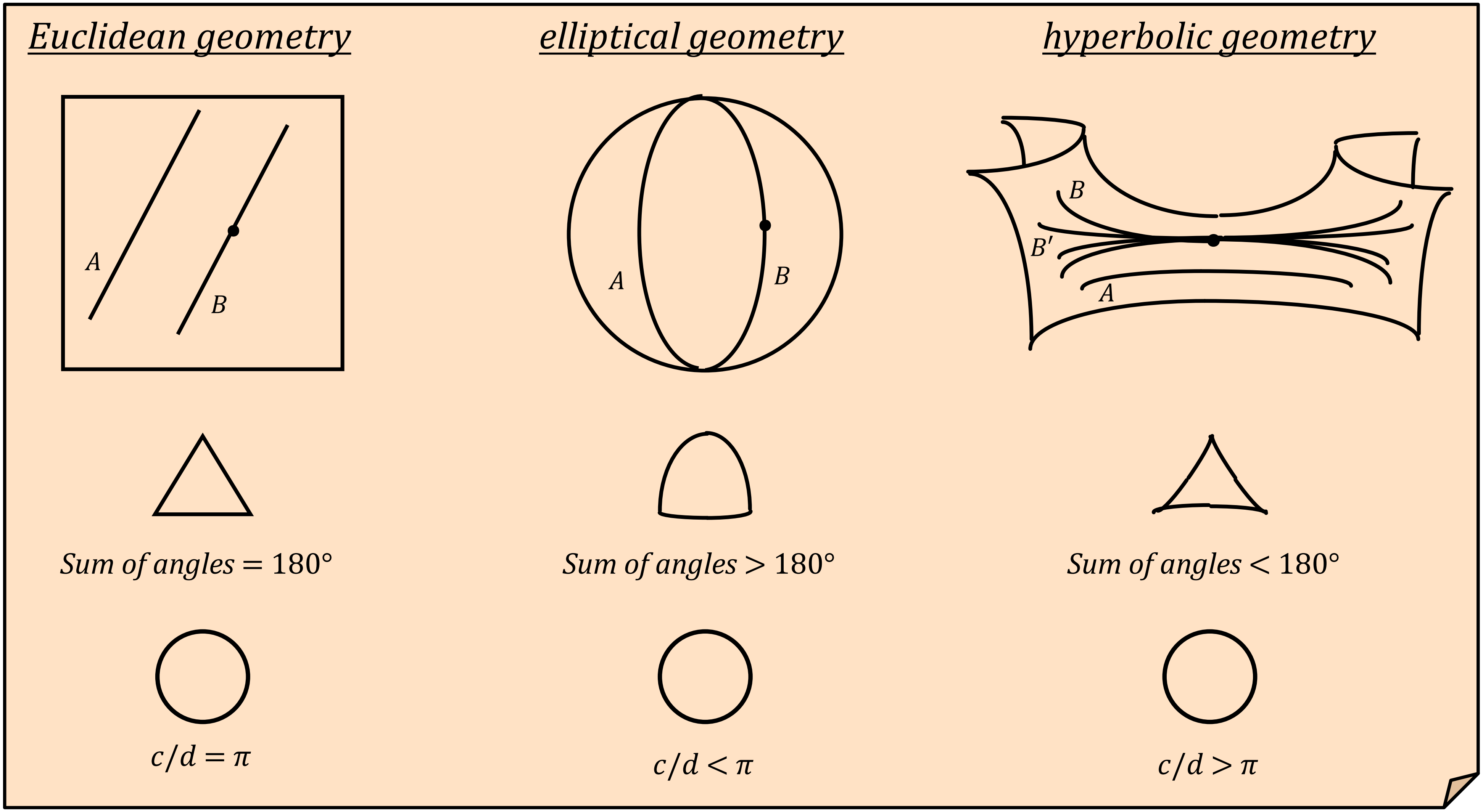
14. Poincaré and the Conventionality of Geometry

- *Recall:*



- All are *logically consistent* (all describe *possible* worlds).
- Which geometry describes the *actual* world?

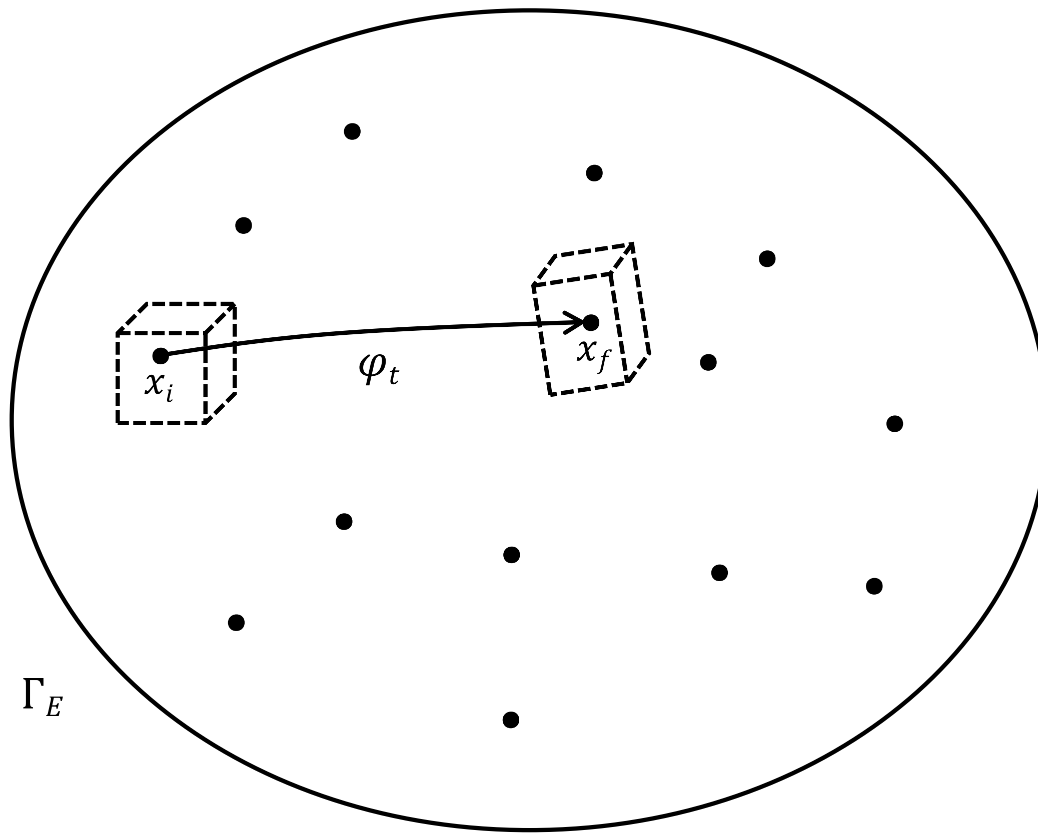
Aside: Poincaré's Recurrence Theorem (1890).

"On the 3-body Problem and the Equations of Dynamics"

Henri Poincaré
(1854-1912)

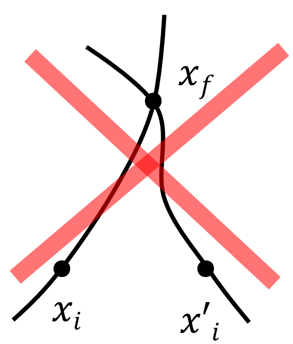
Theorem: For every *mechanical* system with a bounded phase space, *almost every* initial state of the system will, after some finite time, return to a state arbitrarily close to the initial state.

- *Consider*: Gas consisting of N particles governed by Hamilton's equations of motion.
- A *microstate* of the gas is a specification of the position (3 values) and velocity (3 values) for each of its N particles.
  - Γ = phase space = $6N$-dim space of all possible microstates.
  - Γ_E = region of Γ that consists of all microstates with constant energy E.
  - Hamilton's equations define a map $\varphi_t : \Gamma \to \Gamma$ that takes an initial state x_i in Γ_E to a unique final state x_f in Γ_E.
  - Key property: φ_t preserves volumes.



Informal Proof

- *We know*: Trajectories in phase space do not intersect (because φ_t is *deterministic*: any given state cannot have evolved from two separate initial states).
- *And*: Phase space volumes are preserved by φ_t.
- *So*: A tube swept out by φ_t from an initial region A can never cross regions it has already crossed.
- *And*: Assuming phase space is finite and φ_t preserves volumes, at some point, all of phase space will be swept out.
- *Thus*: In order to continue evolving, the tube must connect back to A (otherwise it would intersect a region it's already crossed).



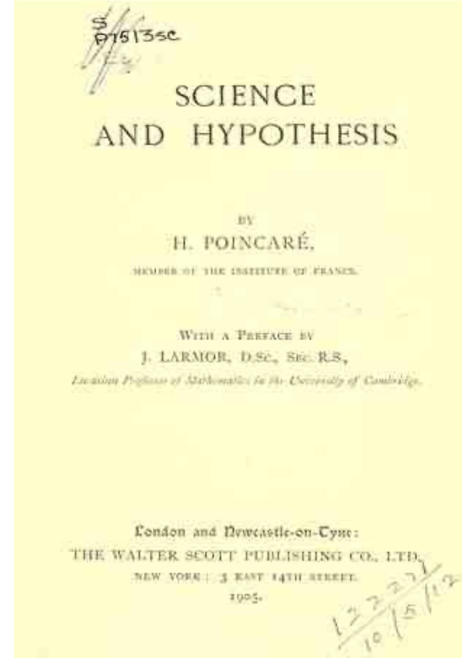
"A theorem, easy to prove, tells us that a bounded world, governed only by the laws of mechanics, will always pass through a state very close to its initial state. On the other hand, according to accepted experimental laws (if one attributes absolute validity to them, and if one is willing to press their consequences to the extreme), the universe tends toward a certain final state, from which it will never depart. In this final state, which will be a kind of death, all bodies will be at rest at the same temperature."

"I do not know if it has been remarked that the English kinetic theories can extricate themselves from this contradiction. The world, according to them, tends at first toward a state where it remains for a long time without apparent change; and this is consistent with experience; but it does not remain that way forever, if the theorem cited above is not violated; it merely stays there for an enormously long time, a time which is longer the more numerous are the molecules. This state will not be the final death of the universe, but a sort of slumber, from which it will awake after millions of millions of centuries. According to this theory, to see heat pass from a cold body to a warm one, it will not be necessary to have the acute vision, the intelligence, and the dexterity of Maxwell's demon; it will suffice to have a little patience."

- Poincare (1893) "Le mécanisme et l'expérience".

Conventionality of Geometry (1904).

"The object of geometry is the study of a particular 'group'; but the general concept of group pre-exists in our minds, at least potentially. It is imposed on us not as a form of our sensitiveness, but as a form of our understanding; only, from among all possible groups, we must choose one that will be the standard, so to speak, to which we shall refer natural phenomenon."



SCIENCE
AND HYPOTHESIS

BY
H. POINCARÉ,

WITH A PREFACE BY
J. LARMOR, D.Sc., Sec. R.S.,

London and Newcastle-on-Tyne:
THE WALTER SCOTT PUBLISHING CO., LTD.

Science and Hypothesis (1904)

"Experiment guides us in this choice, which it does not impose on us. It tells us not what is the truest, but what is the most convenient geometry."

<u>Conventionality of Geometry Thesis</u>

The choice of geometry to describe spatial relations in the world is conventional. There is no objective fact as to what the true geometry of the world is.

"If Lobatschewsky's geometry [hyperbolic] is true, the parallax of a very distant star will be finite. If Riemann's [elliptical] is true, it will be negative. These are the results which seem within the reach of experiment... But what we call a straight line in astronomy is simply the path of a ray of light. If, therefore, we were to discover negative parallaxes, or to prove that all parallaxes are higher than a certain limit, we should have a choice between two conclusions: we could give up Euclidean geometry, or modify the laws of optics, and suppose that light is not rigorously propagated in a straight line."

- *Claim*: Any description of physical spatial relations requires *both* a given geometry *G and* various physical theories *P* that govern measuring instruments:

$$(G \mathbin{\&} P) \Rightarrow \textit{observations}$$

- *So*: Any observed spatial relation can be described by *any* geometry *G*, provided we modify *P* appropriately.

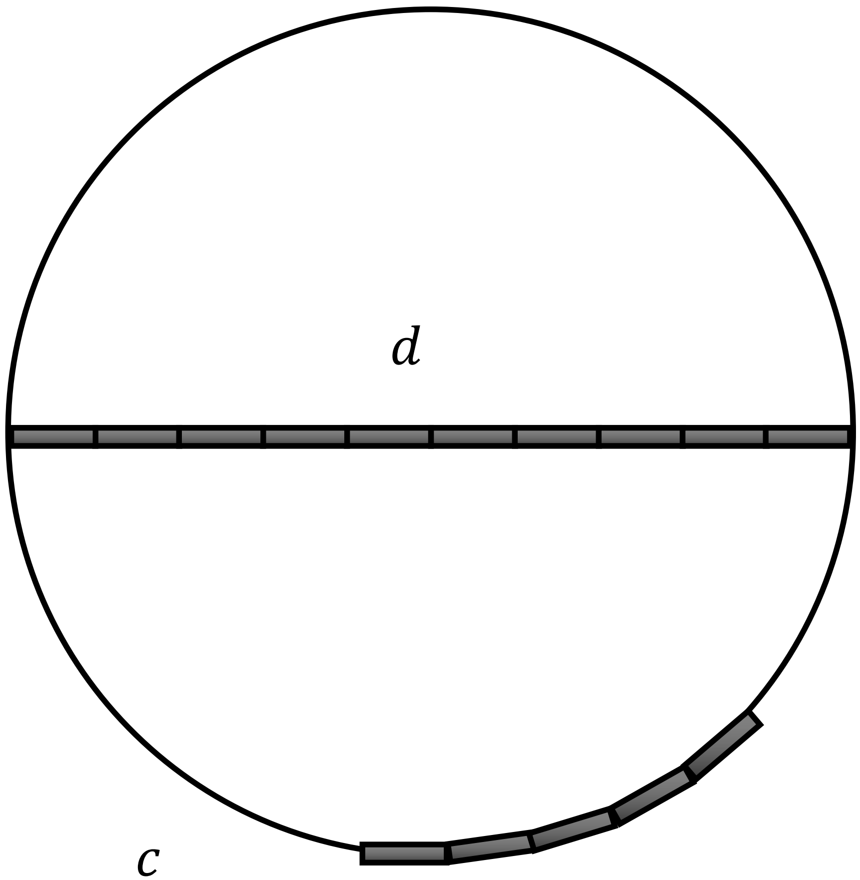
<u>Example</u>: Poincare's Non-Eulcidean world.

"Suppose, for example, a world enclosed in a large sphere and subject to the following laws: -- The temperature is not uniform; it is greatest at the centre, and gradually decreases as we move towards the circumference of the sphere, where it is absolute zero. The law of this temperature is as follows: -- If R be the radius of the sphere, and r the distance of the point considered from the centre, the absolute temperature will be proportional to $R^2 - r^2$. Further, I shall suppose that in this world all bodies have the same coefficient of dilatation, so that the linear dilatation of any body is proportional to its absolute temperature. Finally, I shall assume that a body transported from one point to another of different temperature is instantaneously in thermal equilibrium with its new environment."

"If [the inhabitants of this world] construct a geometry, it will not be like ours, which is the study of the movements of our invariable solids; it will be the study of the changes of position which they will have thus distinguished, and will be 'non-Euclidean displacements', and *this will be non-Euclidean geometry*. So that beings like ourselves, educated in such a world, will not have the same geometry as ours."

<u>*2-dim heated disk:*</u>

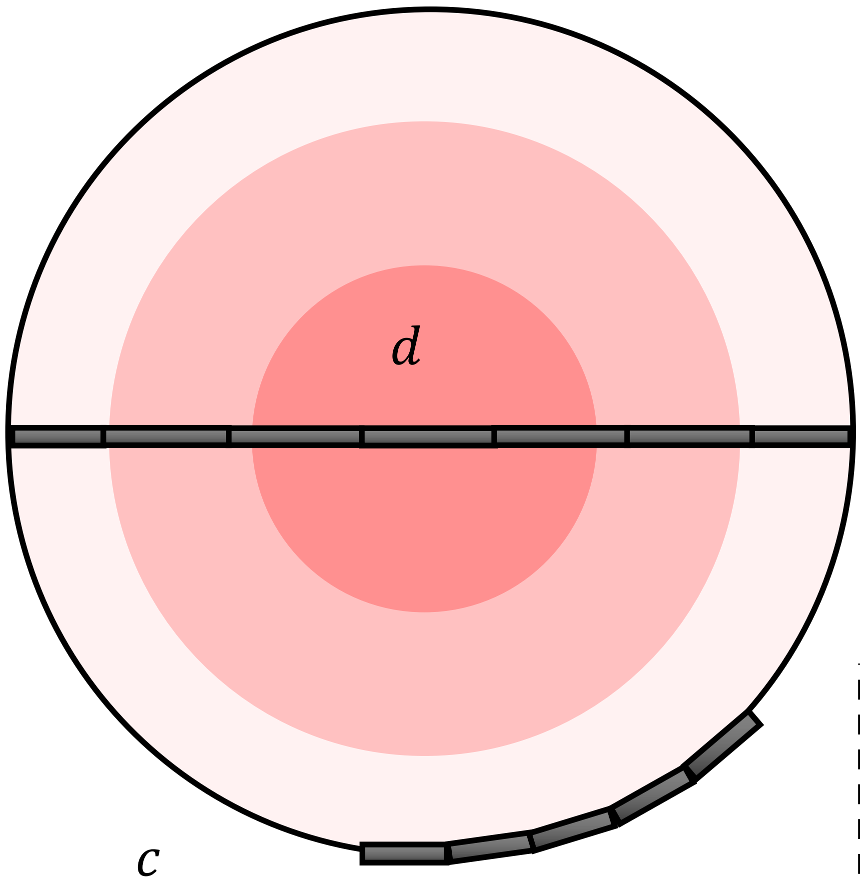
- Consider a physical disk in 2-dim Euclidean space.



- $c/d = \pi$
- Check with rigid measuring rod:
  - Place rod end-to-end across disk to measure d (*ex*., $d = 10$ units).
  - Place rod end-to-end around edge to measure c.

2-dim heated disk:

- Consider a physical disk in 2-dim Euclidean space.
- *Now*: Suppose disk is heated at center, with heat dissipating towards edges.
- *And*: Suppose measuring rods expand when heated.



- *Then*:
  - Diameter reading will be less than unheated disk (*ex*. $d = 7$ units).
  - Circumference reading will remain the same.
- *So*: $c/d > \pi$

- *Let R = disk radius, l_0 = length of rod at center.*
- *Let temperature $T(r)$ be a function of radial distance r from center: $T(r) = R^2 - r^2$.*
- *Let: Length of rod $l(r) = l_0 T(r)/R^2$.*
- *And: For any circle about center with radius $< R$, $c/d > \pi$.*

- Two possible explanations for the $c/d > \pi$ result for the heated disk:

(1) The rod is rigid, and the geometry of space is non-Euclidean (hyperbolic).

(2) The rod is not rigid (it experiences a deformation force), and the geometry of space is Euclidean.

- In the heated disk example, we know (2) is correct: we can detect the deformation force.
- *But*: Suppose the deformation force is *in-principle undetectable*.

Universal force = an in-principle undetectable deformation force.

- *Conventionalist's Claim*: Any deviation from Euclidean geometry can be explained by postulating the existence of a universal force that affects all physical objects in the same way.
  - *In general*: Any geometry can be used to describe physical spatial relations, given the postulation of appropriate universal forces.
- *Consequence*: We cannot be realists about the geometry of physical spatial relations.

Problem of Circularity for Realists With Respect to Spatial Geometry:

(a) We cannot know if universal forces are present without first knowing what the geometry of space is.

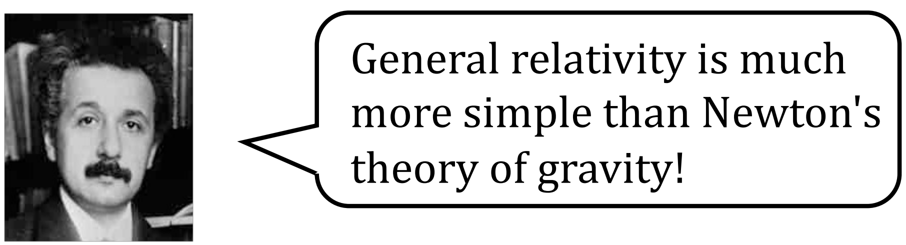
(b) We cannot know what the geometry of space is without first knowing whether universal forces are present.

<u>*Realist Response*</u>:

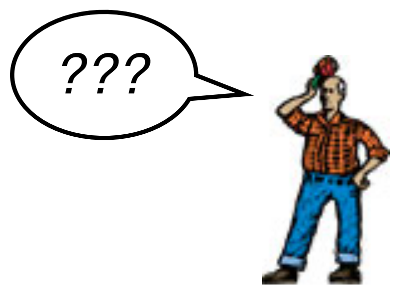
- Agree that there is no observational difference between a Euclidean world with no universal forces, and a non-Euclidean world with universal forces present.
  - *<u>So</u>: If empirical adequacy is the criterion for how one chooses between competing theories, then there's no reason to prefer Euclidean geometry over non-Euclidean geometry as a theory of space.*
  - *<u>But</u>: Why think empirical adqacy is the only criterion of theory choice?*

- Suppose *simplicity* is another criterion of theory choice.
  - *<u>Then</u>*: *We should chose Euclidean geometry to describe physical space, since doing so does not require the additional posit of universal forces.*

- *<u>However</u>*: Simplicity is a highly subjective concept...



Einstein



Average Joe