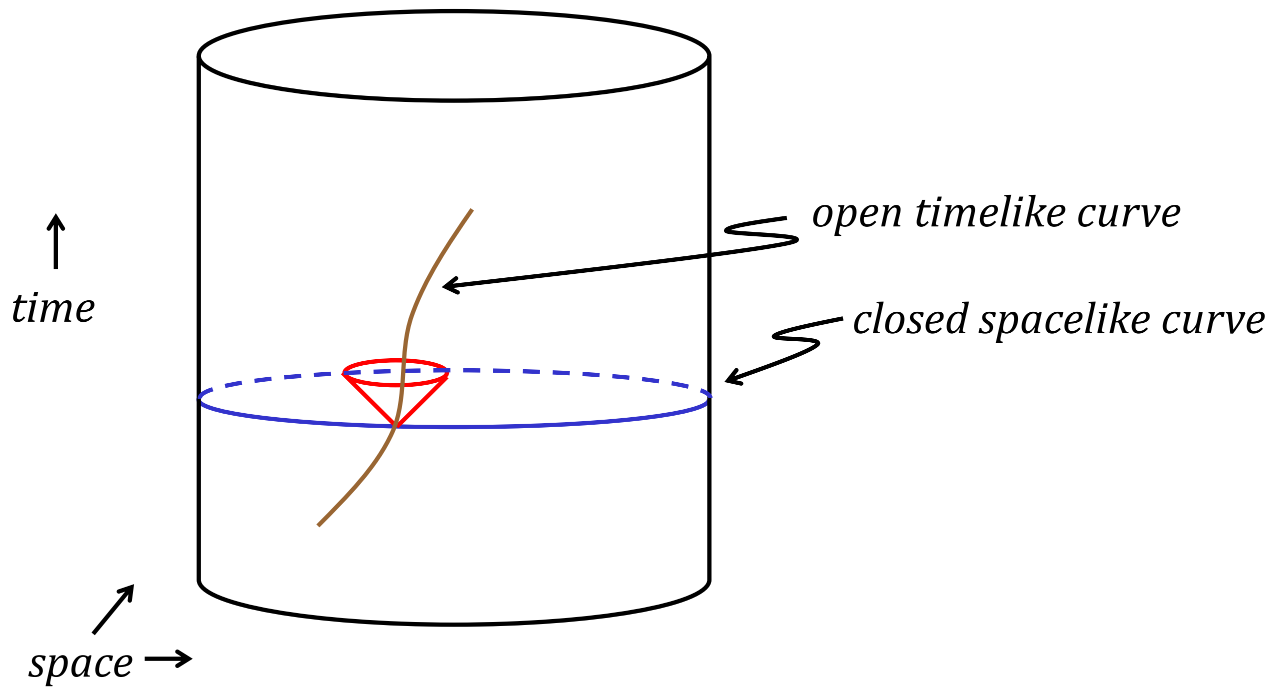
16. Time Travel 1

Topics:
1. Closed Timelike Curves
2. Constraints on Time Machines

1. Closed Timelike Curves and Time Travel

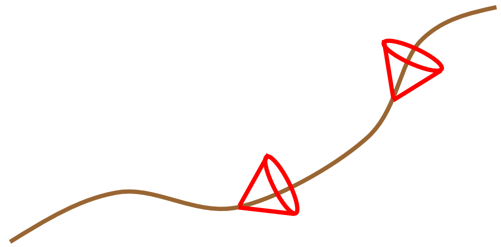
- *Recall*: The Einstein cylinder universe is a solution to the Einstein equations (with cosmological constant) that has closed *spacelike* curves.



Are there solutions to the Einstein equations with closed timelike curves?

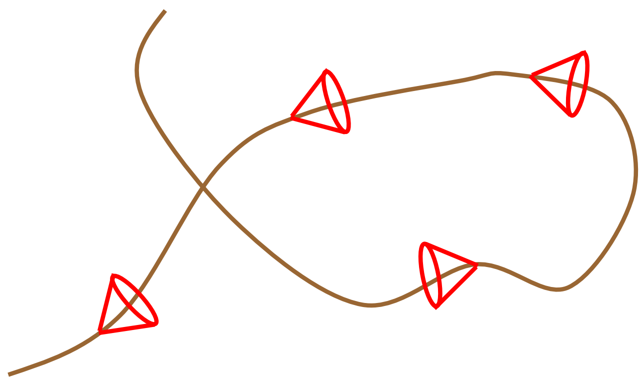
$$G_{\mu\nu}(g_{\mu\nu}) = \kappa T_{\mu\nu}$$

- *Recall*: A *timelike curve* is a worldline of a physical object (an object traveling at a speed $< c$).



Timelike curves "thread" lightcones.

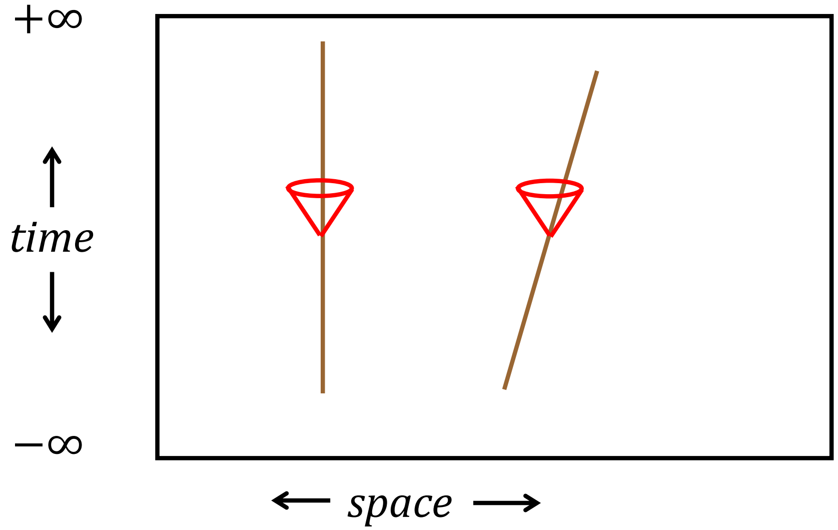
> **Def**. A *closed timelike curve (CTC)* is a worldline of a physical object that intersects itself.



- A physical object following a CTC goes back in time!
  - *At some point in its history, it will reach a point on its worldline that it previously occupied.*

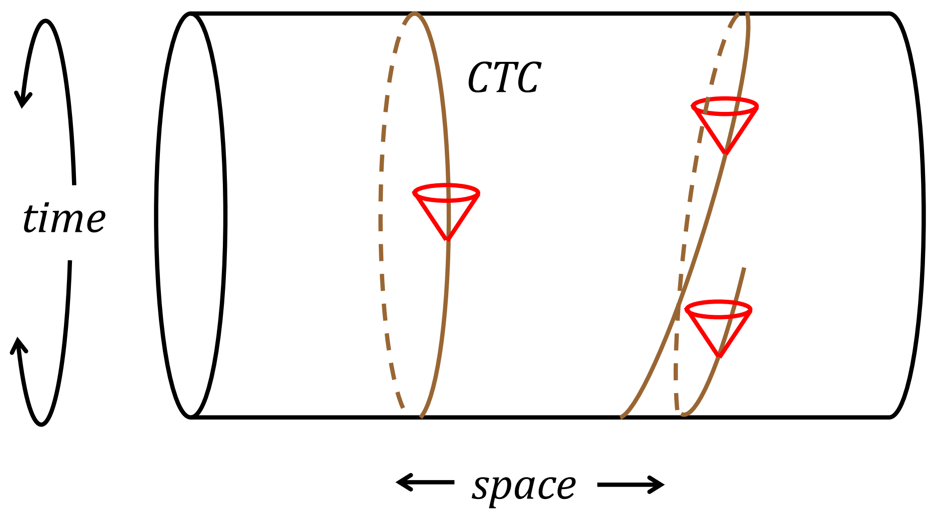
Rolled-Up Minkowski Spacetime

- Simplest solution to the Einstein equations with CTCs.



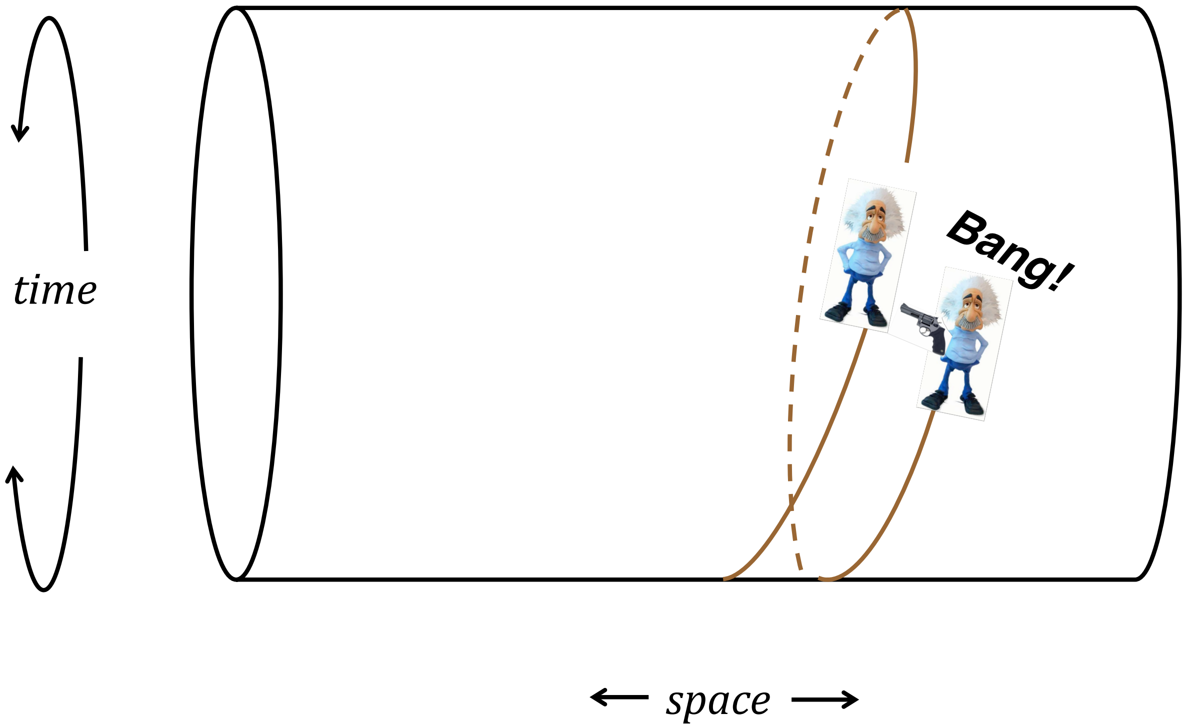
Minkowski Spacetime

- Roll it up: Identify +∞ and −∞.



Rolled-Up Minkowski Spacetime

Time Travel Paradox in Rolled-Up Minkowski Spacetime



- Albert kills himself *if and only if* Albert does not kill himself.
- Possible Resolutions:
  1. Universes with CTCs are physically impossible.
  2. Universes with CTCs are physically possible, but they are constrained in ways that *prevent* violations of causality.

If (2), are such constraints mysterious?

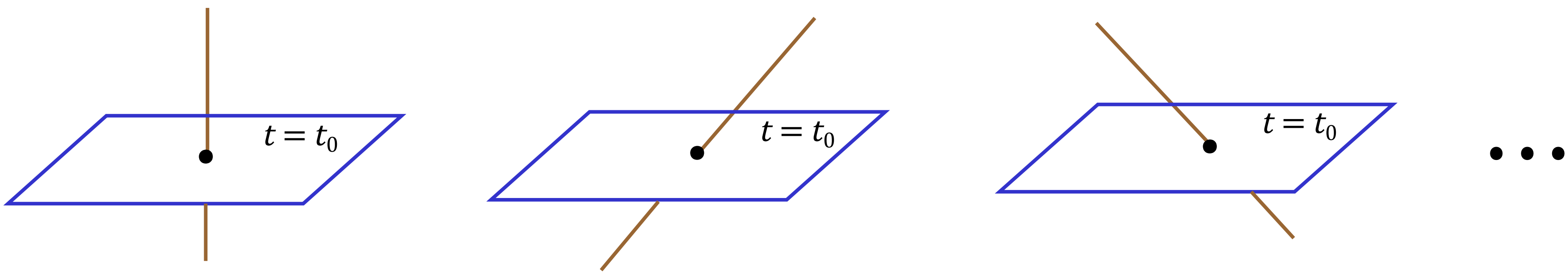
- Any more so than typical initial conditions for physical processes?

<u>*Constraints on Spacetimes with CTCs*</u>

- <u>*Simplest example*</u> - Universe with only 1 particle

<u>*Case 1: Spacetime with no CTCs*</u>

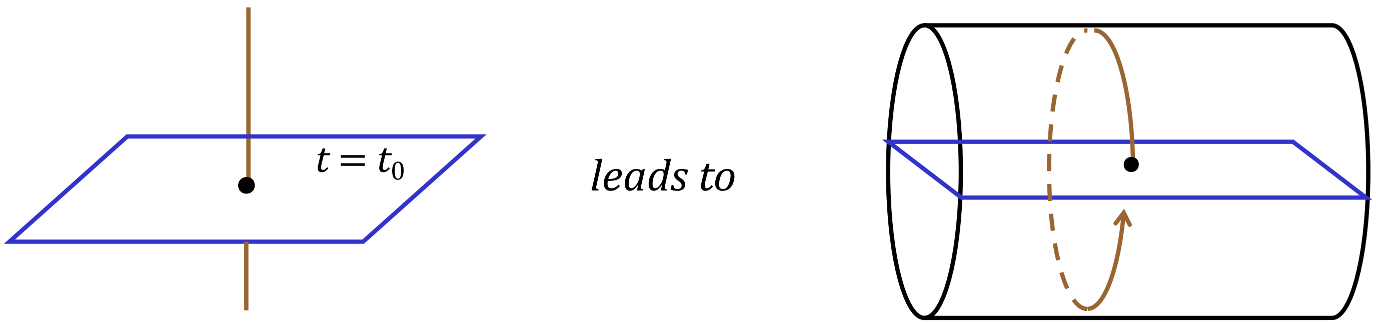
- Particle can have any initial velocity at $t = t_0$ (*w.r.t.* a given observer).



- *All* of these initial conditions lead to physically indistinguishable spacetimes!
  - *Velocities are relative in general relativistic (and Minkowski, and Galilean) spacetimes.*

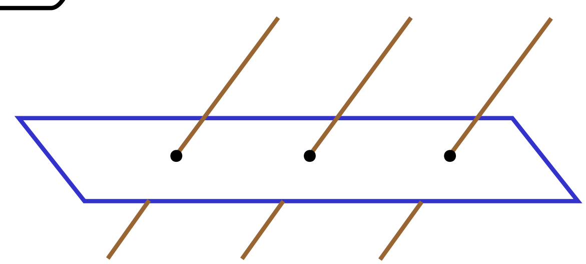
Case 2: Rolled-Up Minkowski Spacetime

- Particle is constrained to have only one initial velocity at $t = t_0$ (*w.r.t.* a given observer).



- Any other initial velocity leads to contradiction!

But: This entails the initial setup is:



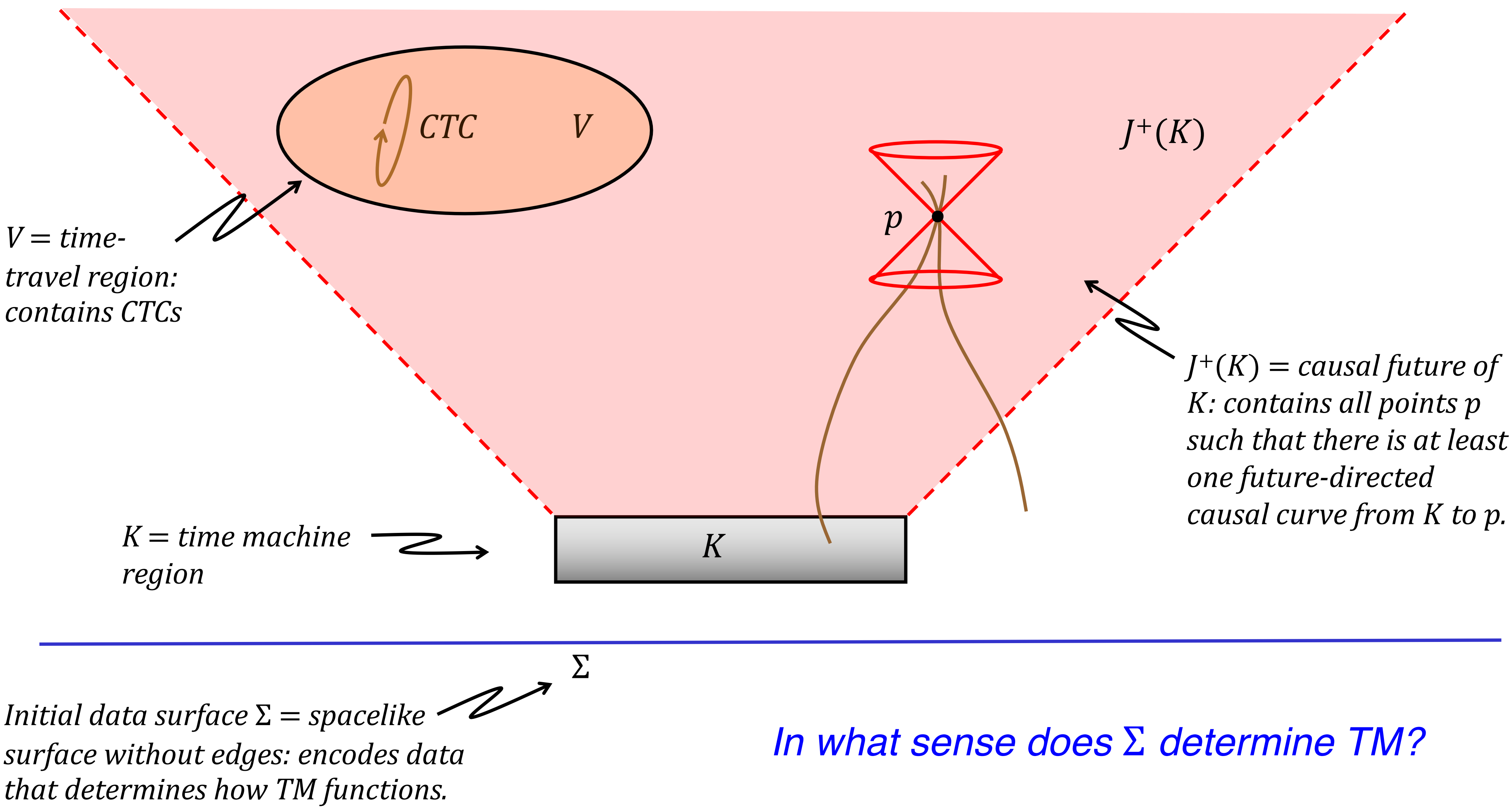
Are there more general constraints that would result in more interesting spacetimes with CTCs?

2. Constraints on Time Machines in General Relativity

Time Machine (TM) Characteristics

(1) TM is confined to finite region of space and operates for a finite time.

(2) TM *causes* CTCs to evolve.



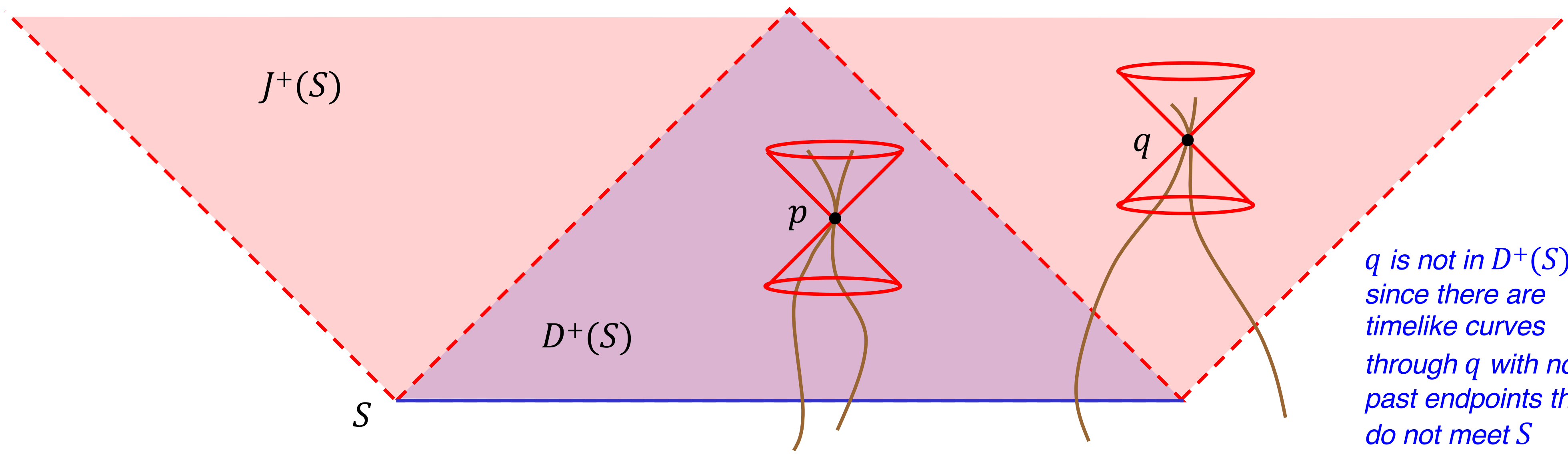
In what sense does Σ determine TM?

- Let S be a spacelike surface (not necessarily without edges).

Def. The *(future) domain of dependence* $D^+(S)$ of S consists of all points p such that *every* timelike curve through p with no past endpoint meets S in one point.

- *Important*: $D^+(S)$ is different from $J^+(S)$.
  - *This means that the points that S determines are in general different from those it can causally interact with!*

$D^+(S)$ contains all points that are uniquely determined by S!

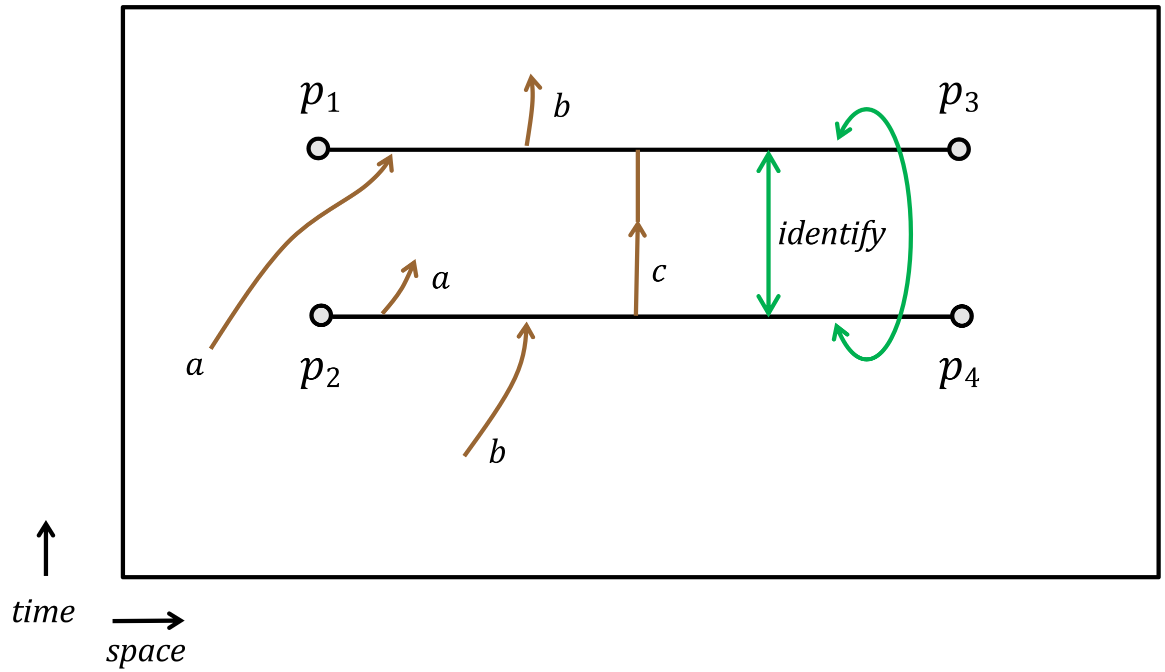


q is not in $D^+(S)$ since there are timelike curves through q with no past endpoints that do not meet S

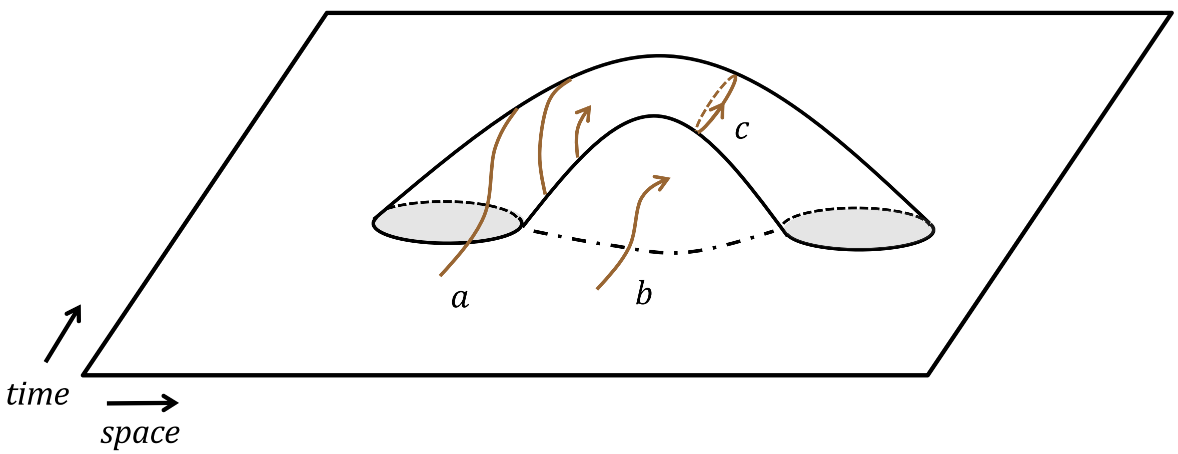
- *So*: To say Σ determines TM is to say $K \subset D^+(\Sigma)$ (K is in the domain of dependence of Σ).

But: Σ cannot uniquely determine what goes on in the time-travel region V caused by the TM!

Deutsch-Politzer Spacetime (Deutsch 1991, Politzer 1992)

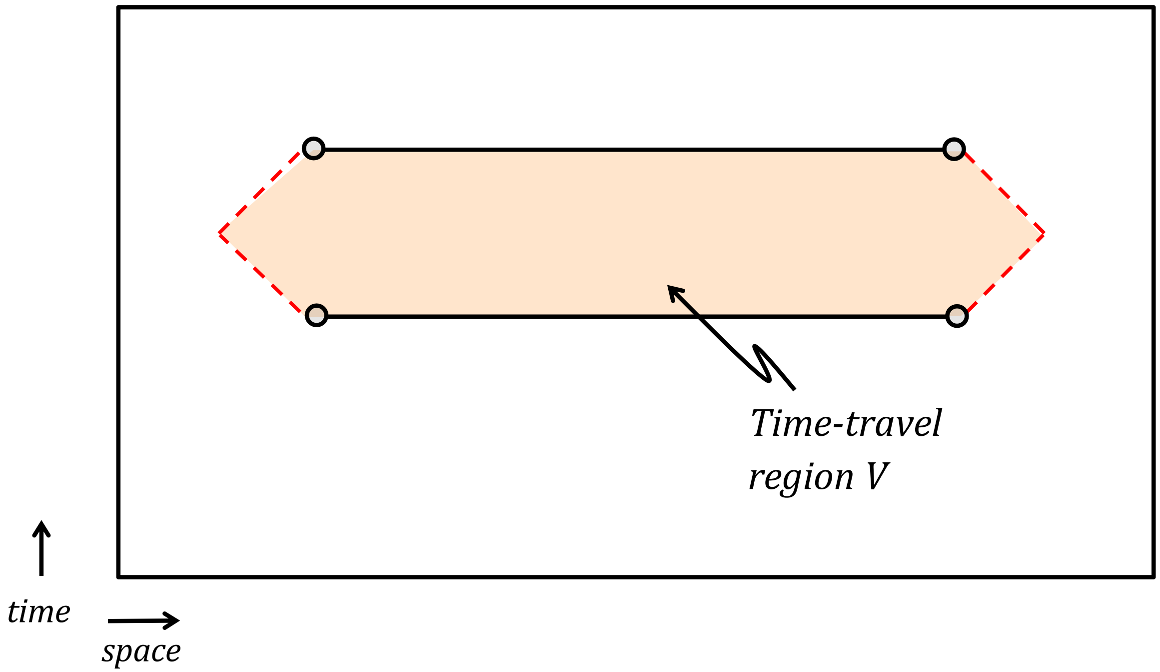


- Remove four points p_1, p_2, p_3, p_4 from Minkowski spacetime.
- Cut two strips between holes.
- Connect upper lip of top strip to lower lip of bottom strip.
- Connect lower lip of top strip to upper lip of bottom strip.



- *Result*: A "handle" in Minkowski spacetime connecting two holes.
- Worldline c is a CTC!
- Worldline a travels back in time!
- Worldline b travels forward in time (as per usual).

Deutsch-Politzer Spacetime (Deutsch 1991, Politzer 1992)



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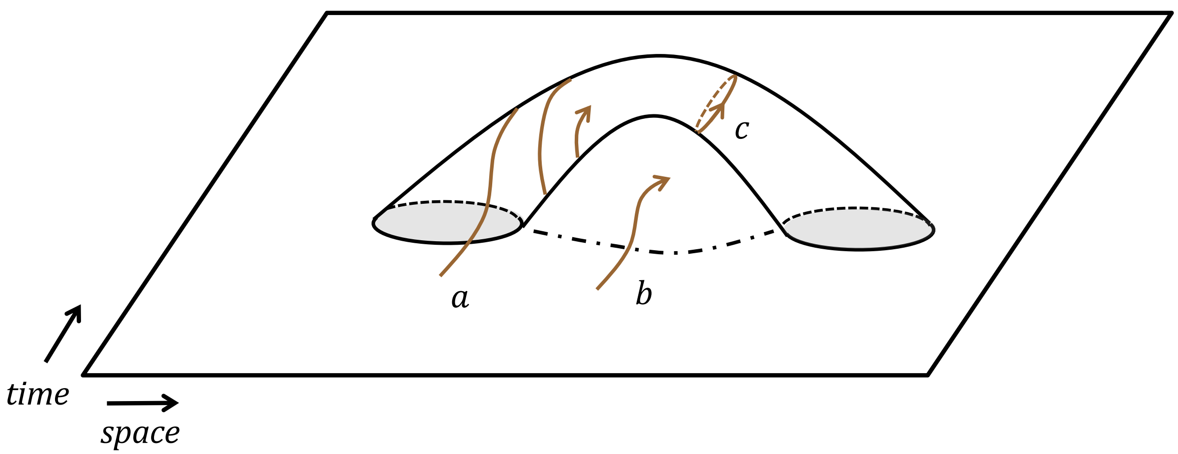

- *Result*: A "handle" in Minkowski spacetime connecting two holes.
- Worldline *c* is a CTC!
- Worldline *a* travels back in time!
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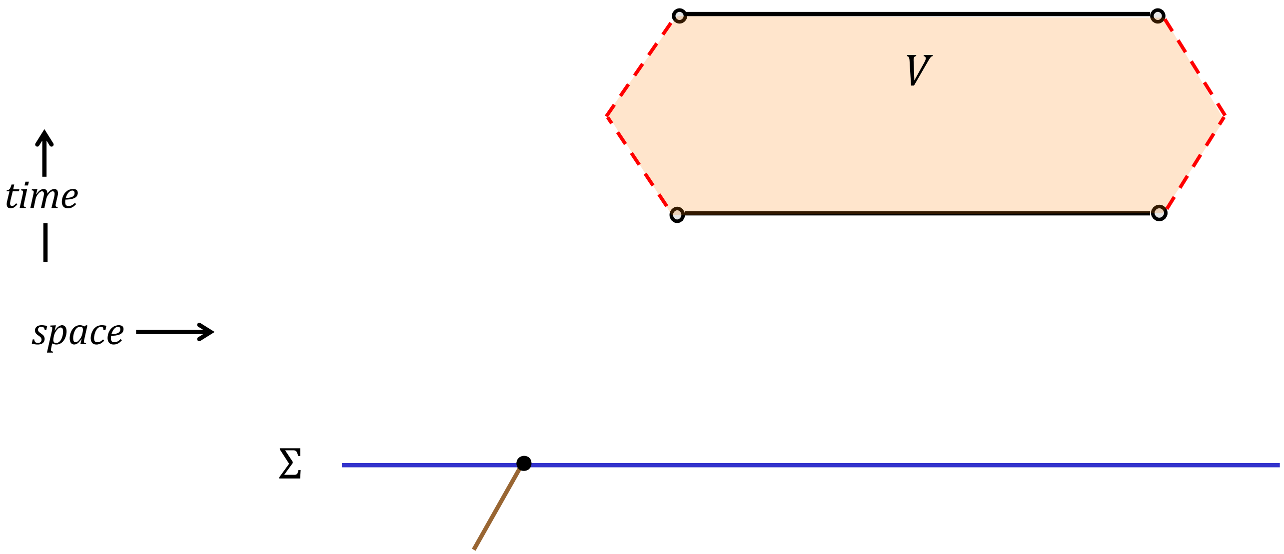
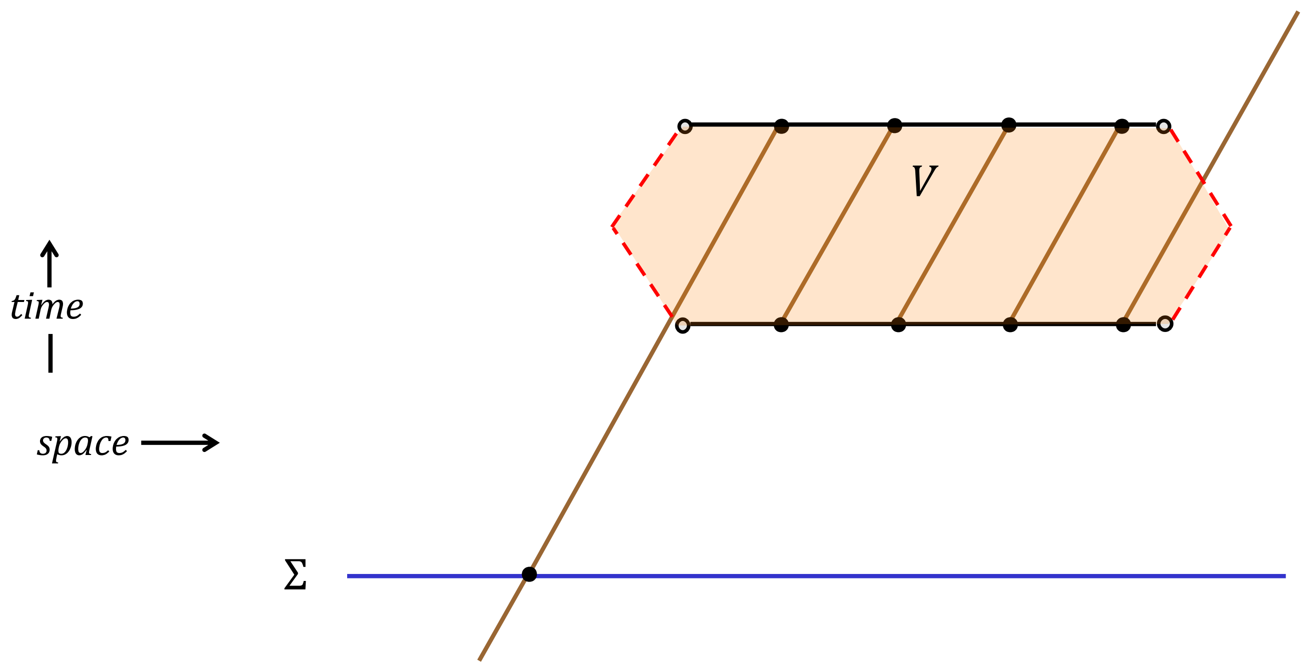
Deutsch-Politzer Spacetime (Deutsch 1991, Politzer 1992)



- Consider a particle traveling to the right at constant speed that registers on Σ and enters *V*.
  - *Its point of intersection with Σ records its position and its velocity.*
  - *This initial data can then be used to predict the path of the particle to the future of Σ.*

Does the initial data on Σ uniquely determine the path of the particle to the future of Σ?

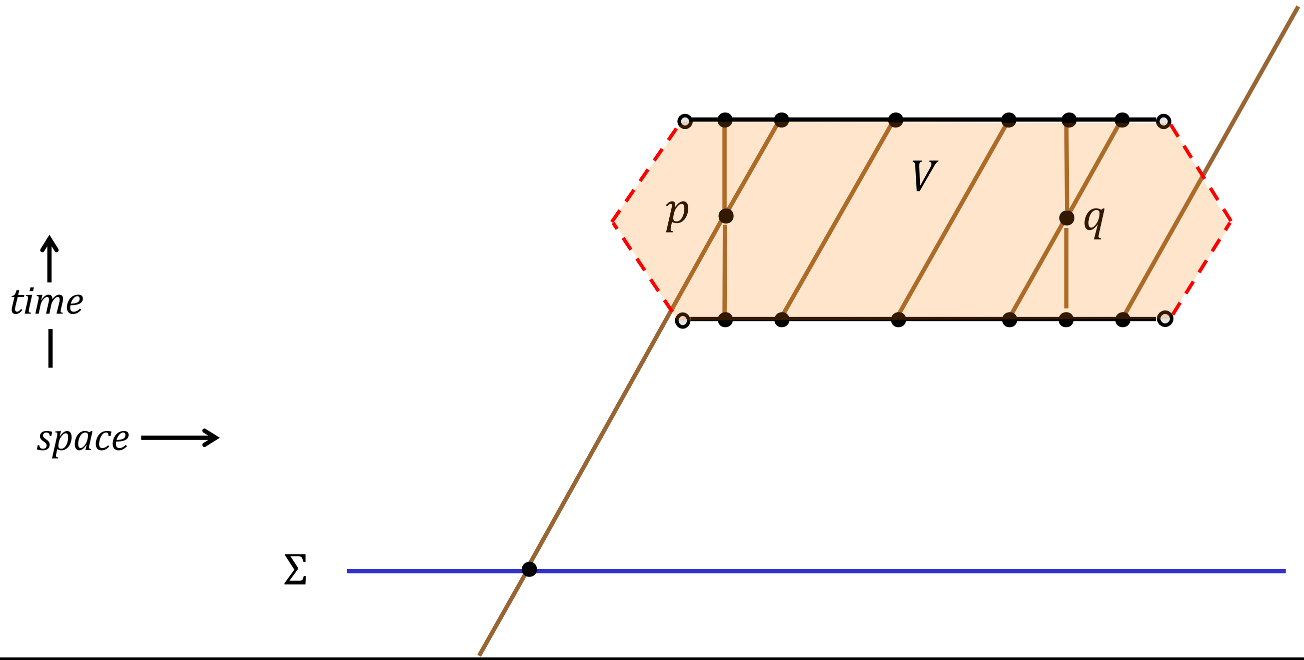
Deutsch-Politzer Spacetime (Deutsch 1991, Politzer 1992)



Prediction I: *No collisions in V.*

1. Particle travels to right with constant speed and enters region *V*.
2. Hits top strip and goes back in time to bottom strip.
3. Continues to right at constant speed.
4. Steps (2) and (3) repeat three more times.
5. Particle leaves region *V*.

Deutsch-Politzer Spacetime (Deutsch 1991, Politzer 1992)

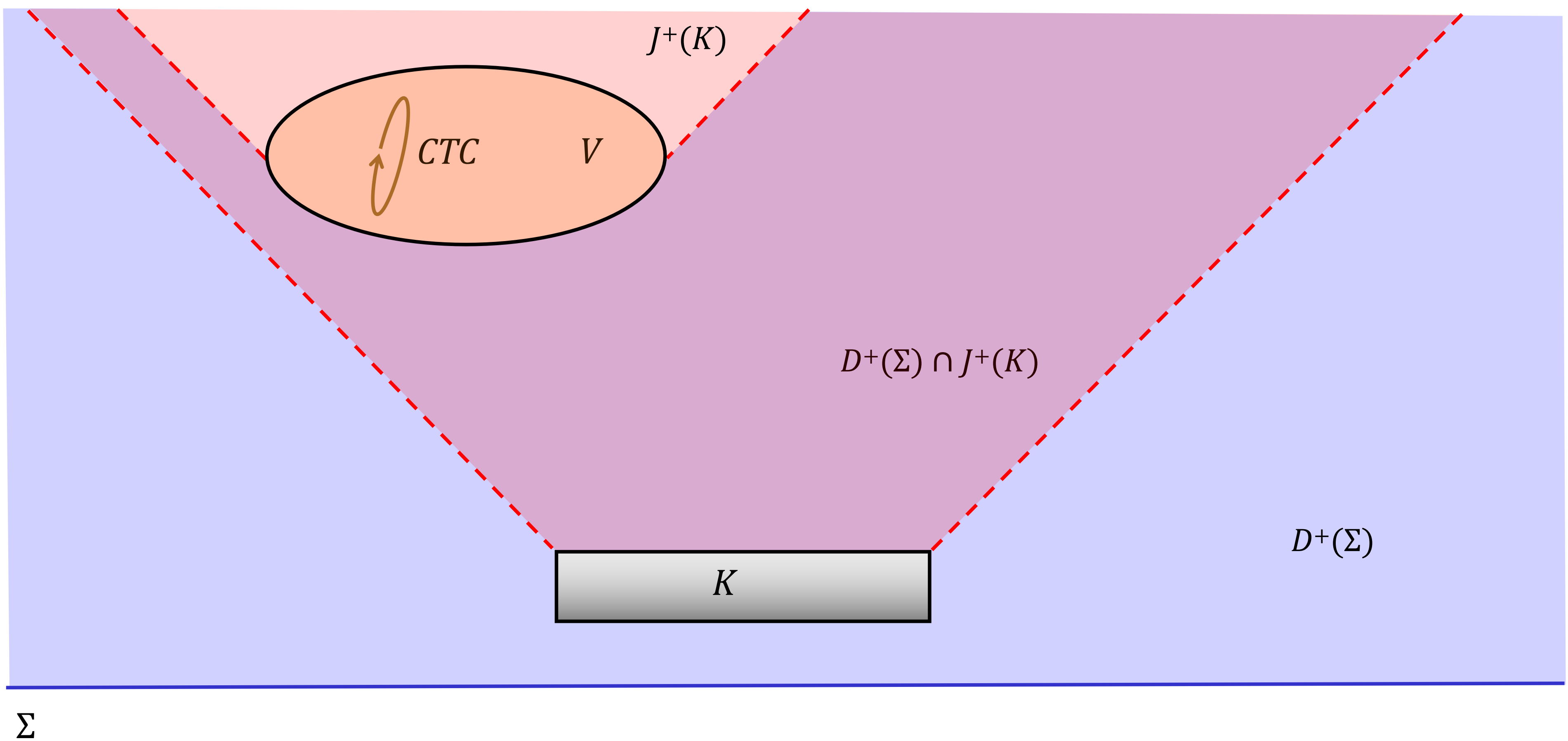


Prediction II (same initial conditions): *Two collisions in V.*

1. Particle travels to right with constant speed and enters region *V*.
2. At *p*, hits an older version of self and comes to rest.
3. Travels back in time and is hit by a younger version of self (again at *p*).
4. Scatters to right at contant speed, travels back in time three more times.
5. At *q*, hits older version of self and comes to rest.
6. Travels back in time and is hit by younger version of self (again at *q*).
7. Scatters to right, travels back in time, and then finally leaves region *V*.

So: Initial conditions on Σ do not uniquely determine what goes on in V.

- *Which means*: V cannot be in $D^+(\Sigma)$.



Recall: Σ has no edges!

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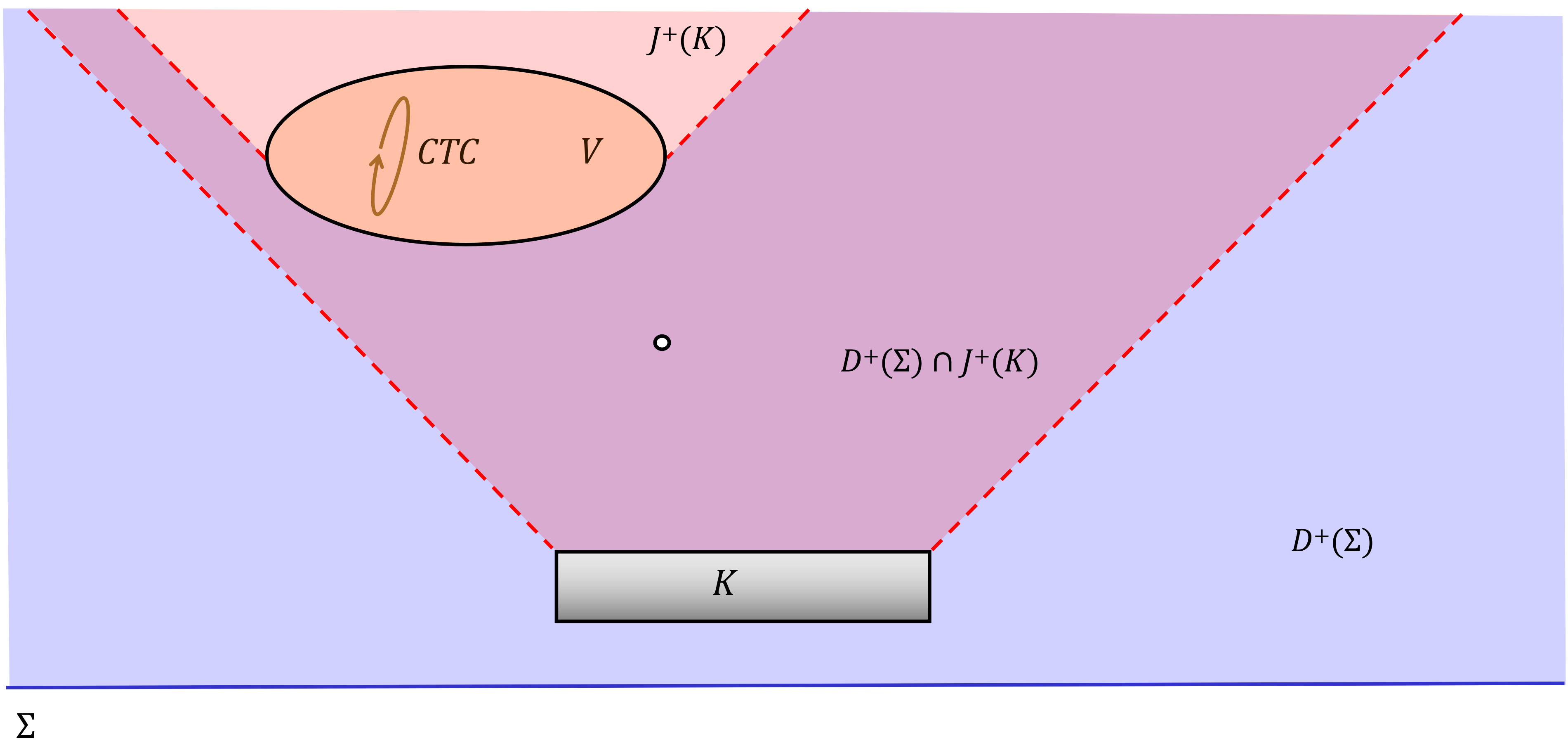
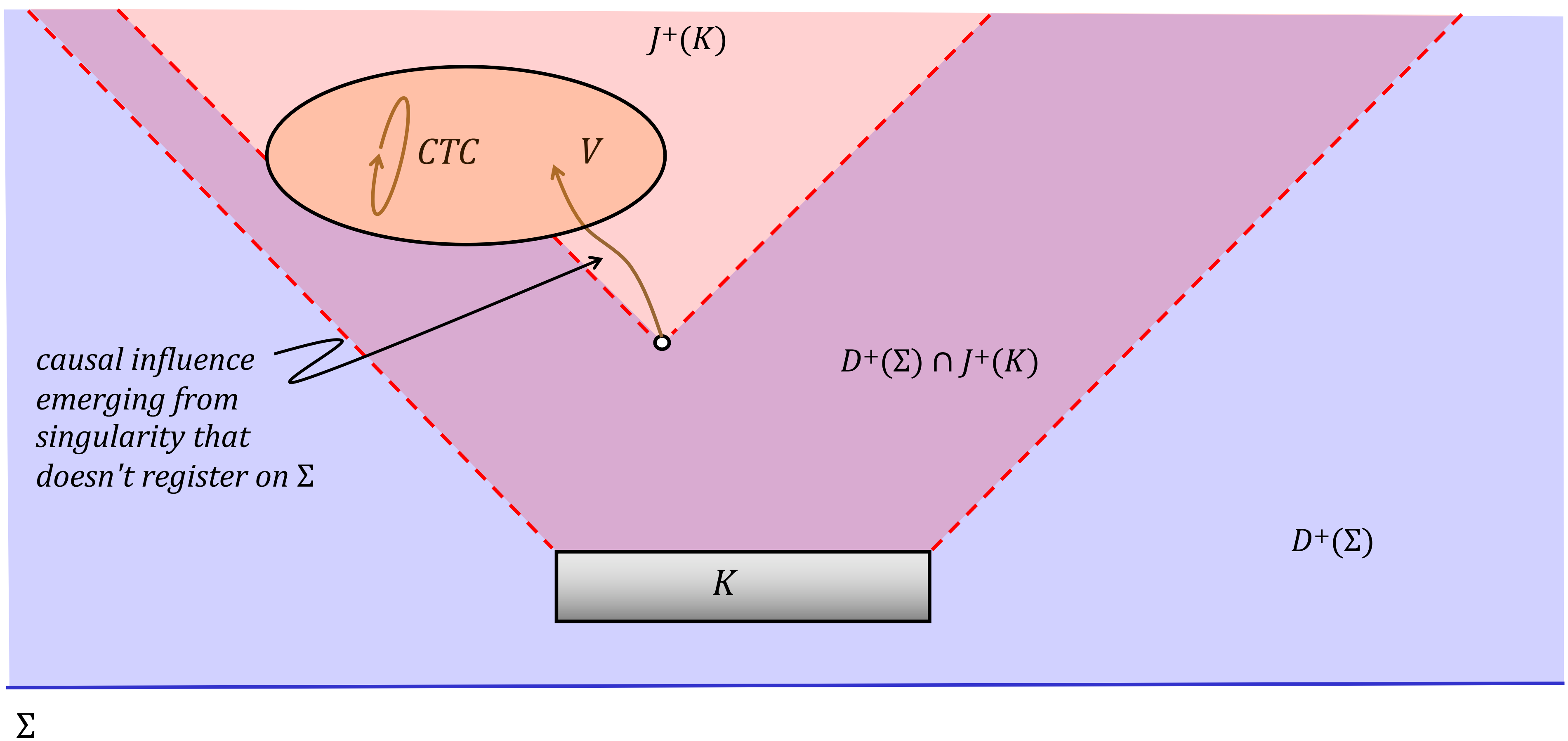
- What happens if there are holes in our TM spacetime?



Recall: Σ has no edges!

So: Initial conditions on Σ do not uniquely determine what goes on in V.

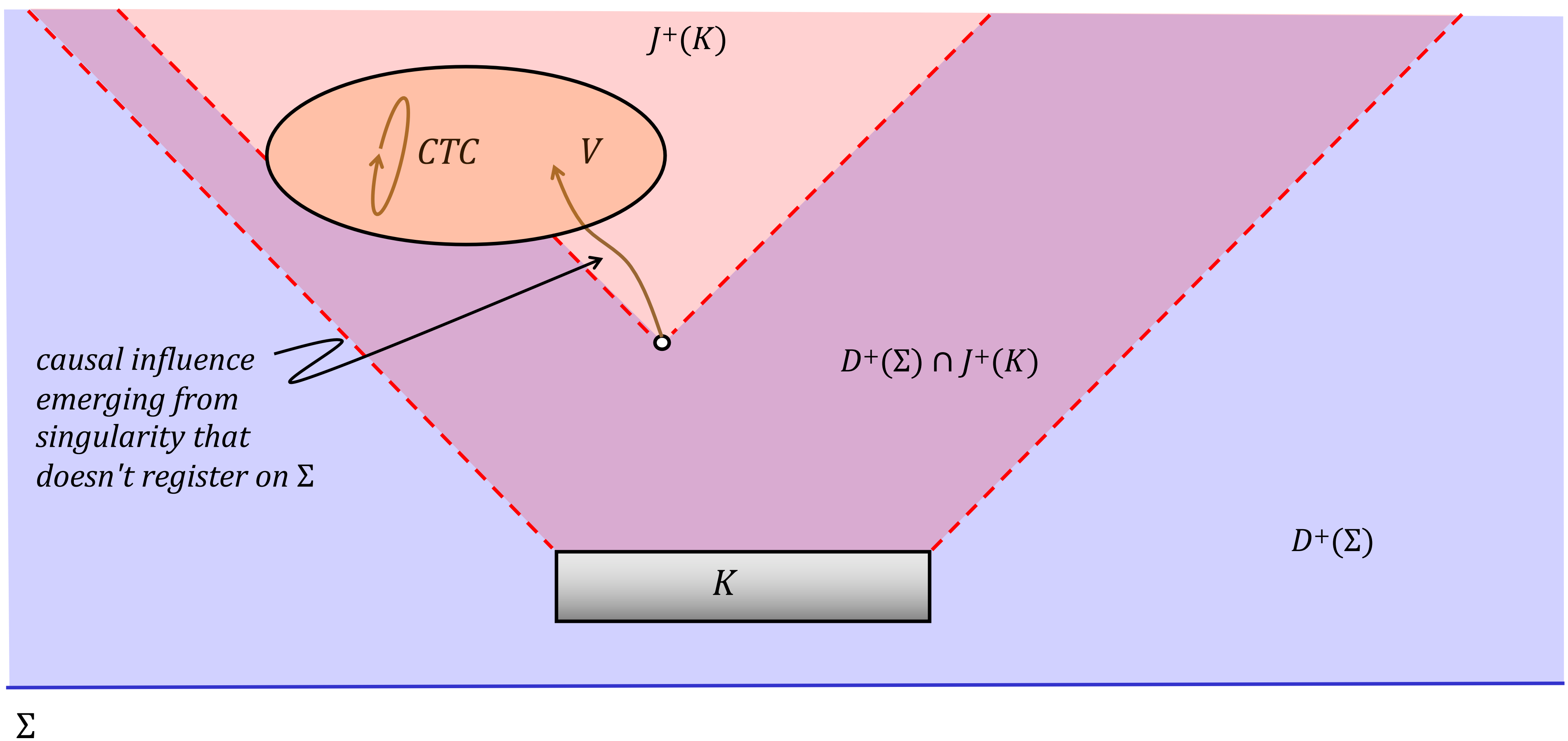
- *Which means*: V cannot be in $D^+(\Sigma)$.
- What happens if there are holes in our TM spacetime?
- V inside $J^+(K)$ and outside $D^+(\Sigma)$ allows for other influences on CTCs besides K.



Recall: Σ has no edges!

Hawking's (1992) Condition for TMs

In order for a TM to exist, the *future Cauchy Horizon* $H^+(\Sigma)$ of Σ must be *compactly generated* (generators cannot emerge from singularities or infinity).



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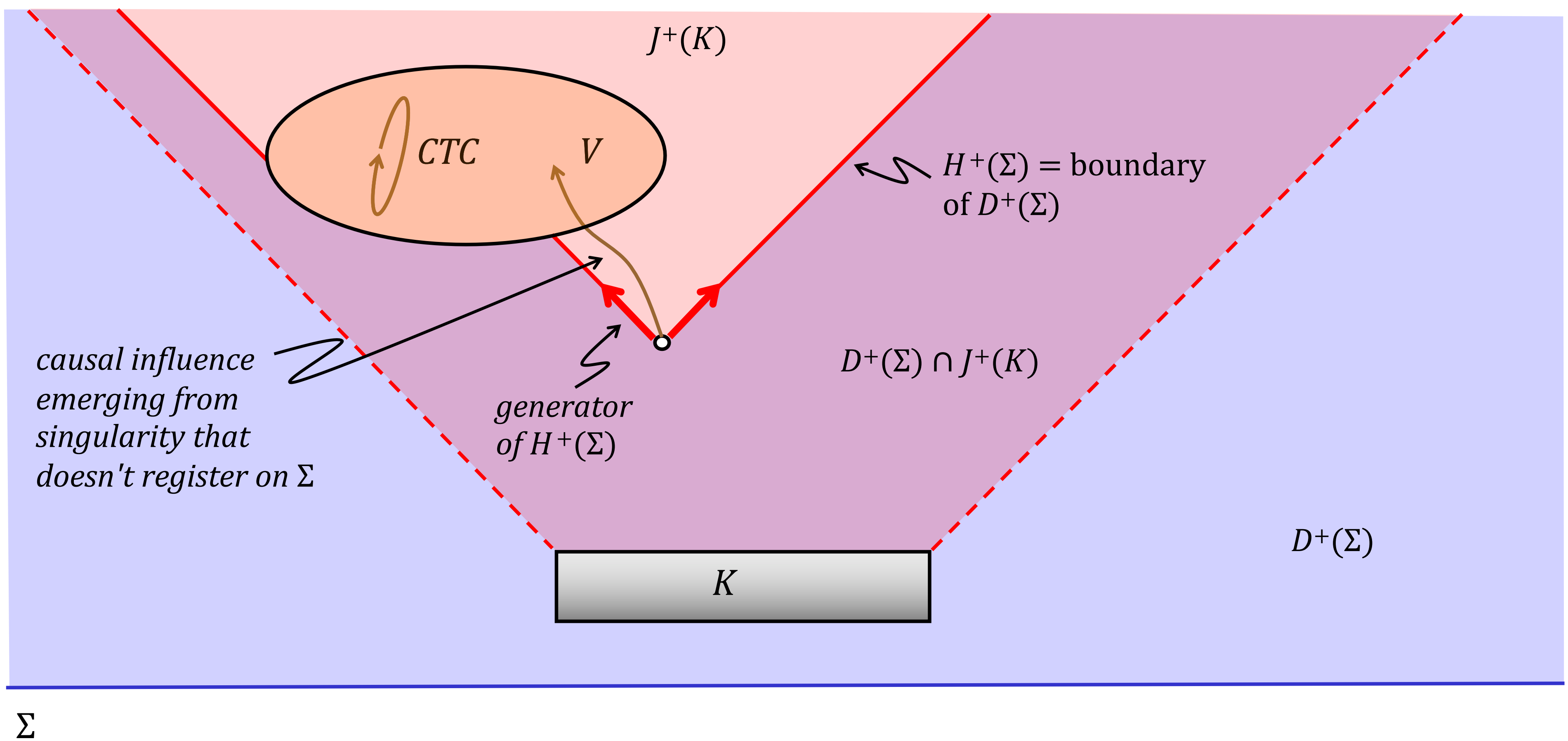
Hawking's (1992) Condition for TMs

In order for a TM to exist, the *future Cauchy Horizon* $H^+(\Sigma)$ of Σ must be *compactly generated* (generators cannot emerge from singularities or infinity).

- *Implication*: If TMs are possible, then $H^+(\Sigma)$ must be compactly generated.
- *Or*: If $H^+(\Sigma)$ is not compactly generated, then TMs are not possible.



Recall: Σ *has no edges!*

No-Go Theorems for TMs in GR: Argue for a particular *TM* condition and then show that it does not hold for physically relevant general relativistic spacetimes.

Hawking's (1992) Chronology Protection Conjecture

If the following hold for a partial Cauchy surface Σ,

(a) Σ is spatially open ("non-compact"),

(b) the Einstein equations hold,

(c) the "weak energy condition" holds (no negative energy),

then $H^+(\Sigma)$ is not compactly generated, and thus (according to *Hawking's Condition*), a TM cannot exist.

Is Hawking's condition necessary and sufficient for TMs?

- *Necessary?*

  If TMs are possible, then $H^+(\Sigma)$ must be compactly generated.

  Or: If $H^+(\Sigma)$ is not compactly generated, then TMs are not possible.

  There may be influences on V that don't register on Σ!

  - *But*: Just because there *may* be influences on V that don't register on Σ, doesn't mean there actually *are* such influences.

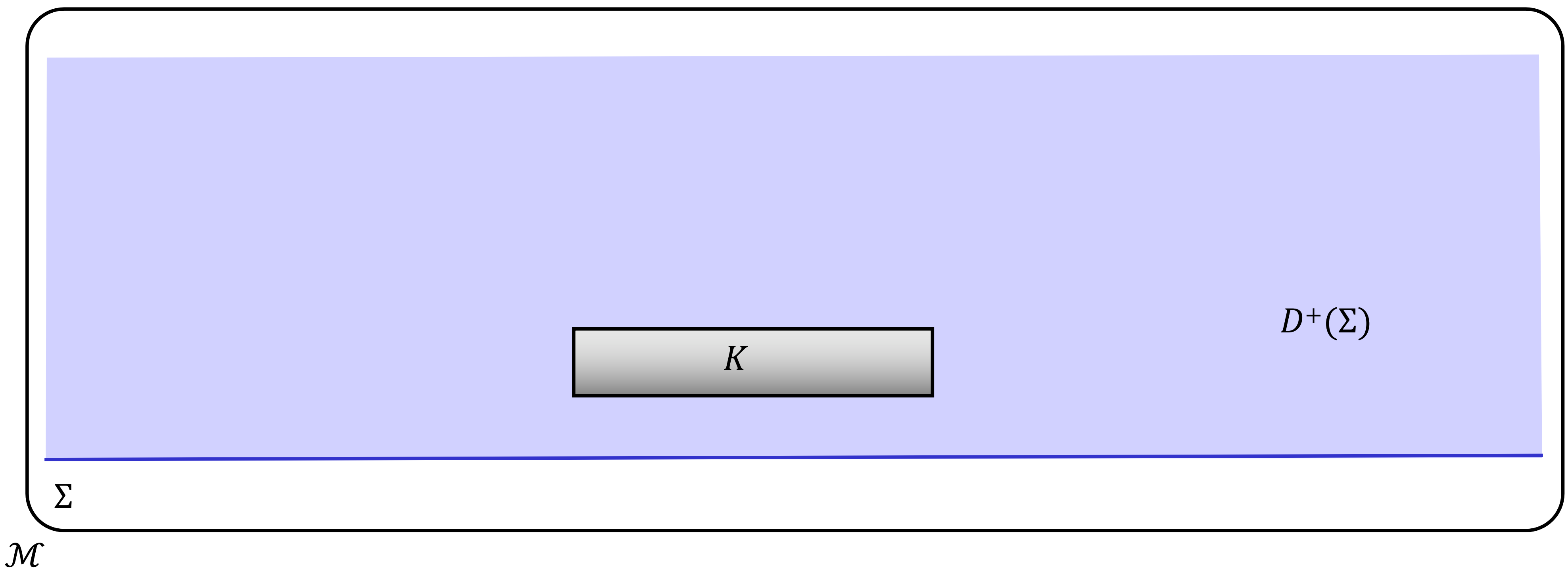
- *Sufficient?*

  If $H^+(\Sigma)$ is compactly generated, then TMs are possible.

  No influences on V get swallowed up by holes!

  - *But*: If no influences on V get swallowed up by holes, does this guarantee that Σ completely determines what goes on in V?

Def. An *extension* of $D^+(\Sigma)$ is an embedding of $D^+(\Sigma)$ as a proper subset of another "larger" spacetime.



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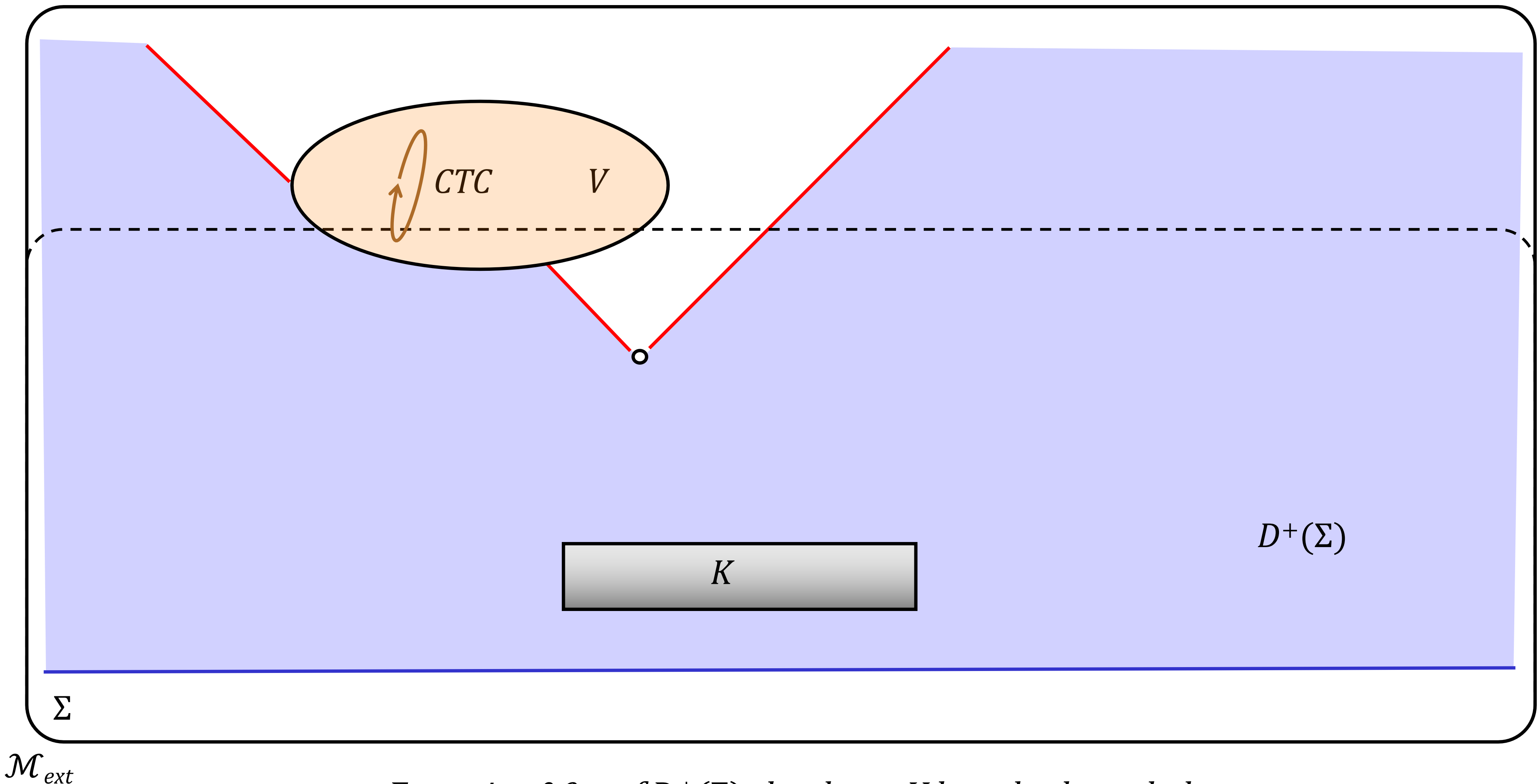
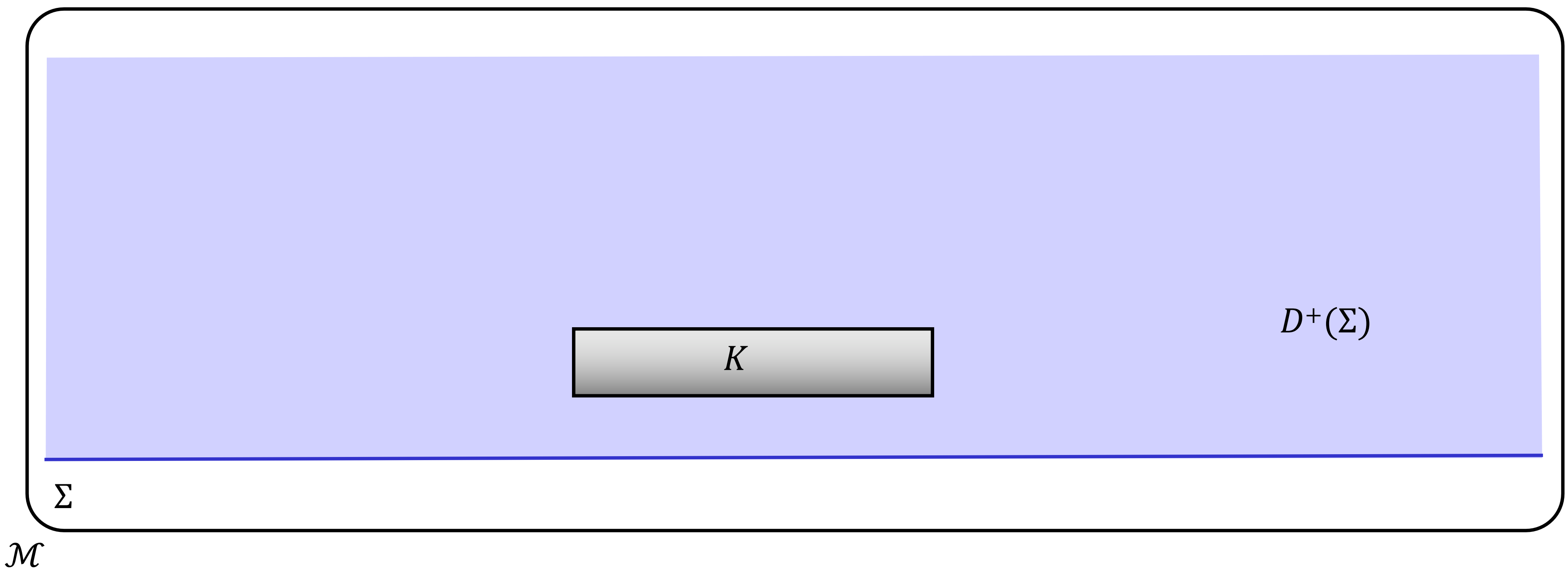


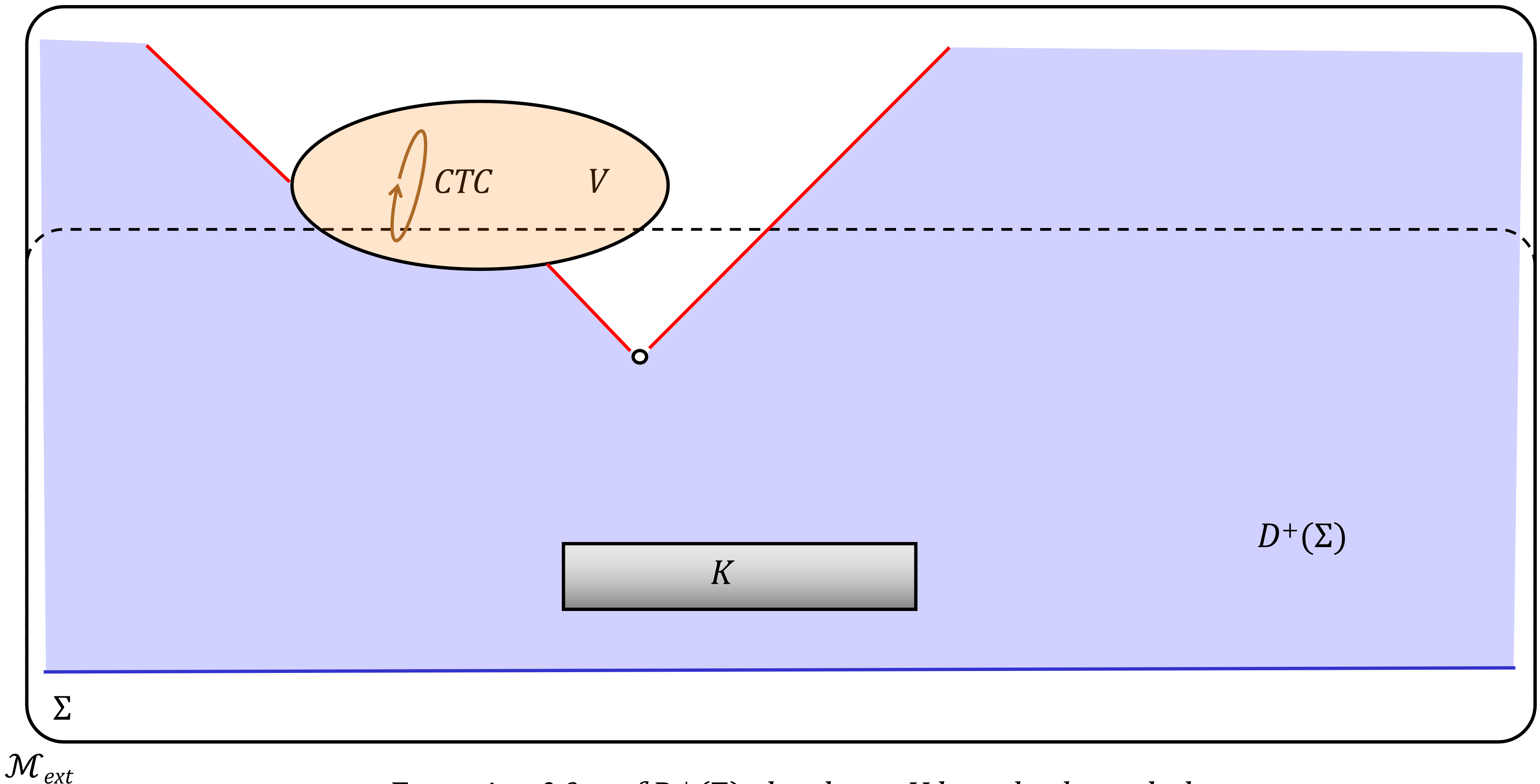
Extension $\mathcal{M}_{ext}$ of $D^+(\Sigma)$ that has a V but also has a hole

Def. An *extension* of $D^+(\Sigma)$ is an embedding of $D^+(\Sigma)$ as a proper subset of another "larger" spacetime.

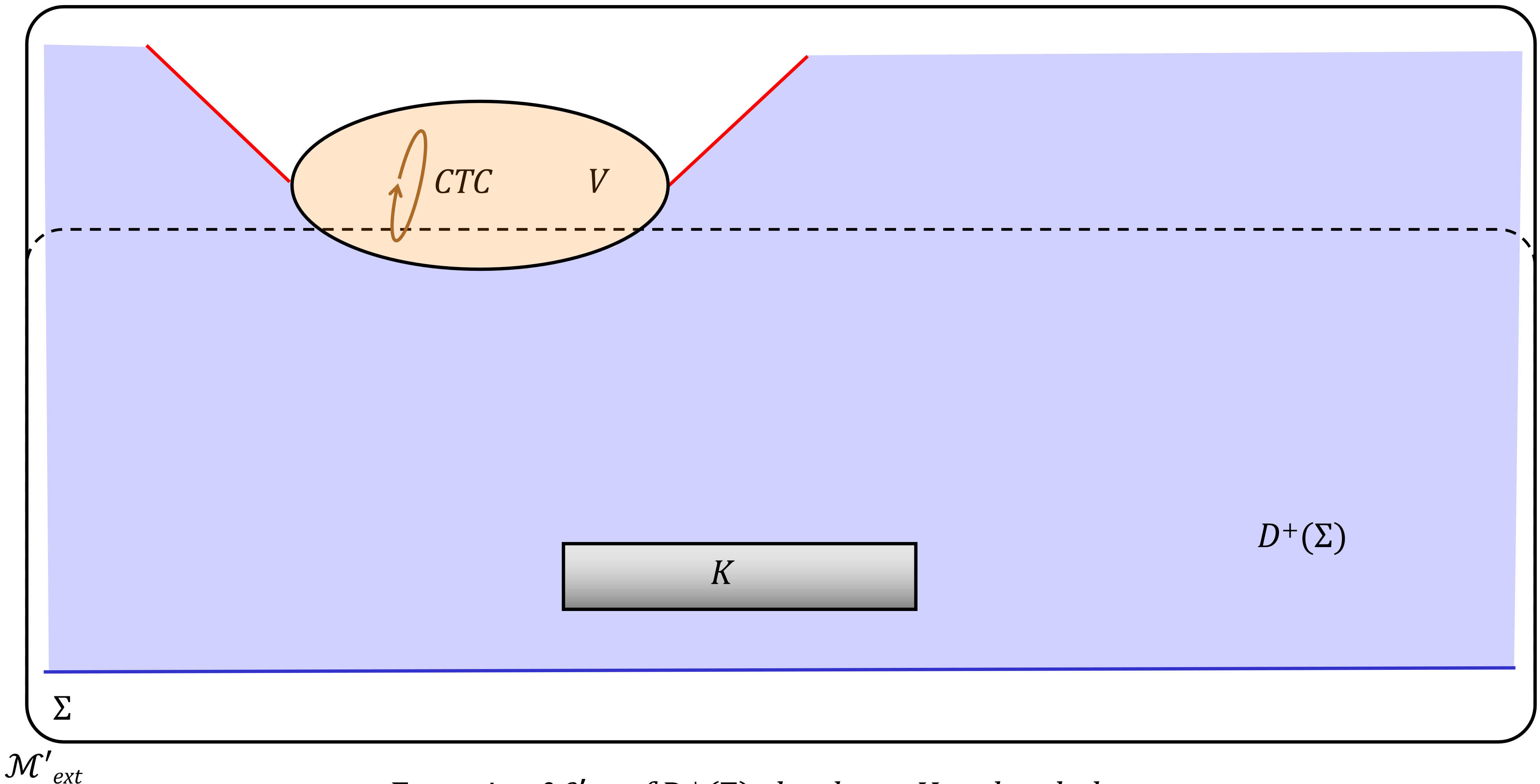
Earman, Smeenk & Wüthrich's (2009) Potency Condition for TMs
In order for a TM to exist, every smooth, maximal, "hole-free" extension of $D^+(\Sigma)$ that satisfies the Einstein equations and energy conditions must contain CTCs.

- There may be many ways of extending $D^+(\Sigma)$ into the future.
  - *Some extensions may contain holes. If so, we can't say conditions on Σ are uniquely responsible for CTCs that may exist.*
  - *Some extensions may not be "maximal"; i.e., they themselves may be further extended.*
  - *A non-maximal extension may contain holes. If so, we can't say conditions on Σ are uniquely responsible for CTCs that may exist.*
- *But*: If *all* maximal and hole-free extensions contain CTCs, then we can reasonably say that conditions on Σ *must* be responsible for these CTCs!

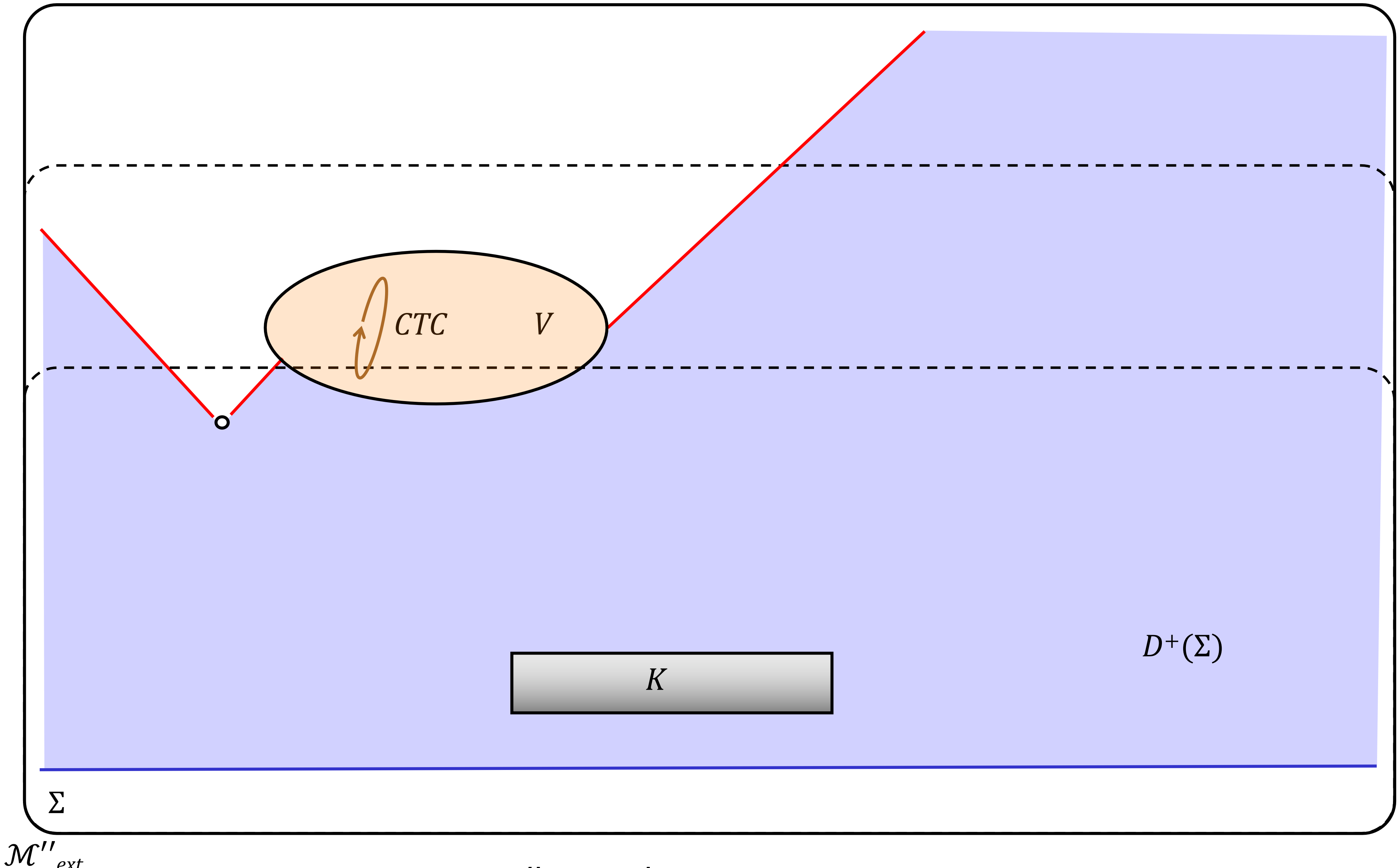

$D^+(\Sigma)$
K
Σ
$\mathcal{M}$



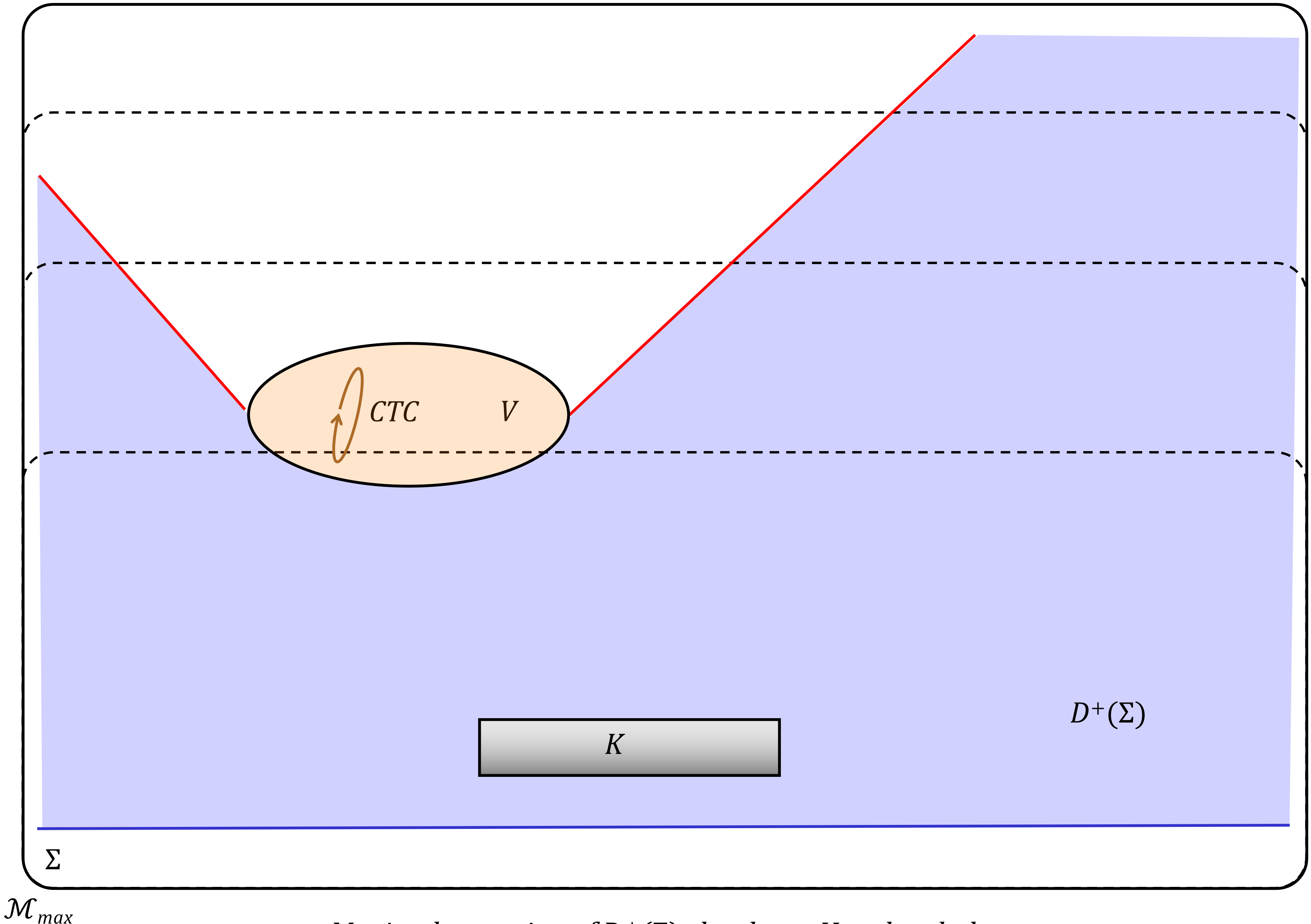
Extension $\mathcal{M}_{ext}$ of $D^+(\Sigma)$ that has a V but also has a hole



Extension $\mathcal{M}'_{ext}$ of $D^{+}(\Sigma)$ that has a V and no holes



Extension $\mathcal{M}''_{ext}$ of $\mathcal{M}'_{ext}$ that has a V and a hole



Maximal extension of $D^+(\Sigma)$ that has a V and no holes