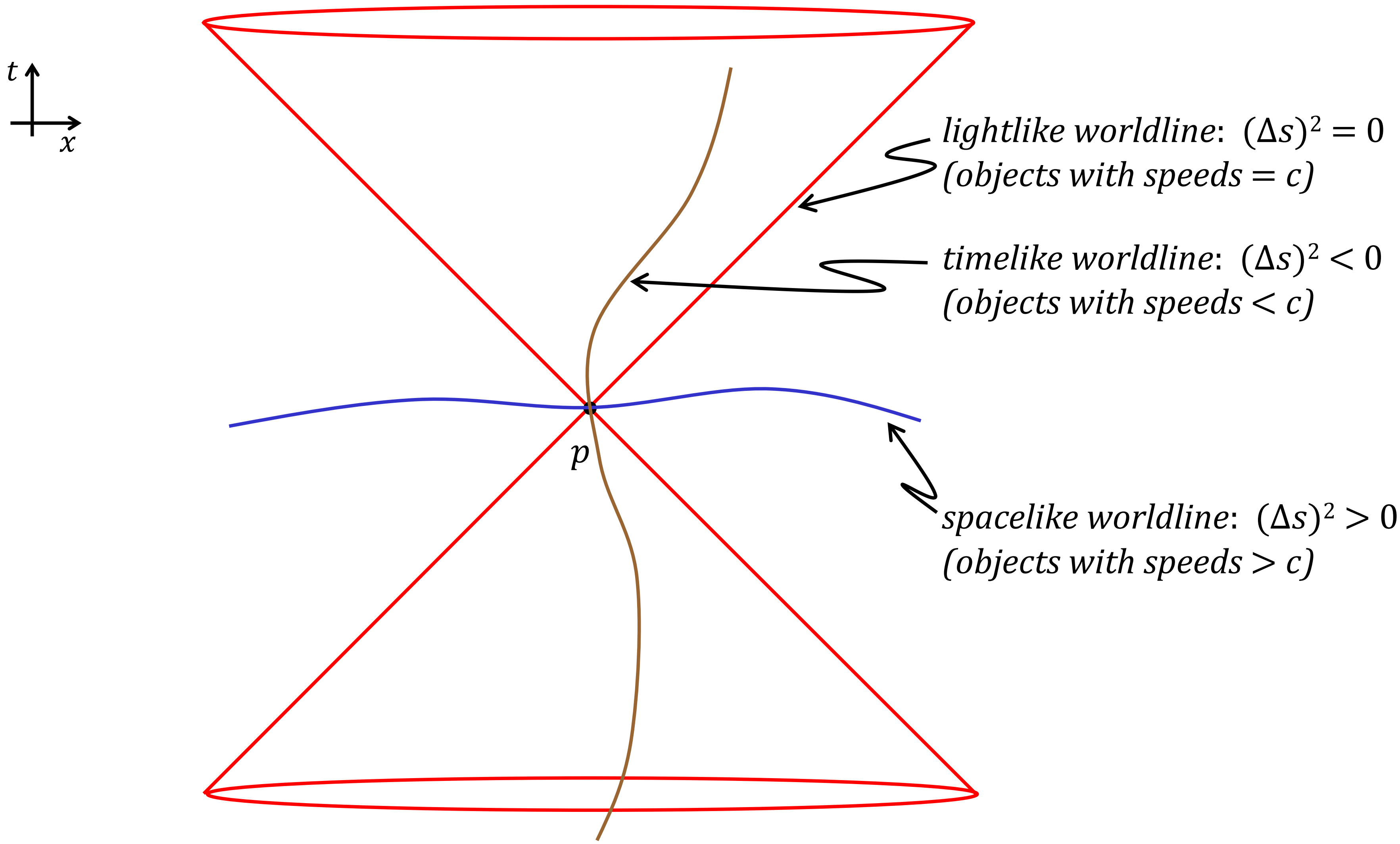
(b) Lightlike. $(\Delta s)^2 = 0$, or: $\dfrac{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}{\Delta t} = c$

(c) Spacelike: $(\Delta s)^2 > 0$, or: $\dfrac{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}{\Delta t} > c$

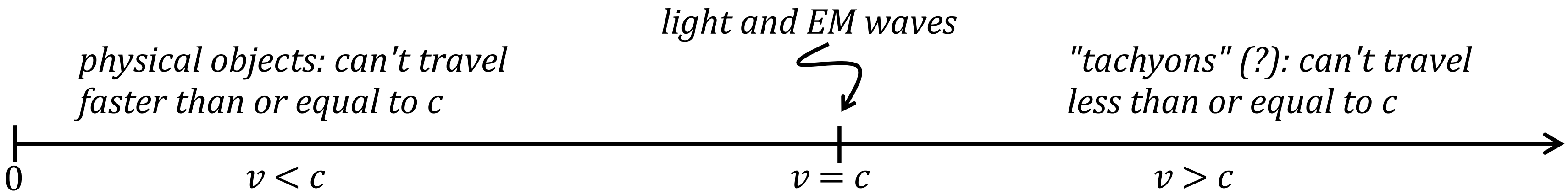
Three different types of worldline in Minkowski spacetime!

- *<u>Absolute distinction</u>*: All inertial frames agree on Δs, so all inertial frames agree on which worldlines are timelike, lightlike, and spacelike!

Hence: The Minkowski metric defines a *lightcone* at any point p.



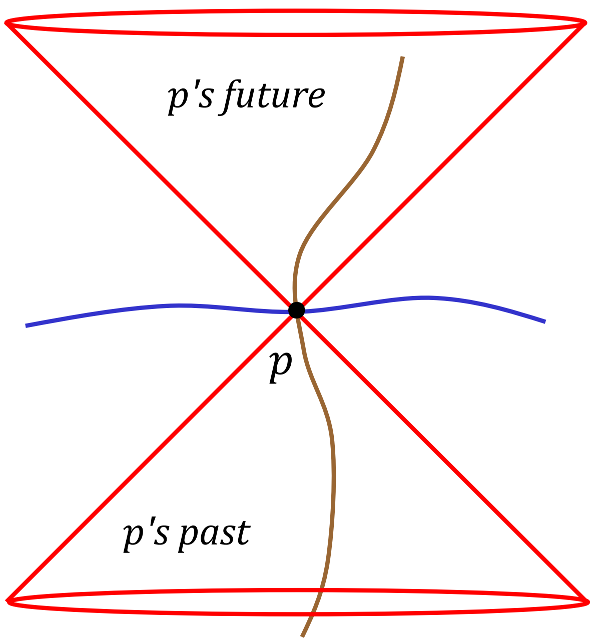
Claim: The distinction between lightlike, timelike, and spacelike worldlines with respect to any point p is *mutually exclusive*.



<u>Claim A</u>: An object traveling at $v < c$ with respect to an inertial frame cannot travel at $v \geq c$ with respect to any other inertial frame.

<u>Claim B</u>: An object traveling at $v = c$ with respect to an inertial frame cannot travel at $v > c$ or $v < c$ with respect to any other inertial frame.

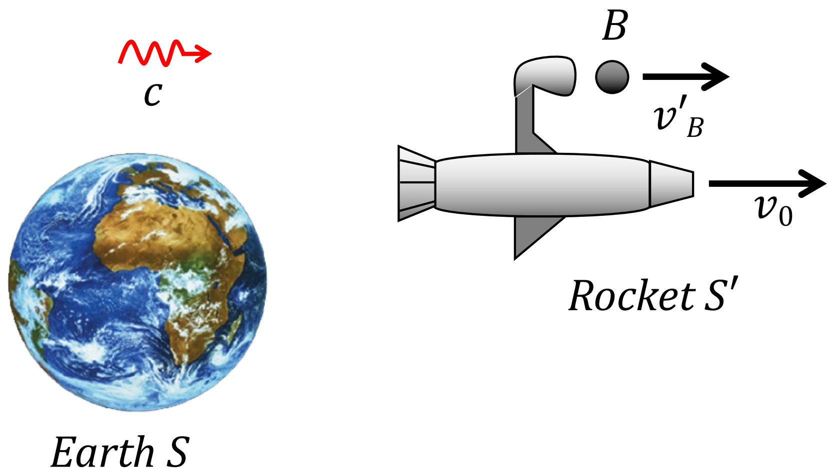
<u>Claim C</u>: An object traveling at $v > c$ with respect to an inertial frame cannot travel at $v \leq c$ with respect to any other inertial frame.



<u>Lightcone at p splits spacetime into 4 regions</u>:

1. Events in p's forward lightcone (future of p).
2. Events in p's backward lightcone (past of p).
3. Events on p's lightcone.
4. Events outside p's lightcone.

Claim A: An object traveling at $v < c$ with respect to an inertial frame cannot travel at $v \geq c$ with respect to any other inertial frame.

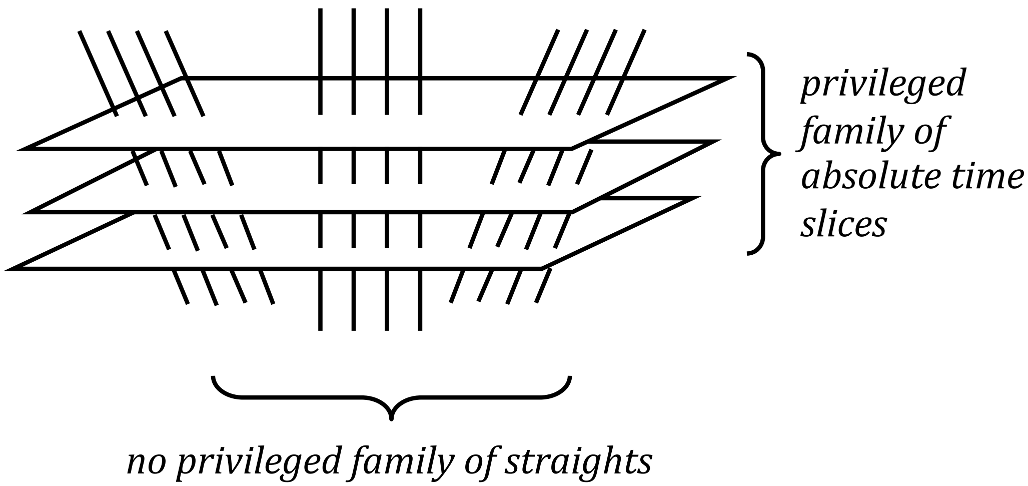


v_0 = speed of S' with respect to S
v'_B = speed of B with respect to S'
v_B = speed of B with respect to S
c = speed of light signal

Proof:

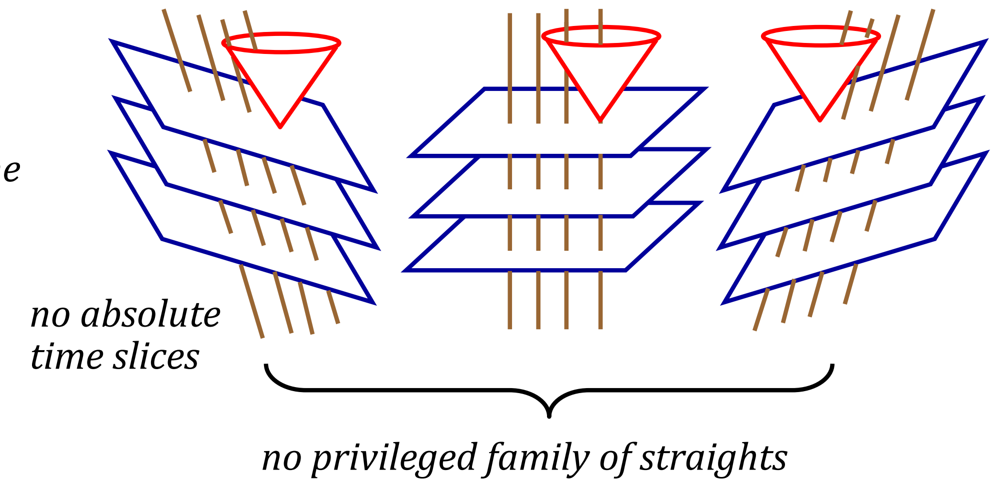
- *Given*: $v'_B < c$
- *Suppose*: $v_B \geq c$
  - *Now*: S and S' measure same speed c for light signal (*Light Postulate*).
  - *But*: S' observes light signal overtaking B.
  - *And*: S observes B pacing ($v_B = c$) or overtaking ($v_B > c$) light signal.
  - *So*: S and S' are observationally distinct: *Violation of Principle of Relativity*.
- *Thus*: If $v'_B < c$, then it cannot be that $v_B \geq c$.

Galilean Spacetime



1. Many inertial frames; none privileged.
2. Velocity is relative.
3. Acceleration is absolute.
4. Simultaneity is absolute.

Minkowski Spacetime



1. Many inertial frames; none privileged.
2. Velocity is relative.
3. Acceleration is absolute.
4. Simultaneity is relative.
5. Invariant light-cone structure at each point.

3. The Conventionality of Simultaneity

Hans Reichenbach
(1891-1953)

<u>*Claim*</u>: Given an event *A*, there is no objective fact of the matter as to what *distant* events at rest with respect to *A* are simultaneous with *A*. The choice is a matter of convention.

<u>*Relativity of simultaneity*</u> = *Different* inertial frames judge the simultaneity of events in different ways. (Entailed by the 2 Postulates.)

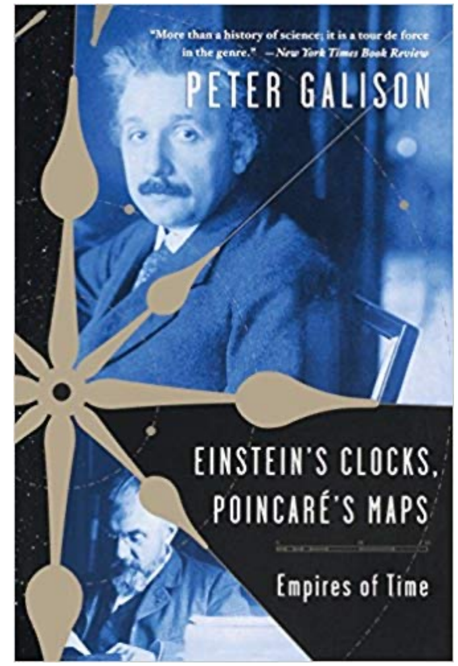
<u>*Conventionality of simultaneity*</u> = Within a *single* inertial frame, the simultaneity of *distant* events is not fixed and can be judged in different ways. (*Not* entailed by the 2 Postulates.)

- How can the simultaneity of distant events in the same inertial frame be established?
  - *Einstein (1905): By setting up synchronized clocks at these events.*

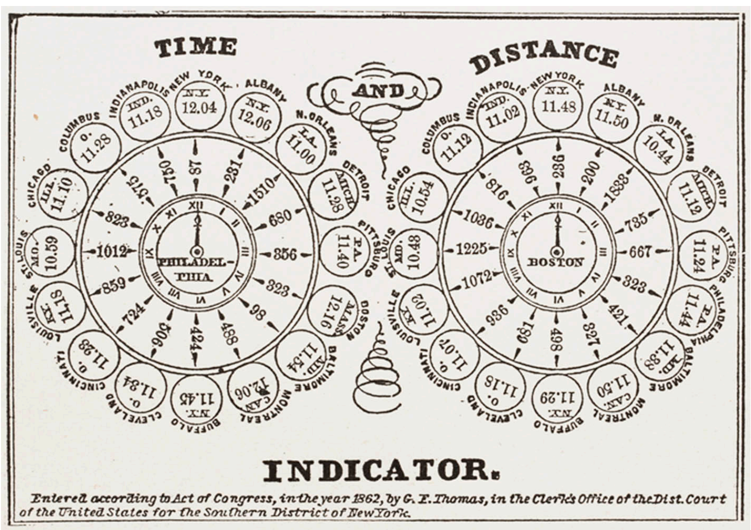
- How can distant clocks in the same inertial frame be synchronized?
  - *Einstein (1905): Use light signals.*

Aside: Why Einstein's focus on clock synchronization?

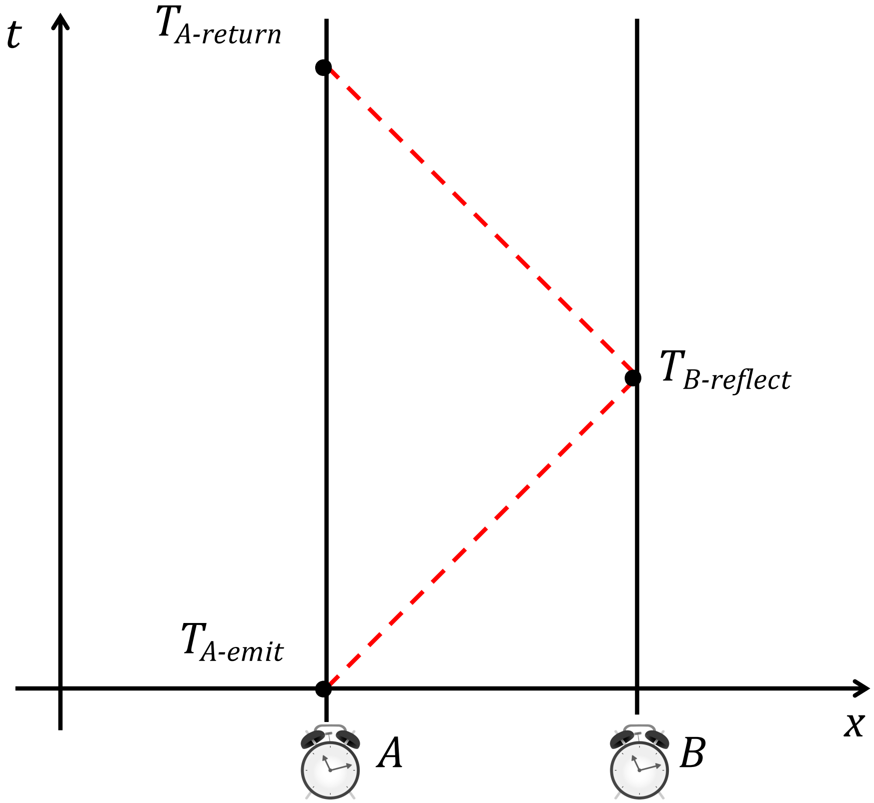
Answer: Clock synchronization was on the cutting edge of technology at the end of the 19th century:



- *Railway technology*: Needed highly accurate (synchronized) clocks for dependable, efficient service.
- *Electrification of clocks*: To synchronize clocks to "railway time", send electric signals from central clock.



- Galison (2003): Example of how technology drives theoretical advances.

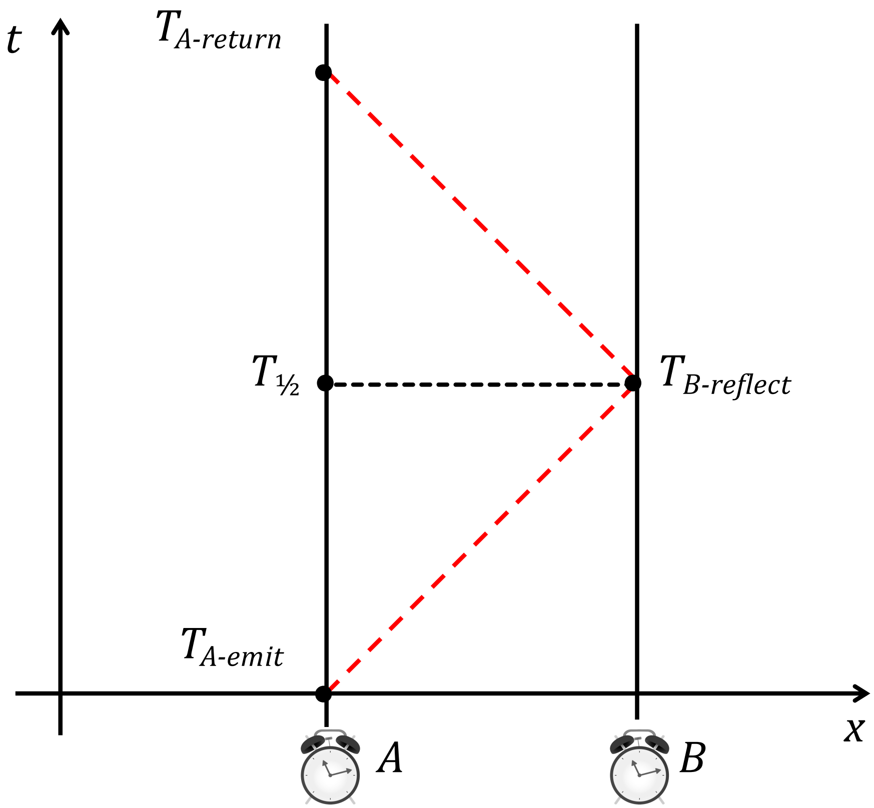


- To synchronize Clock B a given distance from Clock A,

  (1) Emit a light signal from A to B and record the time $T_{A\text{-}emit}$ on A.

  (2) Have B reflect the signal back to A. Record the time on B, $T_{B\text{-}reflect}$.

  (3) Record the time on A, $T_{A\text{-}return}$, when the light signal returns.



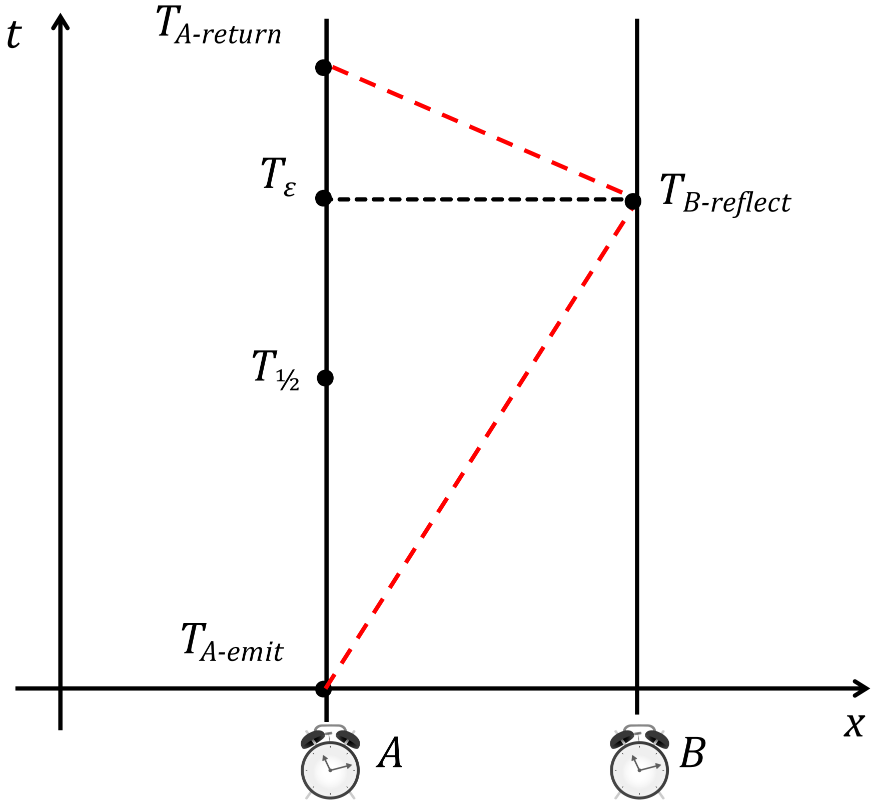
Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at $T_{1/2}$.

- ***Einstein's Stipulation***: *A* and *B* may be said to be in synchrony just when

  $T_{B\text{-}reflect} = T_{1/2} \equiv T_{A\text{-}emit} + \frac{1}{2}(T_{A\text{-}return} - T_{A\text{-}emit})$.

  - ***Assumption***: Light travels at the same speed c in all directions.



Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at $T_{1/2}$.

Non-Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at T_ε.

- *Einstein's Stipulation*: *A* and *B* may be said to be in synchrony just when

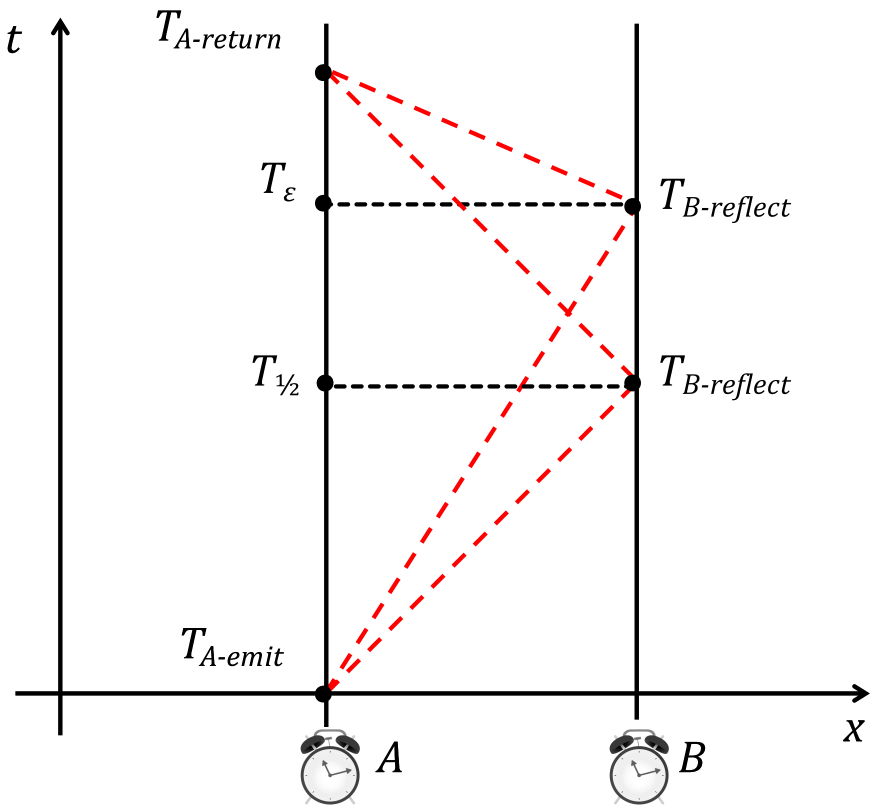
  $T_{B\text{-}reflect} = T_{1/2} \equiv T_{A\text{-}emit} + \frac{1}{2}(T_{A\text{-}return} - T_{A\text{-}emit})$.

  - *Assumption*: Light travels at the same speed *c* in all directions.

- *Reichenbach's Conventionalism*: *A* and *B* may be said to be in synchrony just when

  $T_{B\text{-}reflect} = T_\varepsilon \equiv T_{A\text{-}emit} + \varepsilon(T_{A\text{-}return} - T_{A\text{-}emit})$, for *any* value of ε, where $0 < \varepsilon < 1$.

  - *Assumption*: Light does *not* necessarily travel at the same speed *c* in all directions.



Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at $T_{½}$.

Non-Standard Simultaneity

The event at $T_{B\text{-}reflect}$ is simultaneous with the event at T_ε.

- Who's right: Einstein or Reichenbach?
  - *Does light travel at the same speed in all directions or not?*

How can the "one-way" speed of light be measured?

Reichenbach's Claim:

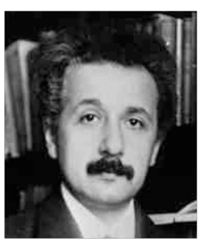
(a) To measure the one-way speed of light, we need synchronized clocks.

(b) But we can only synchronize our clocks if we have prior knowledge of distant simultaneity, which requires prior knowledge of the one-way speed of light.

Realist Response:

- Agree that there is no observational difference between the standard simultaneity relation and any non-standard simultaneity relation.
- *So*: If empirical adquacy (*i.e.*, agreement with observation) is the criterion for how one chooses between competing theories, then there's no reason to prefer the standard relation to any non-standard relation.
- *But*: Why think empirical adquacy is the only criterion of theory choice?

- Suppose *simplicity* is a criterion of theory choice.
- *Then*: We should prefer the standard simultaneity relation, since it assumes light travels at the same speed in all directions.
- *However*: Simplicity is a highly subjective concept...



Einstein

General relativity is much more simple than Newton's theory of gravity!

???

Average Joe

Realist Response:

- Agree that there is no observational difference between the standard simultaneity relation and any non-standard simultaneity relation.
- *So*: If empirical adquacy (*i.e.*, agreement with observation) is the criterion for how one chooses between competing theories, then there's no reason to prefer the standard relation to any non-standard relation.
- *But*: Why think empirical adquacy is the only criterion of theory choice?

> - Suppose *unifying power* is a criterion of theory choice (*i.e.*, we should choose that theory that fits better with other theories).
> - *Then*: We should prefer the standard simultaneity relation, since Friedman-Robertson-Walker spacetimes in general relativity (*i.e.*, "Big Bang" spacetimes) are isotropic in a way that singles out the standard definition.
> - *But*: Adopting such spacetimes as descriptions of our universe requires many assumptions, one of which *just* is isotropy.