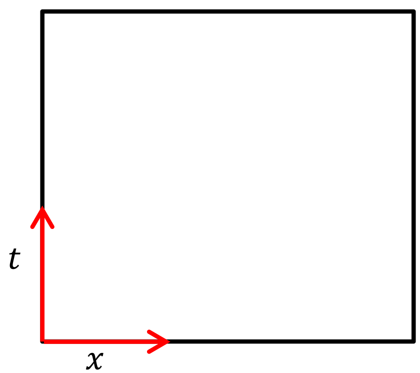
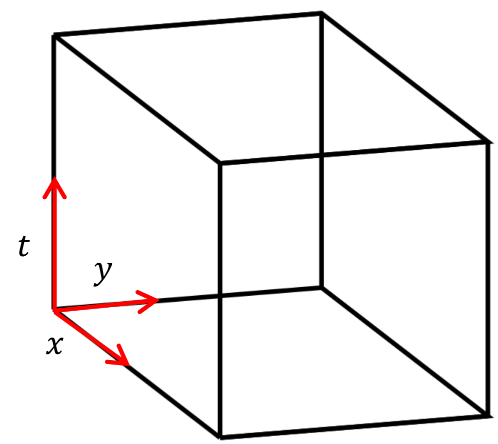
11. Spacetime

1. Types of Spacetimes

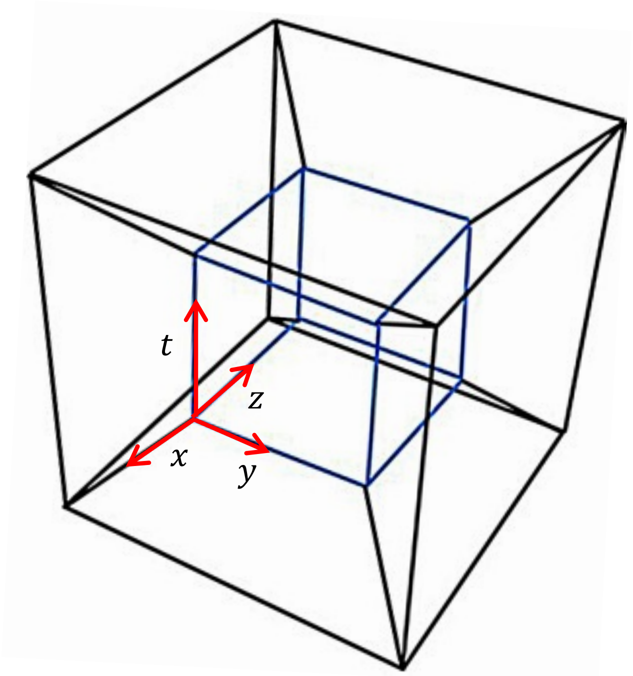
- A *spacetime* is a 4-dim collection of points with additional structure.



2-dim square



"shadow" (projection) of 3-dim cube in 2-dim



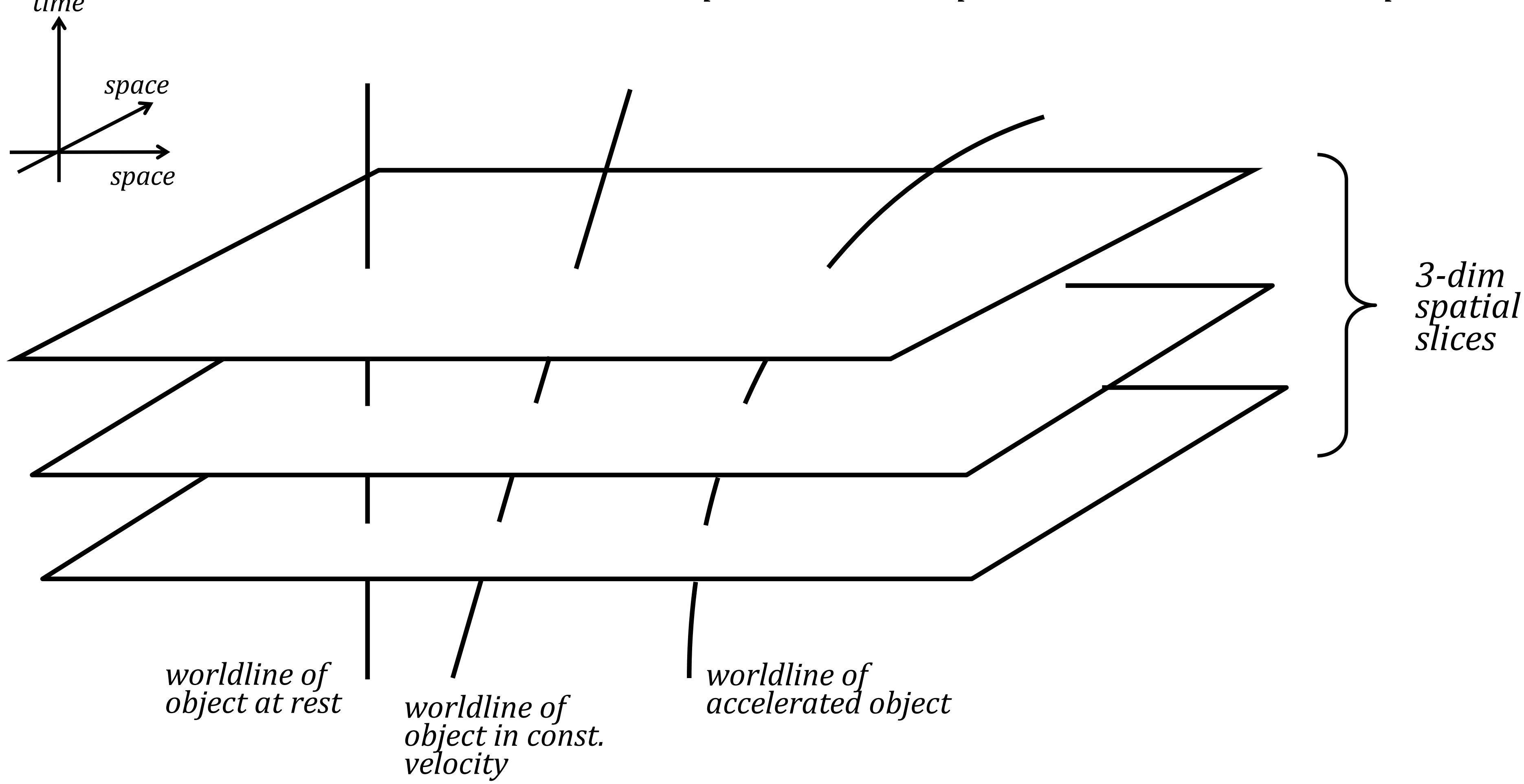
"shadow" (projection) of 4-dim hyper-cube ("tesseract") in 2-dim

11. Spacetime

1. Types of Spacetimes

- A *spacetime* is a 4-dim collection of points with <u>*additional structure*</u>.

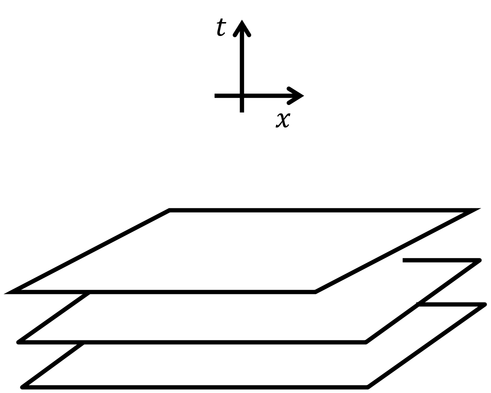
Typically, one or more <u>metrics</u> = a specification of the spatial and temporal distances between points.



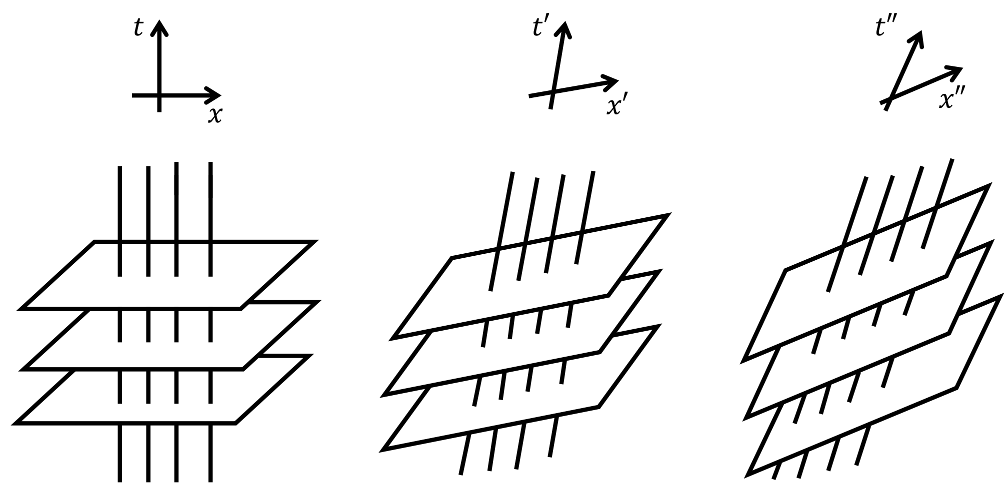
Two ways spacetimes can differ:

(1) Different ways of specifying distances between points yield different types of spacetimes.

- *Classical spacetimes* have *separate* spatial and temporal metrics: only one way to split time from space (spatial and temporal distances are *absolute*).
- *Relativistic spacetimes* have a *single* spatiotemporal metric, and how it gets split into spatial and temporal parts depends on one's inertial reference frame (spatial and temporal distances are *relative*).



Classical spacetime: only one way to split time from space.

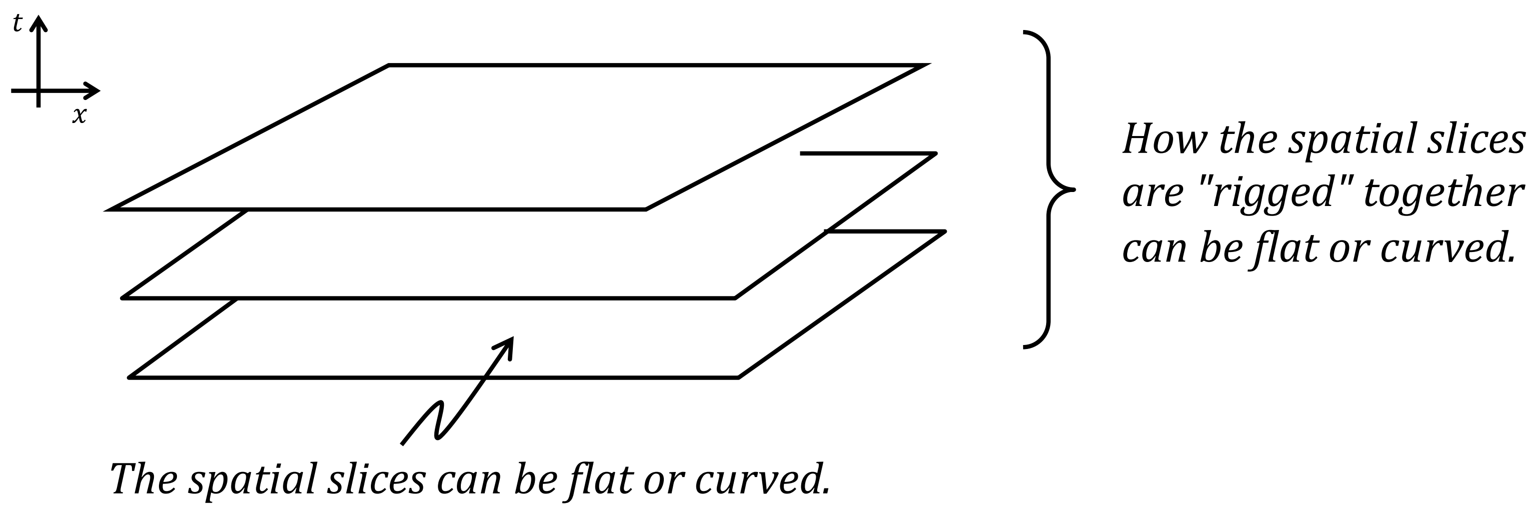


Relativistic spacetime: many ways to split time from space.

(2) Metrics can be *flat* or *curved*: how one specifies the distance between points encodes the curvature of the spacetime.

- *Classical spacetimes* can be flat or curved.
- *Relativistic spacetimes* can be flat (Minkowski spacetime) or curved (general relativistic spacetimes).

<u>*Two ways curvature can manifest itself*</u>



2. Classical Spacetimes

Newtonian spacetime is a 4-dim collection of points such that:

(N1) Between any two points p, q, with coordinates (t,x,y,z) and (t',x',y',z'), there is a definite *temporal interval* $T(p,q) = t' - t$.

(N2) Between any two points p, q, with coordinates (t,x,y,z) and (t',x',y',z'), there is a definite *Euclidean distance*

$$R(p,q) = \sqrt{(x'-x)^2 + (y'-y)^2 + (z'-z)^2}$$

(N1) *and* (N2) *entail*:

(a) All worldlines have a definite *absolute velocity*.

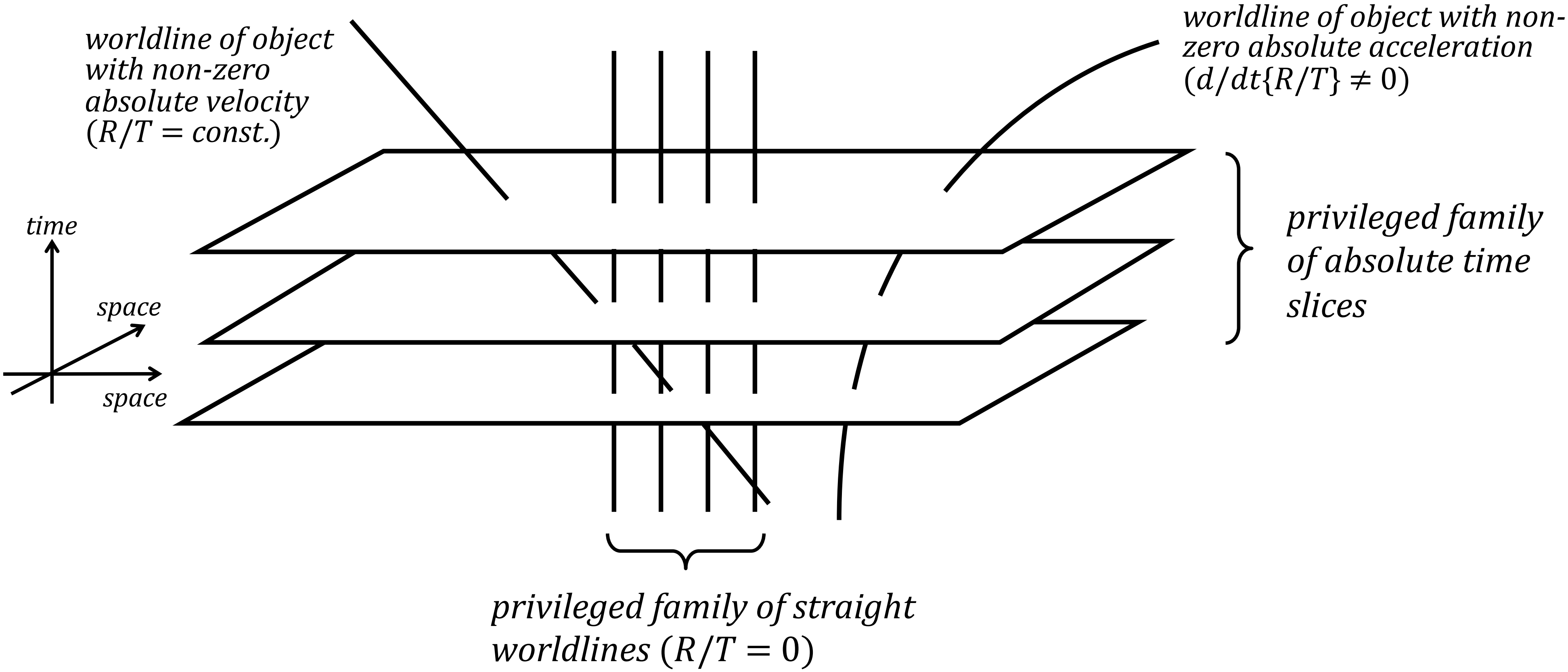
For worldline γ, and any two points p, q on γ, the *absolute velocity* of γ with respect to p, q can be defined by $R(p,q)/T(p,q)$.

(b) There is a privileged collection of worldlines defined by $R(p,q)/T(p,q) = 0$.

(c) All worldlines have a definite *absolute acceleration*.

For worldline γ, and points p, q on γ, the *absolute acceleration* of γ *with respect to* p, q can be defined by $d/dt\{R(p,q)/T(p,q)\}$.

Newtonian Spacetime



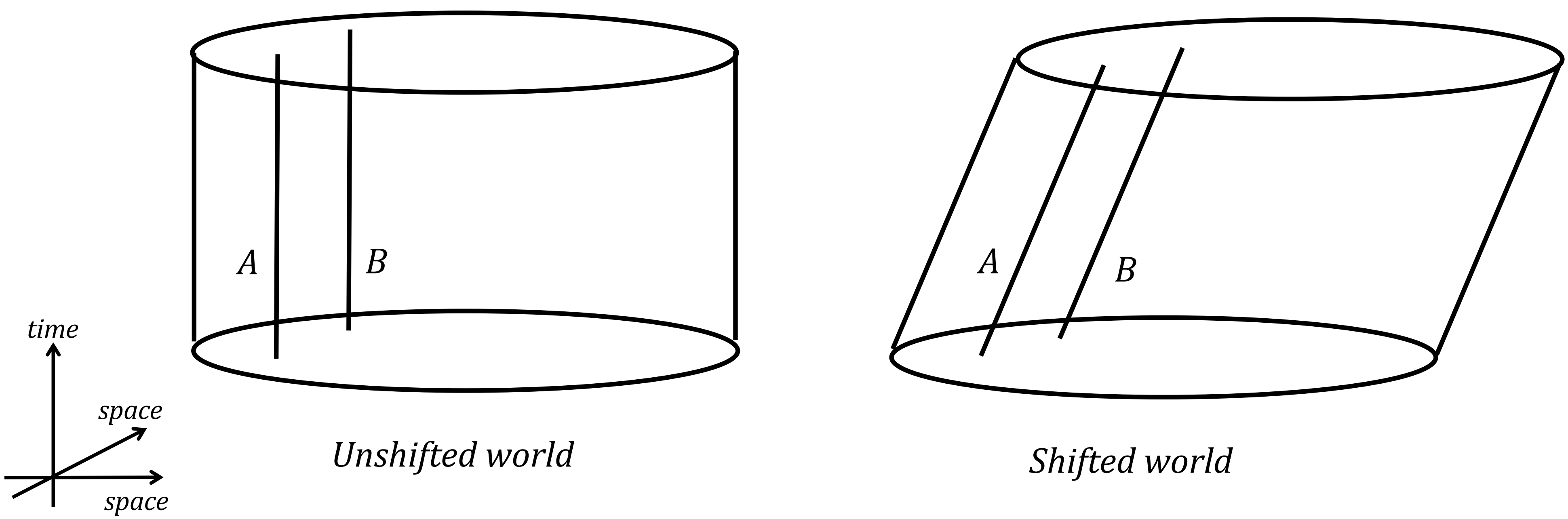
1. Single, privileged inertial frame.
2. Velocity is absolute.
3. Acceleration is absolute.
4. Simultaneity is absolute.

- Newtonian spacetime has enough structure to support the Principle of Inertia.

> *Principle of Inertia in Newtonian Spacetime*
> Objects follow straight worldlines ($R/T = const$) unless acted upon by external forces.

- *But*: Newtonian spacetime also supports Leibiz's Kinematic Shift!

Kinematic Shift in Newtonian Spacetime



- In Newtonian spacetime, the unshifted and shifted worlds are distinct!
  - *Families of straight lines with different slopes can be distinguished from each other.*

Galilean spacetime is a 4-dim collection of points such that:

(G1) Between any two points p, q, with coordinates (t, x, y, z) and (t', x', y', z'), there is a definite *temporal interval* $T(p, q) = t' - t$.

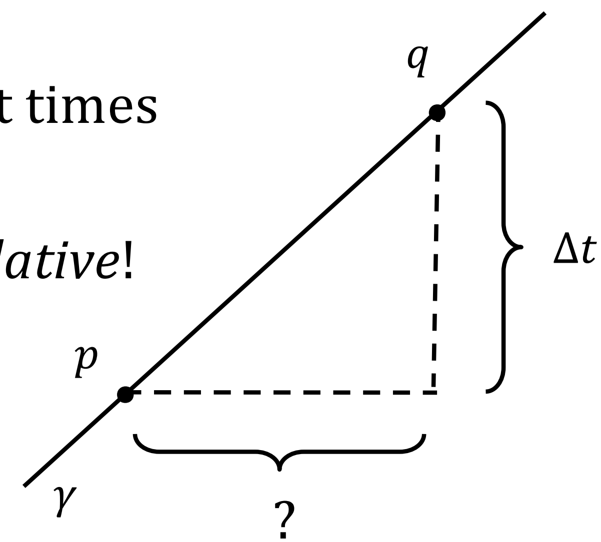
(G2) Between any two *simultaneous* points p_t, q_t, with coordinates (t, x, y, z) and (t, x', y', z'), there is a definite *Euclidean distance*,

$$R(p_t, q_t) = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

(G3) Any worldline γ through point p has a definite curvature $S(\gamma, p)$.

(G2) *entails*:

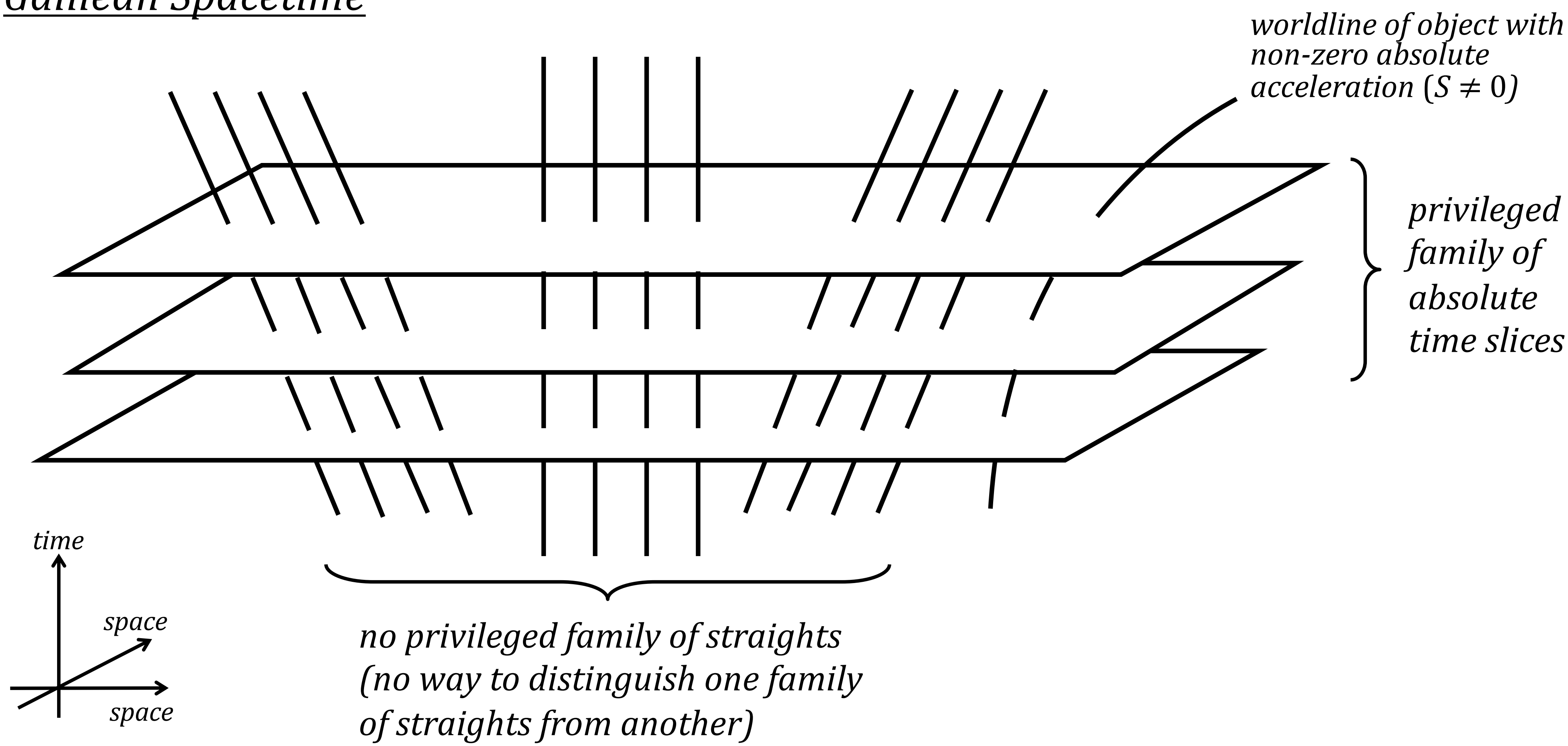
- No absolute spatial distance between points at different times on any worldline γ.
- So: No absolute velocity for any worldline: *velocity is relative*!
- So: No *single* privileged frame of reference.



(G3) *entails*: *acceleration remains absolute!*

For worldline γ and point p on γ, the *absolute acceleration* of γ *with respect to* p is given by $S(\gamma, p)$.

Galilean Spacetime



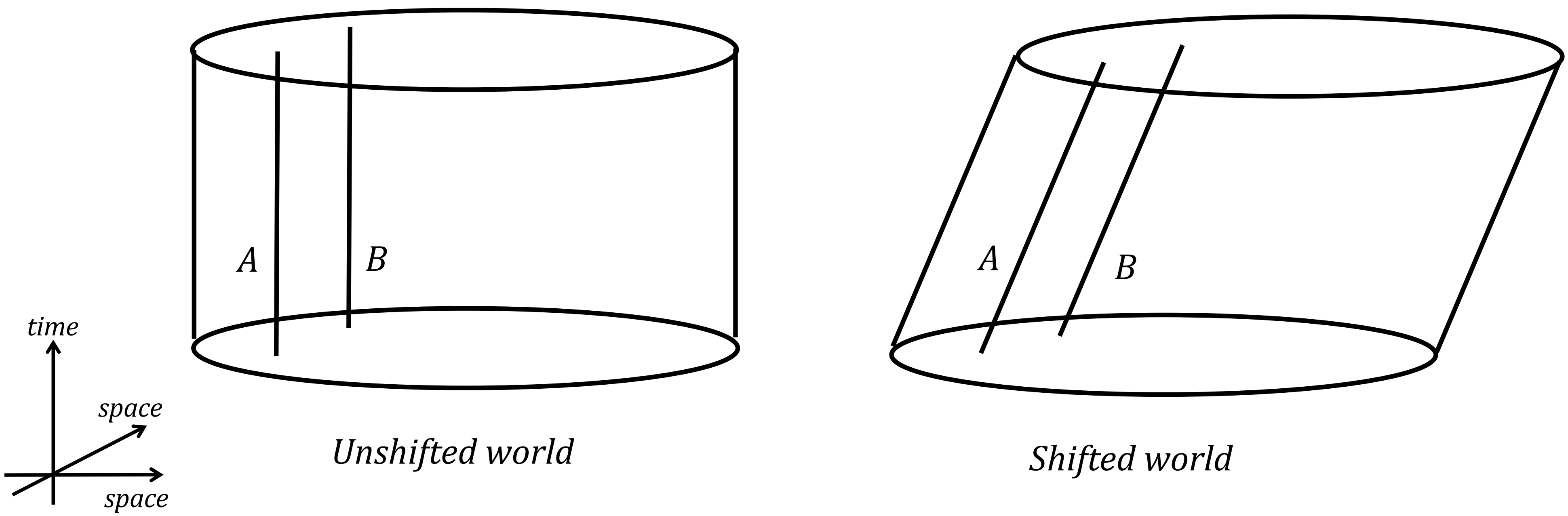
1. Many inertial frames; none privileged.
2. Velocity is relative.
3. Acceleration is absolute.
4. Simultaneity is absolute.

- Galilean spacetime has enough structure to support the Principle of Inertia.

> *Principle of Inertia in Galilean Spacetime*
> Objects follow straight worldlines ($S = 0$) unless acted upon by external forces.

- *And*: Galilean spacetime does *not* support Leibiz's Kinematic Shift!

Kinematic Shift in Galilean Spacetime

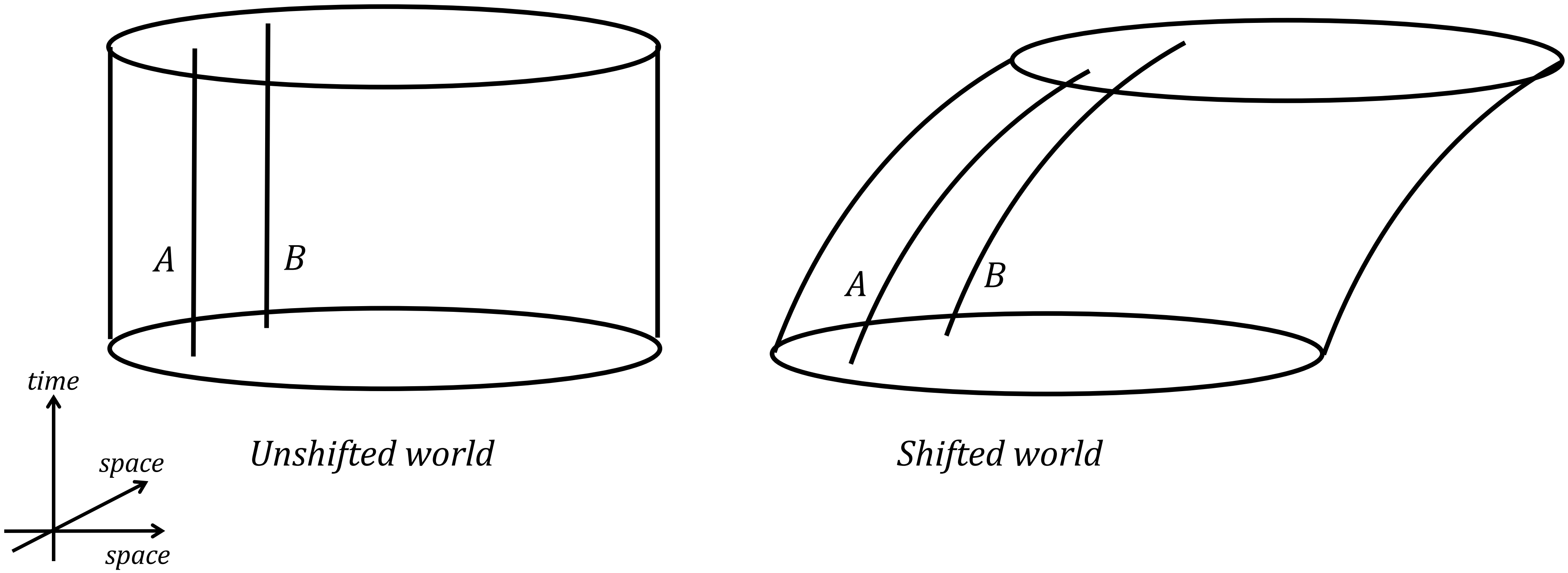


- In Galilean spacetime, the unshifted and shifted worlds are indistinguishable! (Families of straight lines cannot be distinguished from each other.)

What about Clarke's Dynamic Shift?

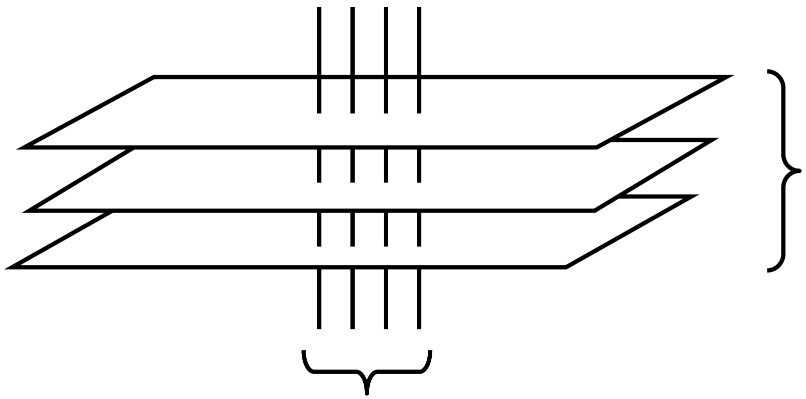
- Both Newtonian and Galilean spacetimes support the Dynamic Shift:

Dynamic Shift in Newtonian and Galilean Spacetimes



- In Newtonian spacetime, the unshifted and shifted worlds differ on their values of absolute acceleration $d/dt\{R/T\}$.
- In Galilean spacetime, the unshifted and shifted worlds differ on their values of absolute acceleration S.
- (In both Newtonian and Galilean spacetime, straight worldlines are distinct from curved worldlines.)

Newtonian Spacetime

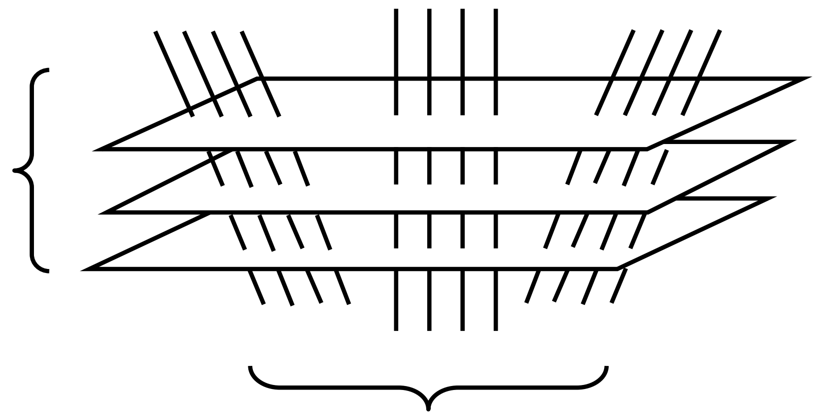


privileged family of absolute time slices

privileged family of straight worldlines ($R/T = 0$)

1. Single, privileged inertial frame.
2. Velocity is absolute.
3. Acceleration is absolute.
4. Simultaneity is absolute.

Galilean Spacetime



no privileged family of straights

1. Many inertial frames; none privileged.
2. Velocity is relative.
3. Acceleration is absolute.
4. Simultaneity is absolute.

- There are no privileged locations in Newtonian and Galilean spacetimes (they are homogeneous).
- *Recall*: Aristotle's cosmos has a privileged location...

Aristotelian spacetime is a 4-dim collection of points such that:

(A1) Between any two points p, q, with coordinates (t, x, y, z) and (t', x', y', z'), there is a definite *temporal interval* $T(p, q) = t' - t$.

(A2) Between any two points p, q, with coordinates (t, x, y, z) and (t', x', y', z'), there is a definite *Euclidean distance*

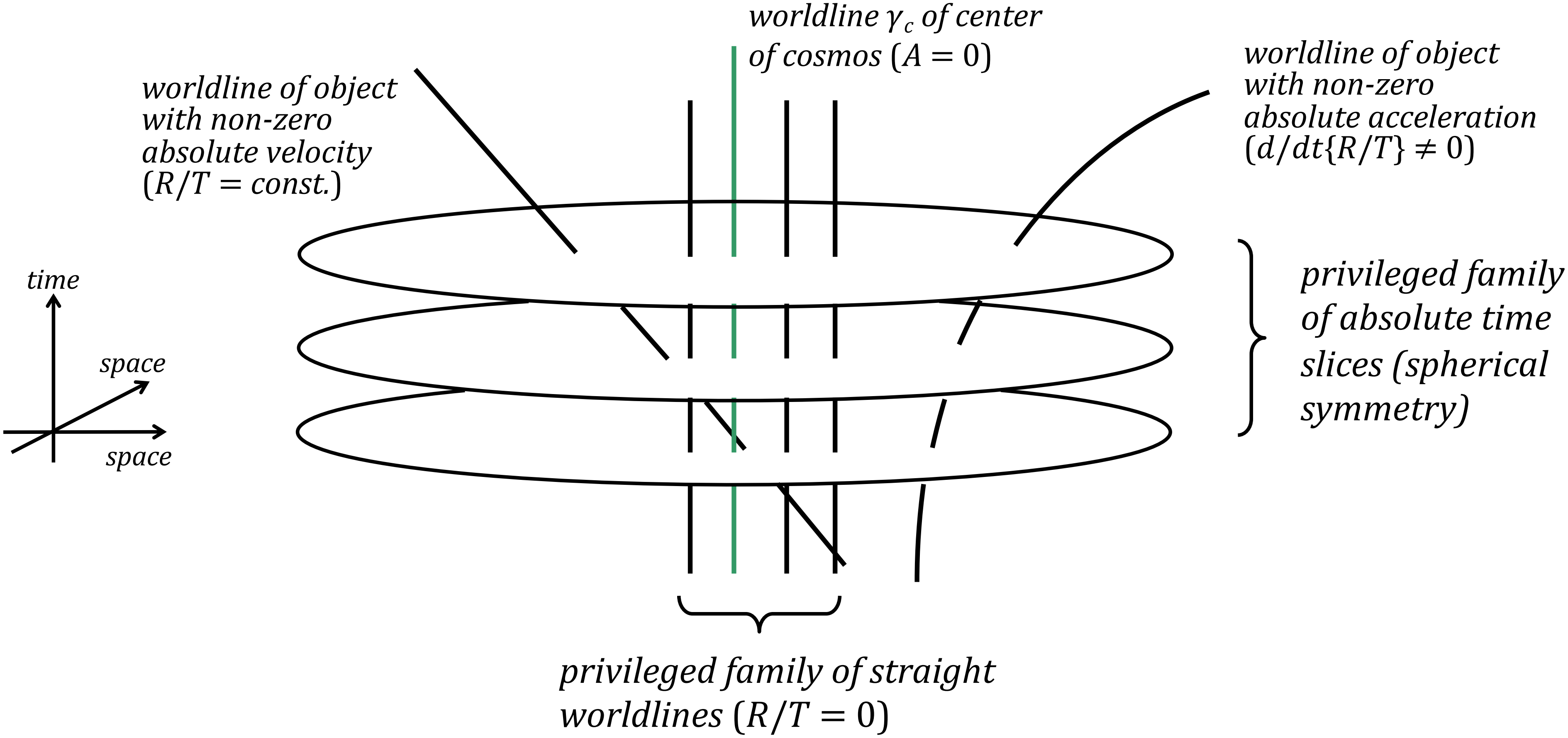
$$R(p, q) = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

(A3) Any worldline γ through point p has a definite location $A(\gamma, p)$.

Consequence of (A3):

- Location is absolute.
- Define the worldline γ_c of the center of the cosmos by $A(\gamma_c, p) = 0$, for all points p on γ_c.

Aristotelian Spacetime



1. Single, privileged inertial frame.
2. Position is absolute.
3. Velocity is absolute.
4. Acceleration is absolute.
5. Simultaneity is absolute.

- Newtonian, Galilean, and Aristotelian spacetimes support the Principle of Inertia (they all can distinguish straight worldlines from curved worldlines).
  - *Galilean spacetime does this minimally: no additional superfluous structure.*
  - *Newtonian and Aristotelian spacetimes have additional superfluous structure.*

- Are there classical spacetimes that do not have enough structure to distinguish straight worldlines from curved worldlines?

James Clerk Maxwell
(1831-1879)

"Acceleration, like position and velocity, is a relative term and cannot be interpreted absolutely." (*Matter and Motion* 1877)

- *But*: With respect to Newton's Bucket Experiment...

"This concavity of the surface depends on the absolute motion of rotation of the water and not on its relative rotation."

- Absolute rotation but *no* absolute (linear) acceleration?

Maxwellian spacetime is a 4-dim collection of points such that:

(M1) Between any two points p, q, with coordinates (t, x, y, z) and (t', x', y', z'), there is a definite *temporal interval* $T(p, q) = t' - t$.

(M2) Between any two *simultaneous* points p_t, q_t, with coordinates (t, x, y, z) and (t, x', y', z'), there is a definite *Euclidean distance*,

$$R(p_t, q_t) = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

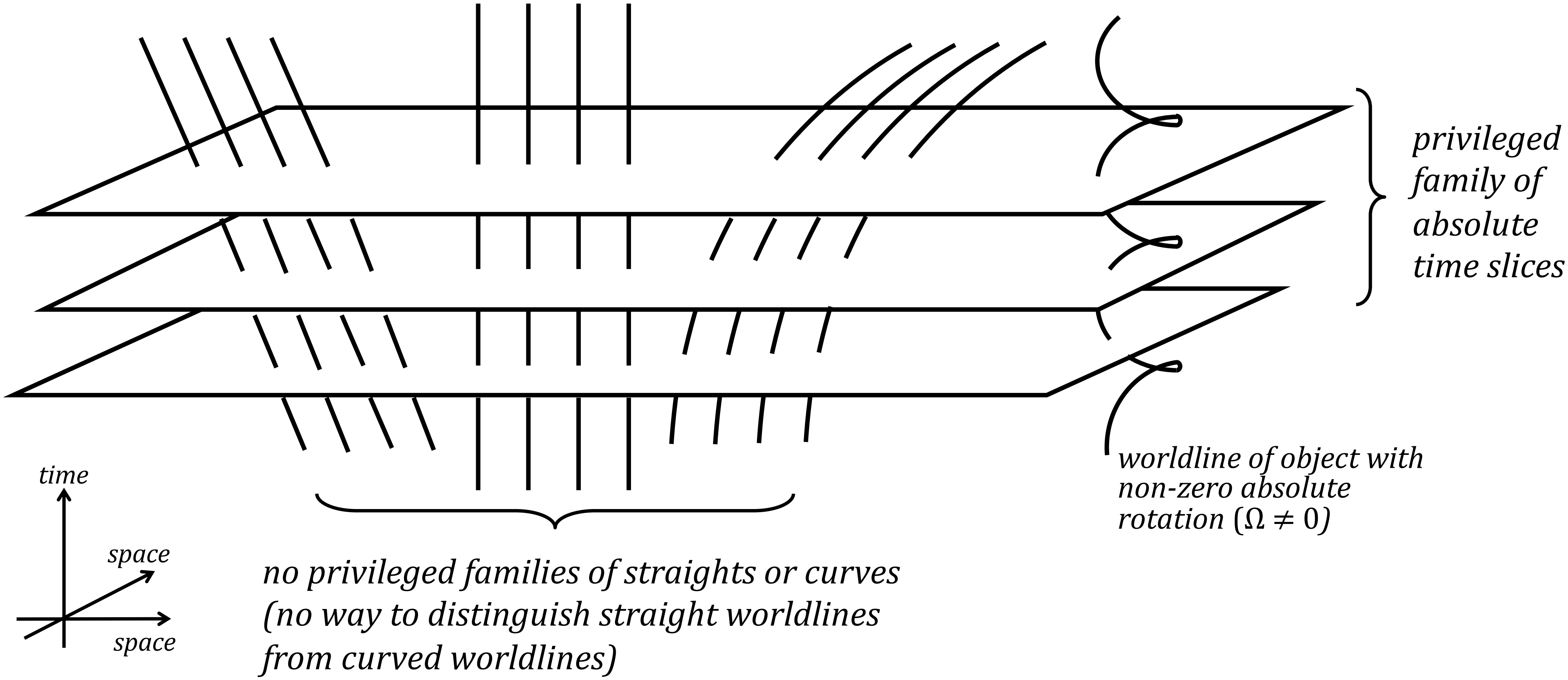
(M3) Any worldline γ through point p has a definite twist $\Omega(\gamma, p)$.

(M3) *entails*:

- *Linear* acceleration is no longer absolute!
  - *There is not enough structure in Maxwellian spacetime to distinguish straight worldlines from curved worldlines.*
- But rotation still is absolute!
  - *M3 allows us to tell when a worldline is "twisted".*

For worldline γ and point p on γ, the *absolute rotation* of γ *with respect to* p is given by $\Omega(\gamma, p)$.

Maxwellian Spacetime



1. No inertial frames.
2. Velocity is relative.
3. Linear acceleration is relative.
4. Rotation is absolute.
5. Simultaneity is absolute.

Leibnizian spacetime is a 4-dim collection of points such that:

(L1) Between any two points p, q, with coordinates (t, x, y, z) and (t', x', y', z'), there is a definite *temporal interval* $T(p, q) = t' - t$.

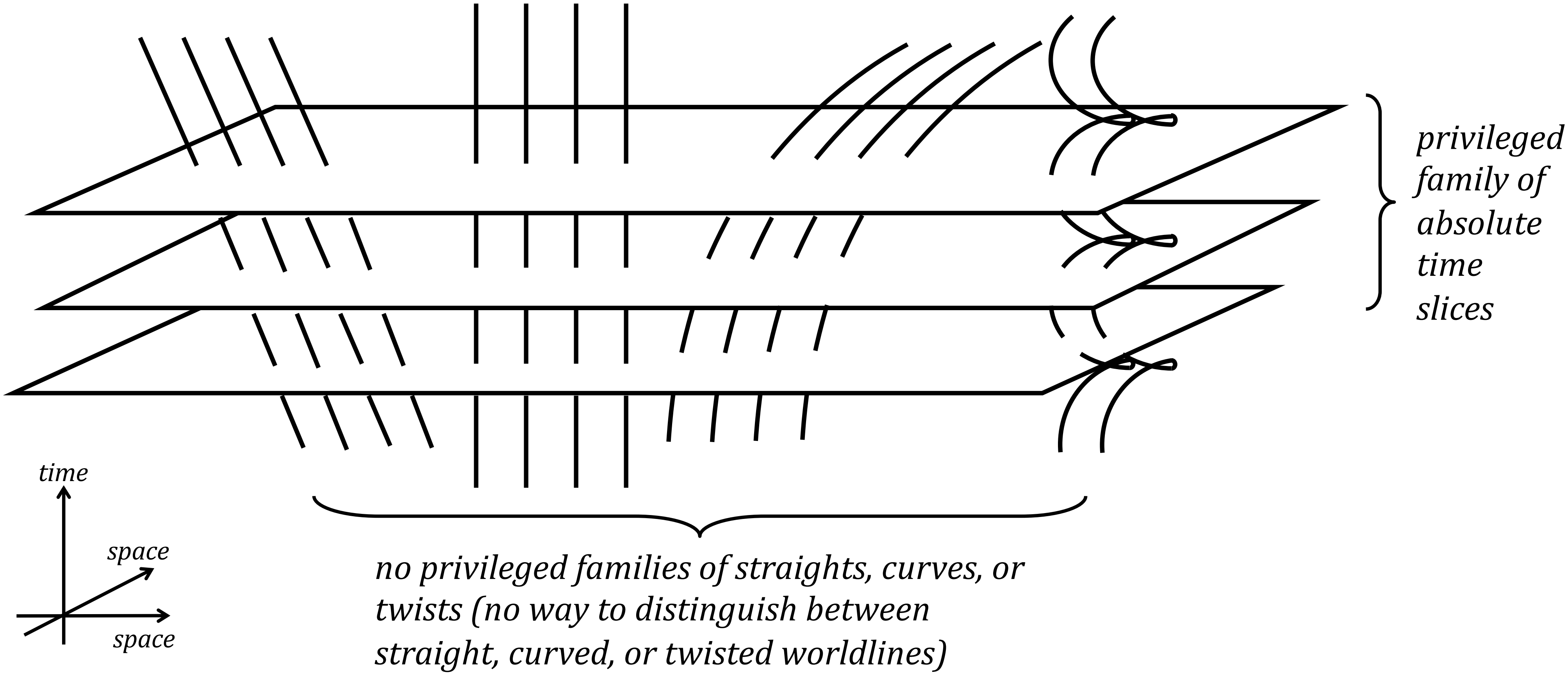
(L2) Between any two *simultaneous* points p_t, q_t, with coordinates (t, x, y, z) and (t, x', y', z'), there is a definite *Euclidean distance*,

$$R(p_t, q_t) = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

- There's still an absolute temporal metric, and an absolute Euclidean spatial metric for the "instantaneous" 3-dim spaces.
  - *Space and time are absolute and Euclidean.*
- *But*: There's no rotation standard, and there's no acceleration standard.
- All motion is relative! (Relationism at last?)

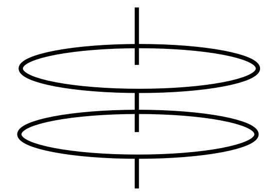
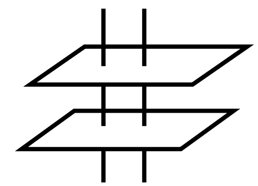
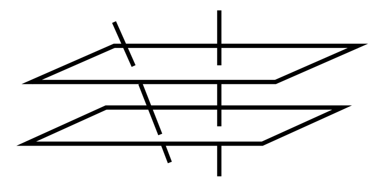
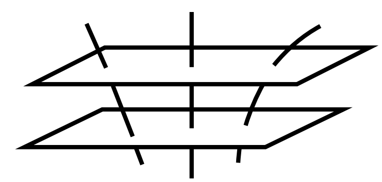
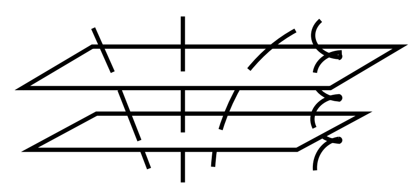
Hmph! But I do concede that absolute acceleration exists...

Leibnizian Spacetime



1. No inertial frames.
2. Velocity is relative.
3. Acceleration is relative.
4. Rotation is relative.
5. Simultaneity is absolute.

A Bestiary of Spacetimes

Spacetime	*Privileged Frames*	*Symmetries*	*Indistinguishable worldlines*
Aristotelian	Rigid Euclidean frame with position at origin, zero velocity, zero acceleration, zero rotation.	$x \to x' = \mathbf{R}x$ $t \to t' = t + const.$ *Rotate in space; translate in time.*	
Newtonian	Rigid Euclidean frame with zero velocity, zero acceleration, zero rotation.	$x \to x' = \mathbf{R}x + const.$ $t \to t' = t + const.$ *Rotate and translate in space; translate in time.*	
Galilean	Rigid Euclidean frames with zero acceleration, zero rotation.	$x \to x' = \mathbf{R}x + \mathbf{v}t + const.$ $t \to t' = t + const.$ *Rotate, translate and boost velocity in space; translate in time.*	
Maxwellian	Rigid Euclidean frames with zero rotation.	$x \to x' = \mathbf{R}x + \mathbf{a}(t)$ $t \to t' = t + const.$ *Rotate, translate, boost velocity and acceleration in space; translate in time.*	
Leibnizian	Rigid Euclidean frames.	$x \to x' = \mathbf{R}(t)x + \mathbf{a}(t)$ $t \to t' = t + const.$ *Rotate in space and time, translate and boost velocity and acceleration in space; translate in time.*	

Aristotelian
$x \to x' = \mathbf{R}x$
$t \to t' = t + const.$

Newtonian
$x \to x' = \mathbf{R}x + const.$
$t \to t' = t + const.$

Galilean
$x \to x' = \mathbf{R}x + \mathbf{v}t + const.$
$t \to t' = t + const.$

Maxwellian
$x \to x' = \mathbf{R}x + \mathbf{a}(t)$
$t \to t' = t + const.$

Leibnizian
$x \to x' = \mathbf{R}(t)x + \mathbf{a}(t)$
$t \to t' = t + const.$

- What transformations preserve Newton's 2nd Law:

$$F = md^2x/\mathrm{d}t^2$$

  - *Answer*: *Aristotelian, Newtonian, and Galilean!*
  - *Which means*: *The most general type of frames that cannot be experimentally distinguished by Newton's 2nd Law are Galilean frames!*

- Consider what happens if we transform Newton's 2nd Law using Leibnizian transformations:

$$F = m\frac{d^2}{dt^2}(\mathbf{R}(t)x + \mathbf{a}(t))$$
$$= m\left(\ddot{\mathbf{R}}(t)x + 2\dot{\mathbf{R}}(t)\dot{x} + \mathbf{R}(t)\ddot{x} + \ddot{\mathbf{a}}(t)\right)$$

  - *The path of an object under no external forces is thus given by:*

$$\ddot{x} + \underbrace{\mathbf{R}^{-1}(t)\ddot{\mathbf{R}}(t)x}_{\text{"centrifugal inertial force"}} + \underbrace{\mathbf{R}^{-1}(t)\ddot{\mathbf{a}}(t)}_{\text{"linear inertial force"}} + \underbrace{2\mathbf{R}^{-1}(t)\dot{\mathbf{R}}(t)\dot{x}}_{\text{"coriolis inertial force"}} = 0$$