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Abstract

In 1989, the Chinese-born American physicist Wen Xiao-Gang first introduced the concept of topological order to describe certain condensed matter systems. Those systems are found to display a new type of order that cannot be described by the classical Landau-Ginsburg theory of matter, which include quantum Hall liquid, topological insulators, and topological superconductors. In recent years, some physicists and philosophers have suggested that certain types of topological order can be understood in terms of a holism that is underwritten by the mechanism of long-range entanglement (LRE). This means that there might exist a notion of emergence that is applicable to all topologically ordered systems. This project seeks to articulate such a notion of emergence that is applicable to condensed matter systems that exhibit this topological order.

What is Topological Order

Topological order is a very new concept that describes quantum entanglement in many body systems [1]. It is due to long-range entanglement, and we can view it simply as a pattern of long-range entanglement [1]. For any physical systems that possess topological order, we call them topologically ordered system. A topologically ordered system is characterized by three components: a topological invariant, a phase transition, and a change in topology. Topological order, associated with long-range entanglements, gives rise to new states of quantum matter [1]. It helps to identify a new type of order that cannot be described by the classical Landau-Ginsburg theory of matter.

Def. 1. A Landau-Ginsburg ordered system is characterized by

- (i) a local order parameter,
- (ii) a phase transition,
- (iii) a spontaneously broken symmetry.

Def. 2. A topologically ordered system is characterized by

- (i) a topological invariant,
- (ii) a phase transition,
- (iii) a change in topology.

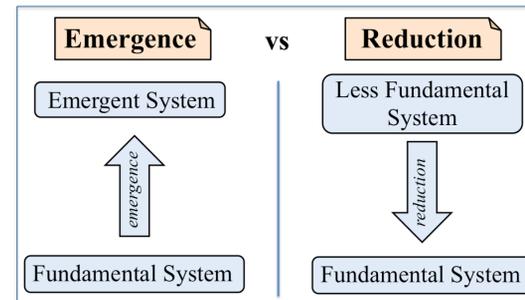
Define Emergence

In the realm of philosophy of physics, we define the word “**emergence**” to be the existence of a certain type of transition from one system (fundamental system) to the other system (emergent system). The two systems should possess two characteristics, which are dependence and independence.

a) **Dependence:** The emergent system is *ontologically determined* by the fundamental system.

b) **Independence:** The emergent system is *novel* with respect to the fundamental system.

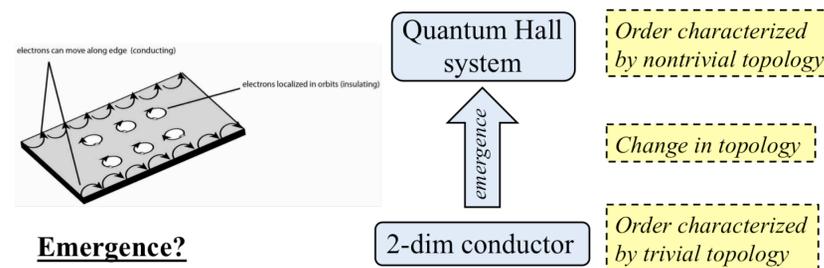
The notion of emergence is always associated with the notion of reduction. The comparison is shown on the right figure.



Emergence Applied to Topological Order

Many physicists and philosophers have claims that Landau-Ginsburg ordered systems exhibit emergence. Here we want to claim:

Topologically ordered systems also exhibit emergence.



Emergence?

(a) **Dependence?** Yes:

- Fundamental system = electrons and lattice atoms of conductor.
- Emergent system = electrons and lattice atoms of QH system.

(b) **Independence?** Yes: Novel properties: Persistent edge current, insulating interior, quantization of Hall resistance.

Works Cited

- [1] Wen, X.-G. (2013) 'Topological Order: From Long-Range Entangled Quantum Matter to a Unified Origin of Light and Electrons', *ISRN Condensed Matter Physics* 2013, 1.
- [2] Laughlin, R. and D. Pines (2000) 'The Theory of Everything', *Proc Nat Acad Sci* 97, 28.

Other Types of Topological Order

In addition to quantum Hall systems, there are also other types of systems that can be applied our notion of emergence. Those systems can be divided into two categories, which are Symmetry Protected Topological (SPT) order and Intrinsic topological order.

Symmetry Protected Topological (SPT) Order	Intrinsic Topological Order
<ul style="list-style-type: none"> • Integer quantum Hall systems • Topological insulators • Topological superconductors 	<ul style="list-style-type: none"> • Fractional quantum Hall systems • Chiral spin liquids • Z_2 spin liquids

Non-interacting "one-body" systems

Paradigm cases

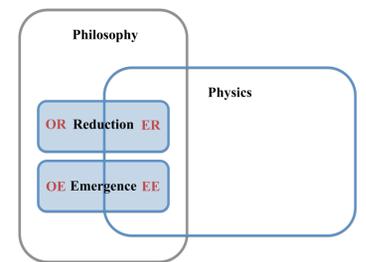
Interacting "many-body" systems

We anticipate that our definition of emergence is applicable to all systems listed above.

The Future Work

The history of physics has shown that higher organizing principles are best identified in the limiting case in which the competition is turned off, and the key breakthroughs are almost always associated with the serendipitous discovery of such limits [2]. We can view topological order as one of those emergent principles. There are definitely more worth to study and investigate on this topic, such as to understand the mechanism of long-range entanglement (LRE), to distinguish LRE from the symmetry-based notion of order, and to explore more possible notions of emergence.

Furthermore, during the study, a more general notion of emergence comes out and provide a possible way to distinguish philosophy and physics (Figure on the right). More is worth to investigate and to summarize for emergence and reduction.



- ER = Epistemological Reduction
- OR = Ontological Reduction
- EE = Epistemological Emergence
- OE = Ontological Emergence