15. Black Hole Thermodynamics

Topics:

1. Laws of B.H. Mechanics
2. Area & Entropy
3. Surface Gravity & Temp
4. Hawking Radiation

General Properties of Relativistic Black Holes

- *No Hair Conjecture*: A black hole is completely characterized by its mass M, charge Q, and angular momentum J.

Four types of black hole:

	nonrotating ($J = 0$)	*rotating* ($J \neq 0$)
uncharged ($Q = 0$)	Schwarzschild	Kerr
charged ($Q \neq 0$)	Reissner-Nordström	Kerr-Newman

- *Radius of event horizon*: $R_h = M + [M^2 - Q^2 - (J/M)^2]^{½}$
- *Area of event horizon*: $A = 4\pi[{R_h}^2 + (J/M)^2]$
  - *So*: A small change in mass δM will correspond to small changes in area δA, charge δQ, and angular momentum δJ.

Hawking (1971) Area Theorem: $\delta A \geq 0$ in any process.

- *Ex*. Suppose two black holes with areas A_1, A_2 collide to form black hole with area A_3. Then $A_3 \geq A_1 + A_2$.

Stephen Hawking
(1942-2018)

Looks like 2nd Law of Thermodynamics!

1. The Laws of Black Hole Mechanics. (Bardeen, Carter, Hawking 1973)

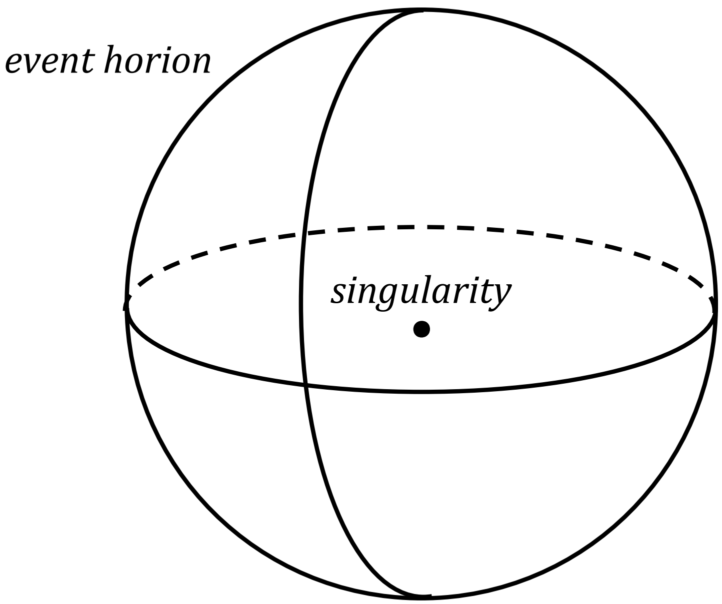
	Black Hole Mechanics	*Thermodynamics*
0th Law	Surface gravity κ is constant over the event horizon of a stationary black hole. *κ = acceleration needed to keep an object at event horizon.*	Temperature T is constant throughout a body in thermal equilibrium.
1st Law	$\delta M = (1/8\pi)\kappa\delta A + \cdots$	$\delta E = T\delta S + \cdots$ *E = energy*
2nd Law	$\delta A \geq 0$ in any process.	$\delta S \geq 0$ in any process. *S = entropy*
3rd Law	$\kappa = 0$ is not achievable by any process.	$T = 0$ is not achievable by any process.

- Formally identical if $A/4 = S$ and $(1/2\pi)\kappa = T$.

Is this merely a formal equivalence, or does it have a physical basis?

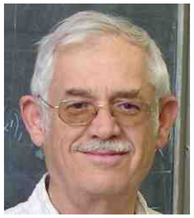
2. Area and Entropy.

- *Question*: What happens when a physical system with a large amount of entropy is thrown into a black hole?



- *Entropy of coffee cup disappears!*
- *Violation of 2nd Law for closed system of black hole + coffee cup?*

- *Bekenstein (1973)*: Suppose black holes have an entropy S_{bh} proportional to their area: $S_{bh} = f(A) = A/4$.



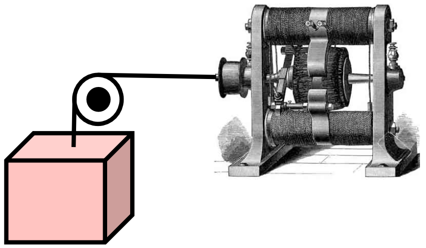
Jacob Bekenstein
2012 Wolf Prize
(NYU-Tandon grad!)
(1947-2015)

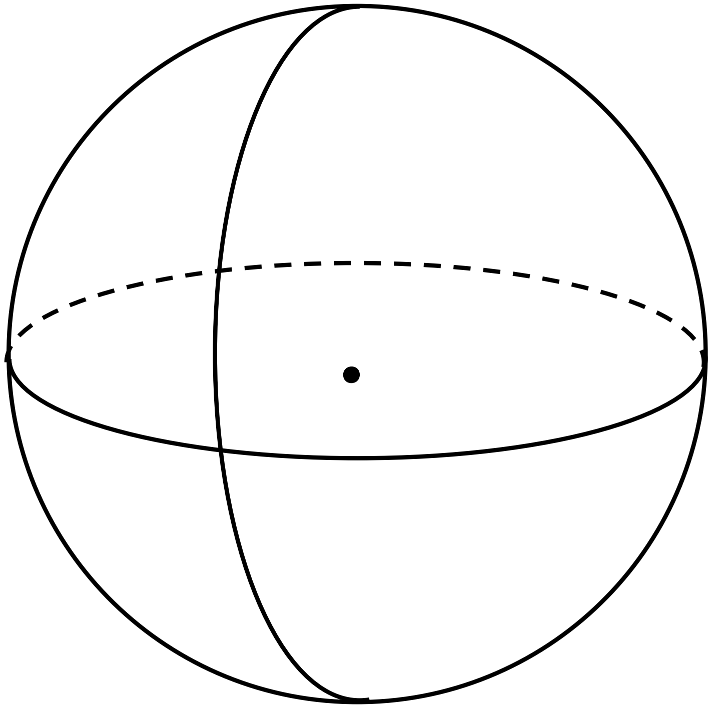
Generalized Second Law of Thermodynamics (GSL)

$$\underbrace{\delta S_{bh}}_{\text{Always positive!}} + \underbrace{\delta S}_{\text{Negative for coffee cup!}} \geq 0$$

Problem (Geroch 1971)

1. Lower box of radiation with high entropy toward event horizon.
2. Use weight to generate work.





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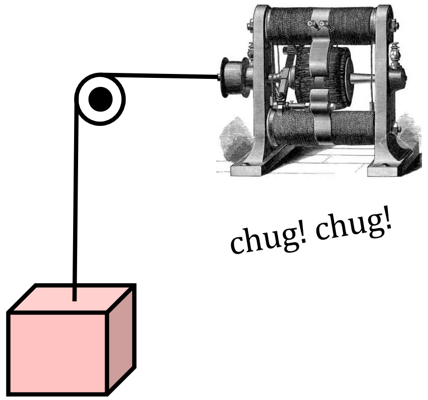

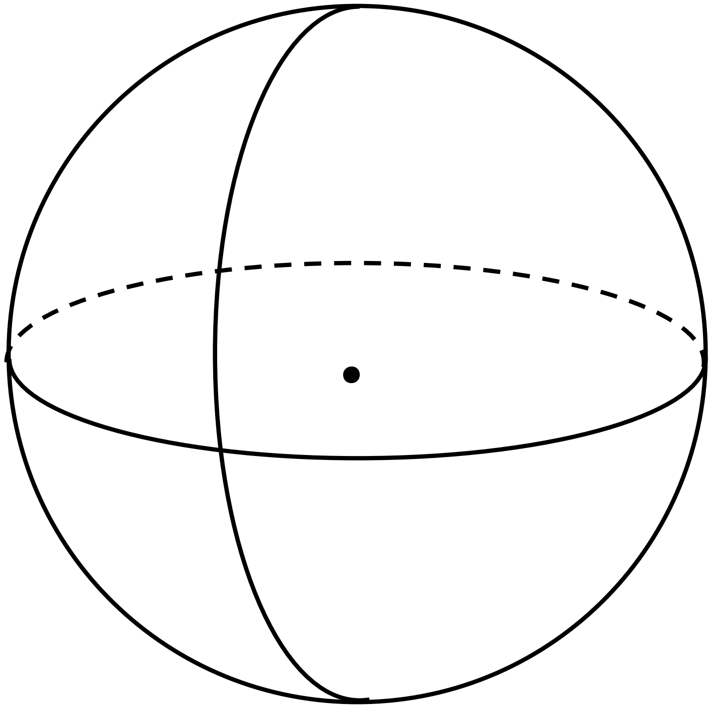

Problem (Geroch 1971)

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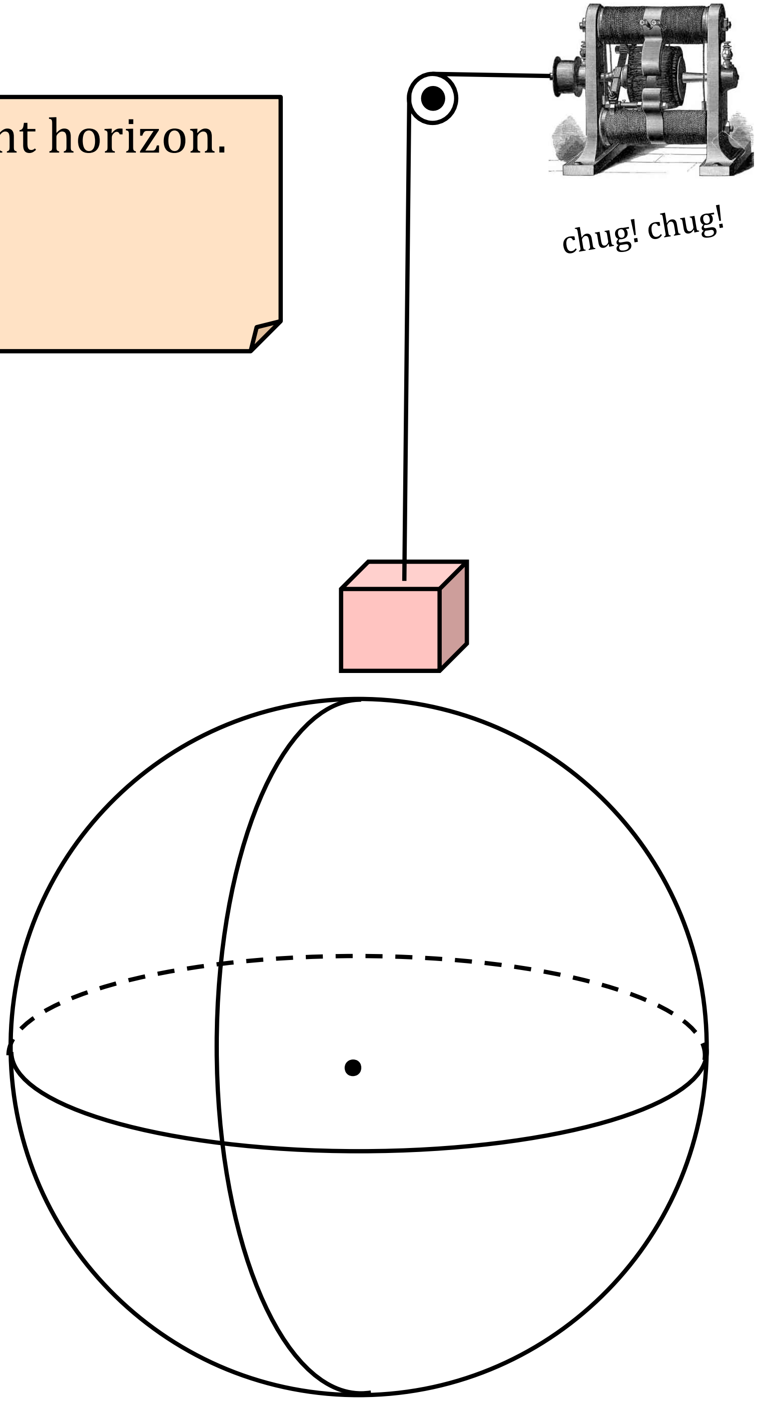
3. At event horizon dump radiation in.



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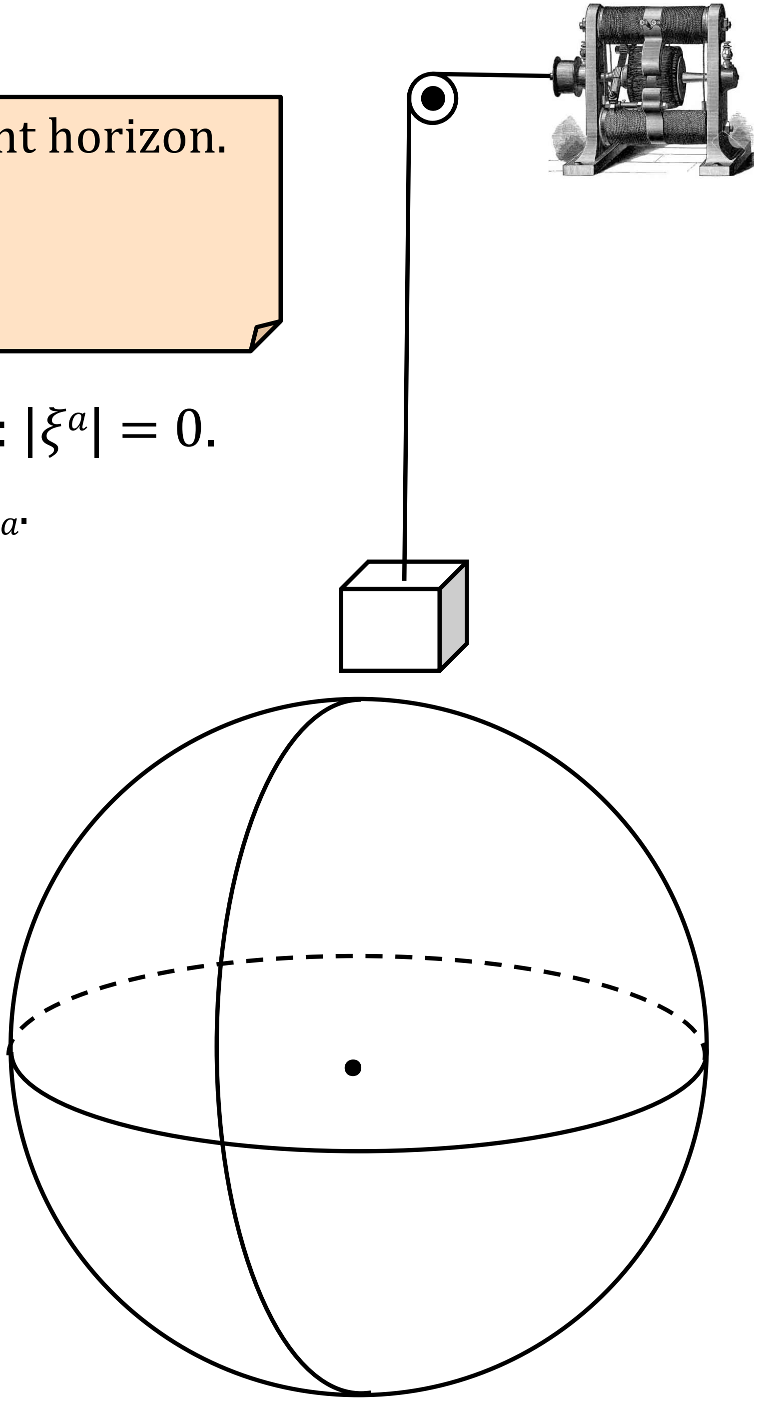


- At event horizon, time-translation vector ξ^a is *null*: $|\xi^a| = 0$.
  - *So*: *At event horizon, the box has zero energy,* $E = -\xi^a p_a$.
- *So*: If box can reach horizon, then no increase in area at Step (3).
  - *Thus*: $\delta S_{bh} = 0$.
  - *But*: $\delta S < 0$.
  - *Thus*: $\delta S_{bh} + \delta S < 0$. *Violation of GSL!*
- *Bekenstein conjecture (1973)*: Box has finite size, so can't reach horizon.

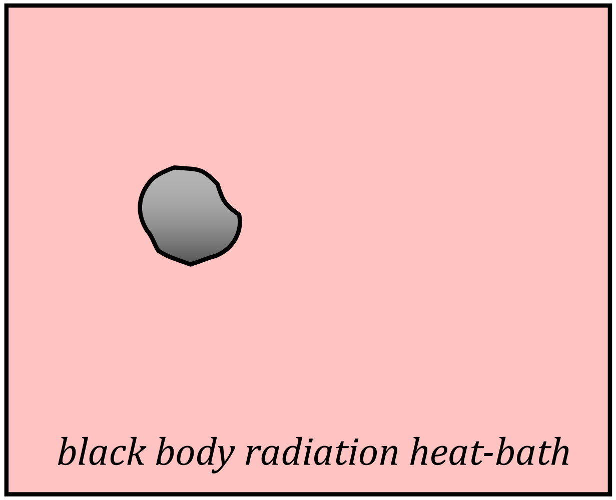
3. Surface Gravity κ and Temperature T.

- *Recall*: Laws of Black Hole Mechanics look like Laws of Thermodynamics if we equate surface gravity κ with temperature: $(1/2\pi)\kappa = T$.

  How seriously should we take this?

- *Claim*: A black hole should be assigned *zero* absolute temperature!

- *Black body* = object that absorbs all incident radiation.
- *Black body radiation* = radiation emitted by a black body in thermal equilibrium.
- *Effective temperature* of an object = temperature of a black body that would emit the same total amount of radiation as the object.
- *How to measure effective temp*: Put object in thermal equilibrium with black body radiation and measure temperature of latter.



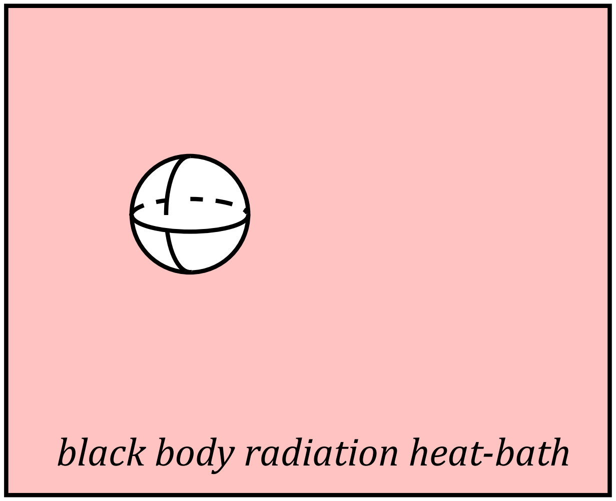
- *Object in equilibirum with heat bath.*
- $T_{object} = T_{heat\text{-}bath}$

Refined Claim: The *effective temperature* of a black hole is absolute zero.

> *"Proof"*: "...a black hole cannot be in equilibrium with black body radiation at any non-zero temperature, because no radiation could be emitted from the hole whereas some radiation would always cross the horizon into the black hole." (Bardeen, Carter, Hawking 1973, pg. 168.)

- *Conclusion*: "In classical black hole physics, κ has nothing to do with the physical temperature of a black hole..." (Wald 1994, pg. 149.)
- *But*: This argument depends on quantum mechanics (black body radiation can only be characterized quantum-mechanically).

Planck's (1900) quantum-mechanical formula for energy distribution of black body radiation: $E(\nu) = h\nu/(e^{h\nu/kT} - 1)$.



Is there a "classical" proof of Claim A?

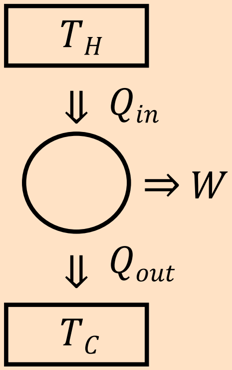
- *Black hole in heat bath.*
- *Equilibrium cannot be established.*

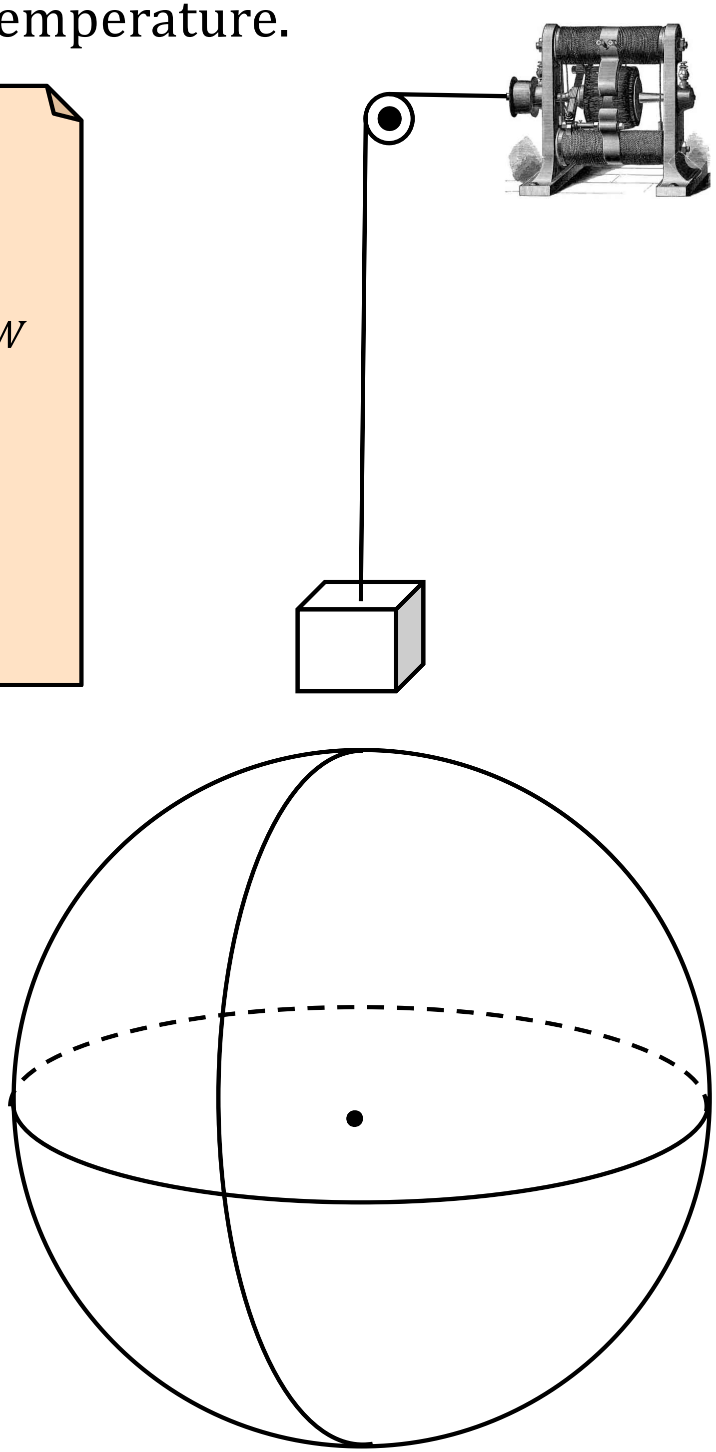
Claim: A black hole should be assigned *zero* absolute temperature.

Classical Proof: Consider "Geroch heat engine":

- T_H = temperature of box at initial position.
- T_C = temperature of black hole.
- *Efficiency* $= W/Q_{in} = (Q_{in} - Q_{out})/Q_{in}$

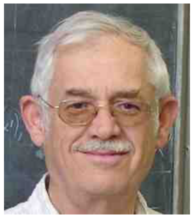
$$\begin{aligned} &= 1 - Q_{out}/Q_{in} \\ &= 1 - T_C/T_H \\ &= 1 \quad (\textit{if}\text{ all energy of box goes into work}) \end{aligned}$$





- *So*: $T_C = 0$, *if* all energy of box goes into work.
- *In other words*: $T_C = 0$, *if* box can reach horizon.

But:

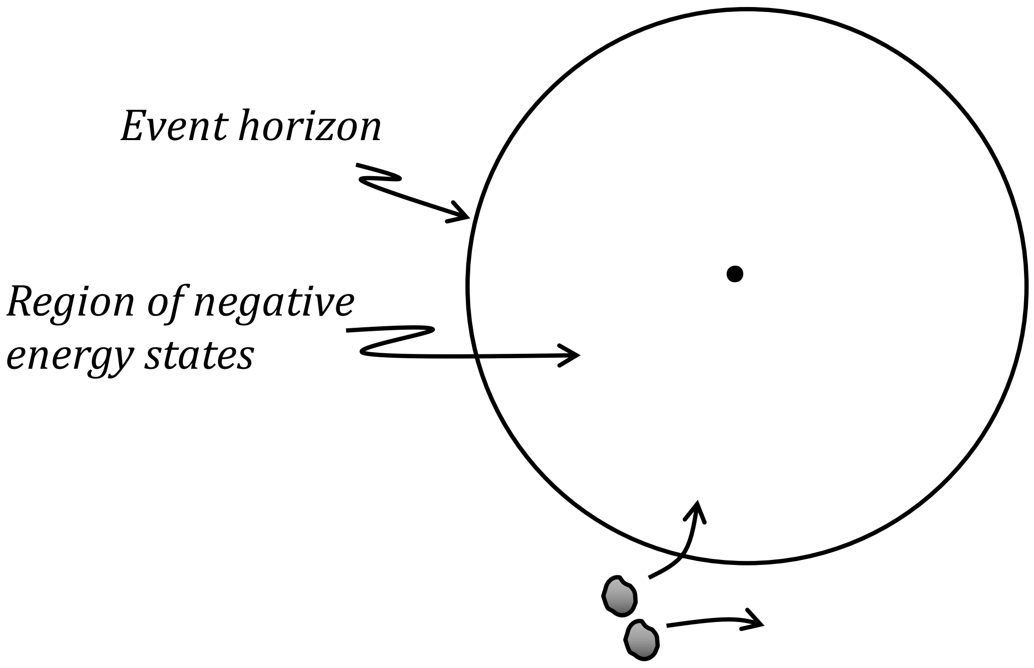


Finite box can't reach horizon!
(Bekenstein 1973)

4. Hawking Radiation.

Hawking (1975): Black holes emit radiation at the same rate that a black body would at temperature $T = (1/2\pi)\kappa$*!*

"One might picture this...in the following way. Just outside the event horizon there will be virtual pairs of particles, one with negative energy and one with positive energy. The negative particle is in a region which is classically forbidden but it can tunnel through the event horizon to the region inside the black hole where the Killing vector which represents time translations is spacelike. In this region the particle can exist as a real particle with a timelike momen-tum vector even though its energy relative to infinity as measured by the time translation Killing vector is negative. The other particle of the pair, having a positive energy, can escape to infinity where it constitutes a part of the thermal emission described above. The probability of the negative energy particle tunnelling through the horizon is governed by the surface gravity κ since this quantity measures the gradient of the magnitude of the Killing vector or, in other words, how fast the Killing vector is becoming spacelike."

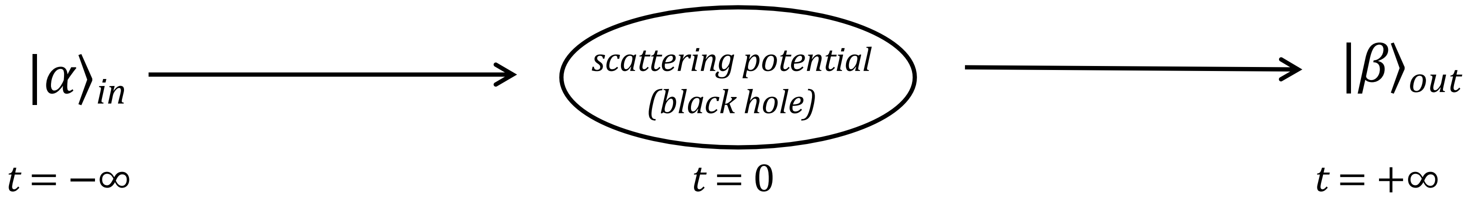


- *Particle/antiparticle pair production in quantum vacuum near event horizon.*
- *Negative energy antiparticle falls through event horizon and falls into singularity, decreasing black hole's area.*
- *Positive energy particle escapes in form of thermal radiation.*

"It should be emphasized that these pictures of the mechanism responsible for the thermal emission and area decrease are heuristic only and should not be taken too literally... The real justification of the thermal emission is the mathematical derivation..."

Technical Aside. *"...the mathematical derivation..."*

Black hole acts as scattering potential for particle states of a quantum field φ.



$|\alpha\rangle_{in} \longrightarrow$ *scattering potential (black hole)* $\longrightarrow |\beta\rangle_{out}$

$t = -\infty$ $\quad t = 0$ $\quad t = +\infty$

Particle states in distant past:

- Expand φ in basis $\{f_\omega\}$ of positive frequency solutions with respect to past:
  $$\varphi = \int d\omega(a_\omega f_\omega + a_\omega^\dagger f_\omega^*)$$
- $a_\omega^\dagger$, a_ω are raising/lowering operators for "in" particle states.
- "In" vacuum $|0\rangle_{in}$ = state with no "in" particles.

Particle states in distant future:

- Expand φ in basis $\{p_\omega, q_\omega\}$, where p_ω are +freq solutions w.r.t. future, and q_ω are solutions w.r.t. event horizon:
  $$\varphi = \int d\omega(b_\omega p_\omega + b_\omega^\dagger p_\omega^* + c_\omega q_\omega + c_\omega^\dagger q_\omega^*)$$
- $b_\omega^\dagger$, b_ω are raising/lowering operators for "out" particle states.
- "Out" vacuum $|0\rangle_{out}$ = state with no "out" particles.

The Main Result:

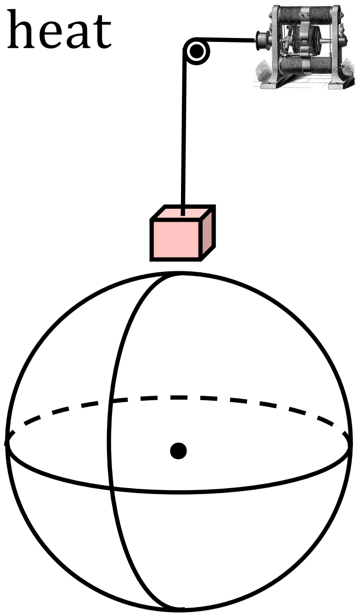
$${}_{in}\langle 0|b_\omega^\dagger\, b_\omega|0\rangle_{in} = \frac{\Gamma_\omega}{e^{2\pi\omega/\kappa} - 1}$$

number of "out" particles in "in" vacuum

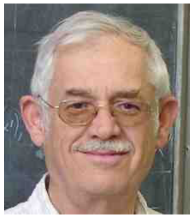
energy distribution of black body radiation with temperature $\kappa/2\pi$

- *So*: "In" vacuum of a quantum field in region of black hole is full of black body radiation!
  - *But*: $|0\rangle_{in}$ and $|0\rangle_{out}$ belong to *(unitarily) inequivalent representations* of the quantum field.
  - *Which means*: It's mathematically incoherent to write ${}_{in}\langle 0|b_\omega^\dagger\, b_\omega|0\rangle_{in}$.

Claim (Unruh and Wald 1982): Hawking radiation prevents Geroch heat engine from violating Generalized Second Law.



- *Recall*: *If* box can reach horizon, then $\delta S_{bh} = 0$, $\delta S < 0$, and thus $\delta S_{bh} + \delta S < 0$. *Violation of GSL!*
- *But*: Hawking radiation generates *buoyancy* that prevents box from reaching horizon!
  - *Hawking radiation justifies Bekenstein's conjecture!*



Finite box can't reach horizon!
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