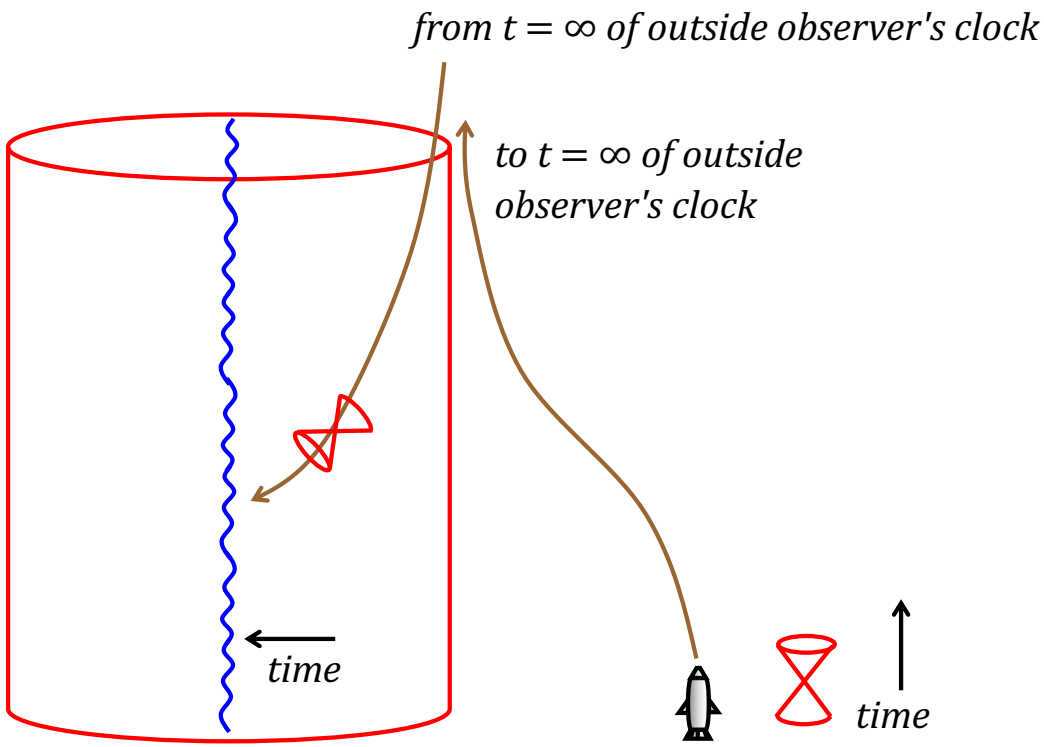
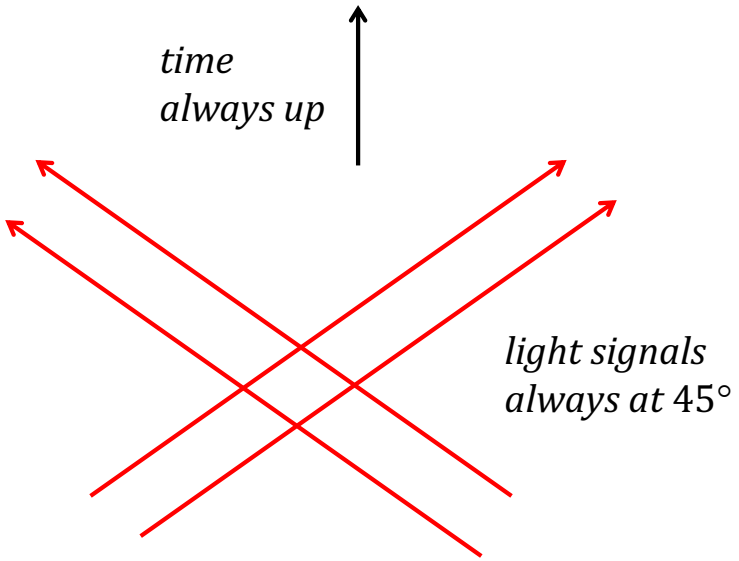
14. Black Holes 2

1. Conformal Diagrams of Black Holes

Topics:
1. Conformal Diagrams
2. Schwarzschild Black Hole
3. Charged Rotating Black Hole
4. Cosmic Censorship Hypothesis

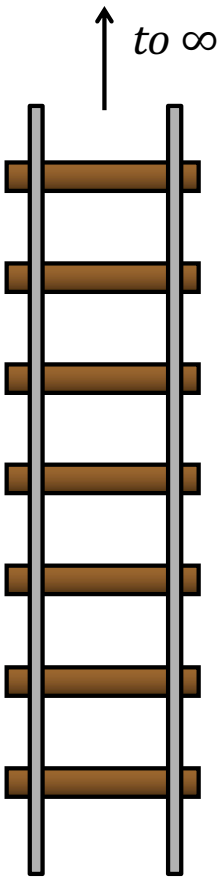


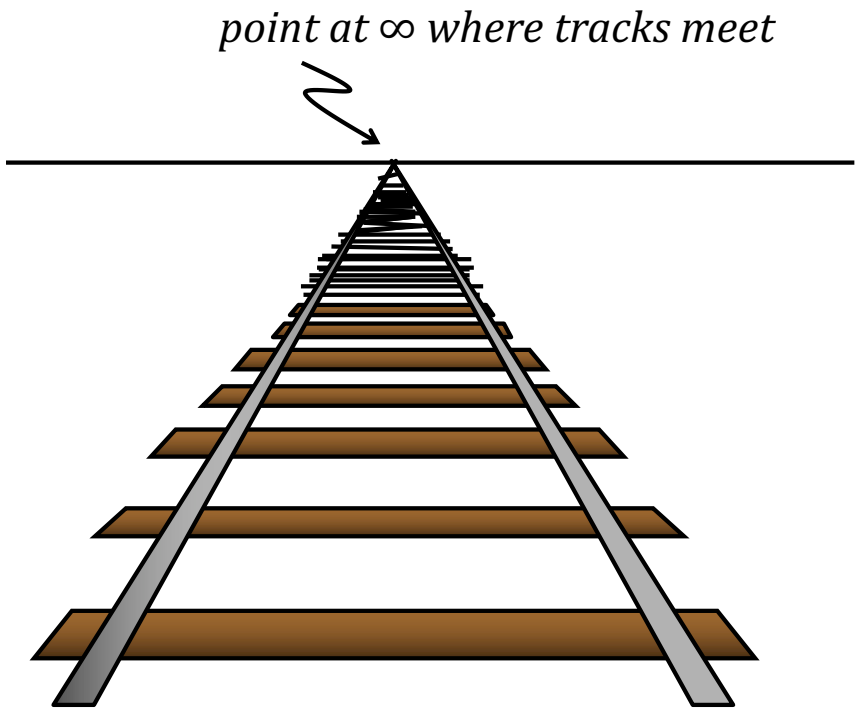


- What is the *causal structure* of this diagram?
  - *Lightcones tip and time points in different directions!*
- Could an outside observer swoop in very close to event horizon and somehow see inside?

Need a diagram in which causal structure is invariant.

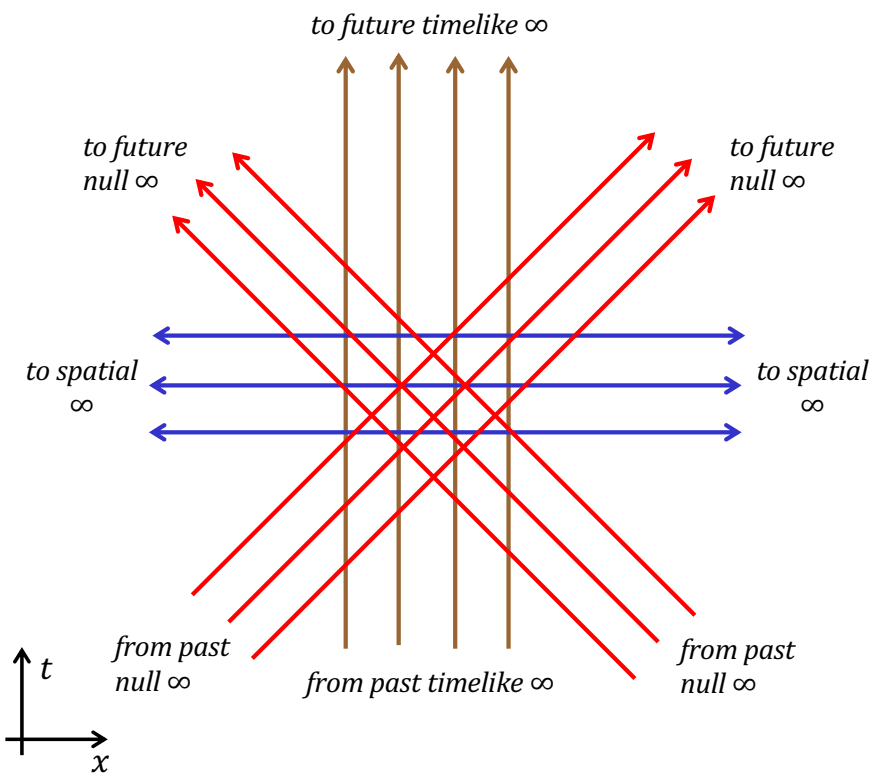
How conformal diagrams work





- Train tracks to ∞ on flat plane.
- *Now*: Change perspective to represent ∞ *in* diagram.
- Do the same thing for *Minkowski spacetime.*

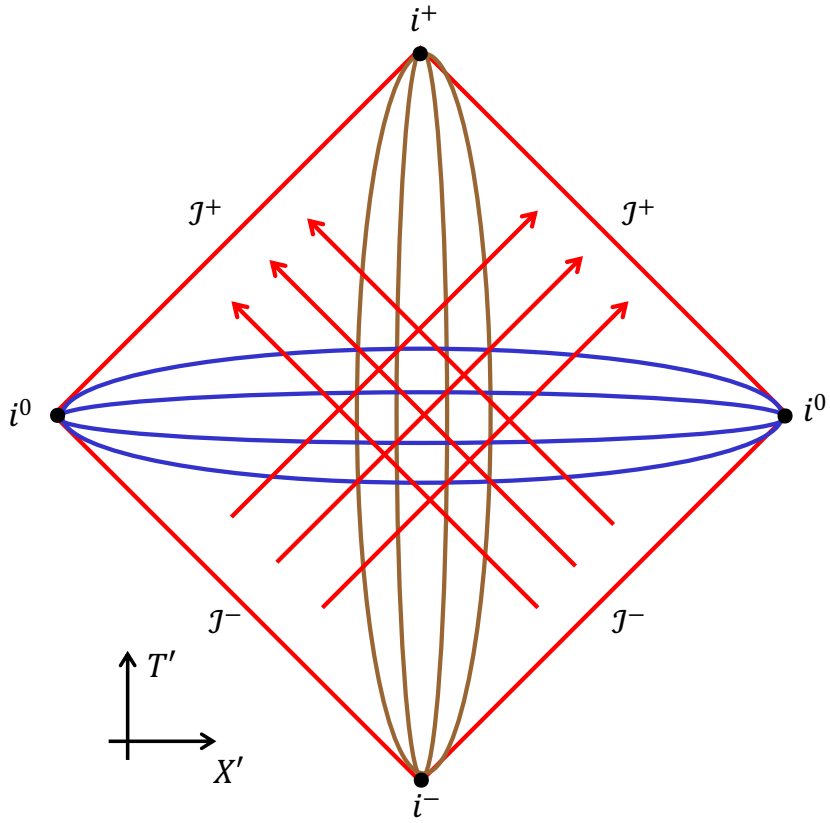
Major Difference: There are three distinct types of infinity in Minkowski spacetime: timelike, spacelike, and lightlike!



Regular Spacetime Diagram of Minkowski spacetime in (t, x) coordinates

$(t, x) \rightarrow (T', X')$

conformal coordinate transformation



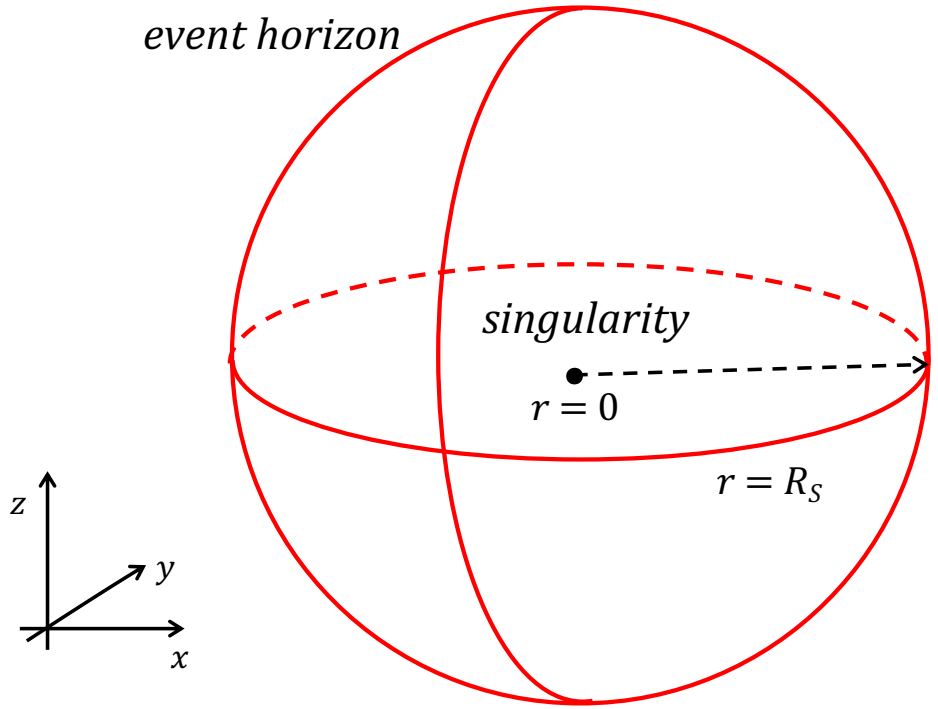
Conformal Diagram of Minkowski spacetime in (T', X') coordinates

Conformal Diagram of Minkowski Spacetime:

- Surfaces: $\mathcal{I}^+$ = future null infinity; $\mathcal{I}^-$ = past null infinity.
- Points: i^0 = spatial infinity; $i^\pm$ = future/past timelike infinity.
- *Important feature*: Lightlike worldlines are everywhere at 45°.

2. Schwarzschild Black Hole

- No charge or rotation (Schwarzschild 1916).

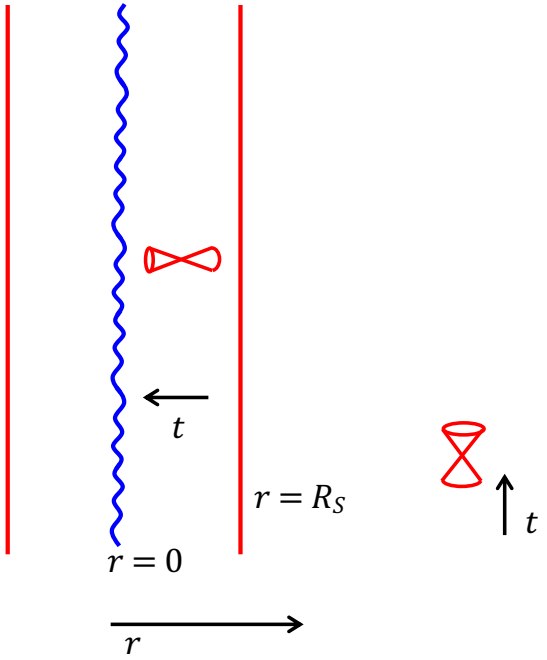


R_S = "Schwarzschild radius"
= radius of event horizon

- Spatial Diagram in (x, y, z) coordinates.
- Can transform to spherical coordinates (r, θ, ϕ), or just r if we suppress the angular coordinates.

Now add time coordinate to get a regular spacetime diagram in (t, r) coordinates...

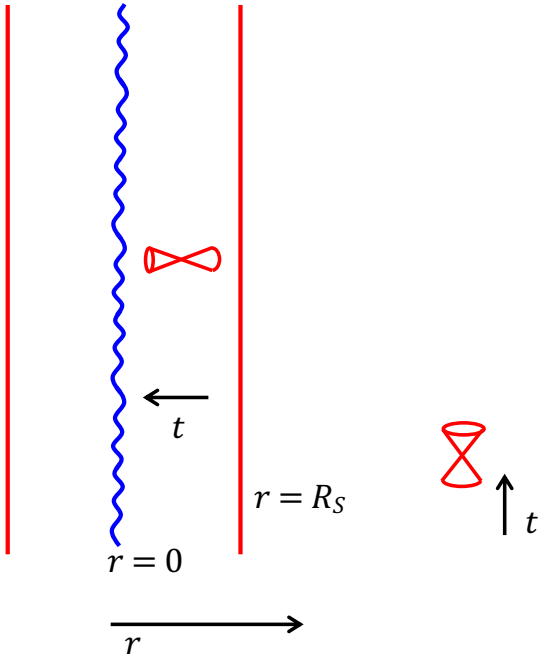
2. Schwarzschild Black Hole



Regular spacetime diagram
in (t, r) coordinates

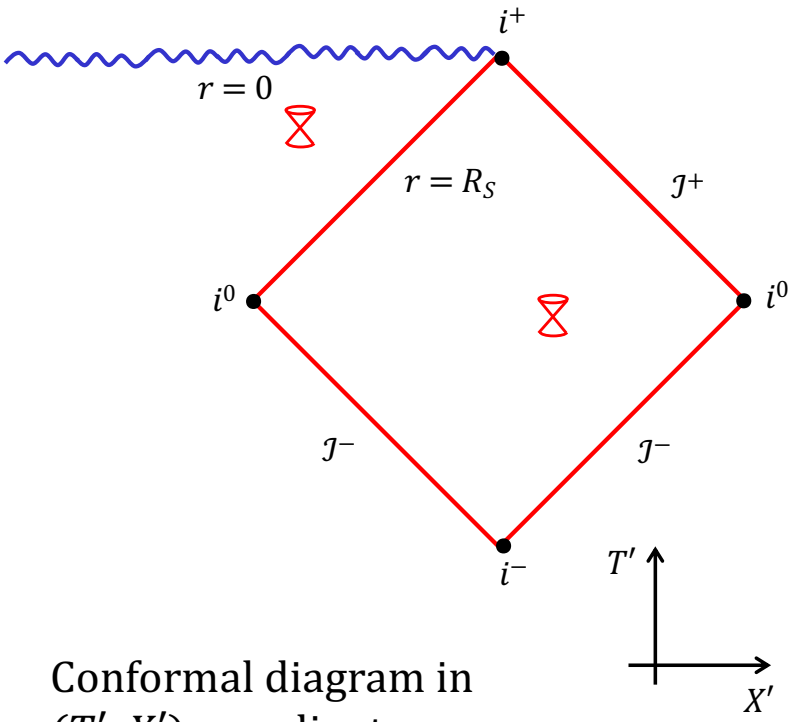
- Singularity is a *spacelike* surface and it is the future of all timelike worldlines within the event horizon.
- Event horizon is a *lightlike* surface.
- Now perform conformal coordinate transformation $(t, r) \rightarrow (T', X')$.

2. Schwarzschild Black Hole



Regular spacetime diagram in (t, r) coordinates

$(t, r) \rightarrow (T', X')$



Conformal diagram in (T', X') coordinates

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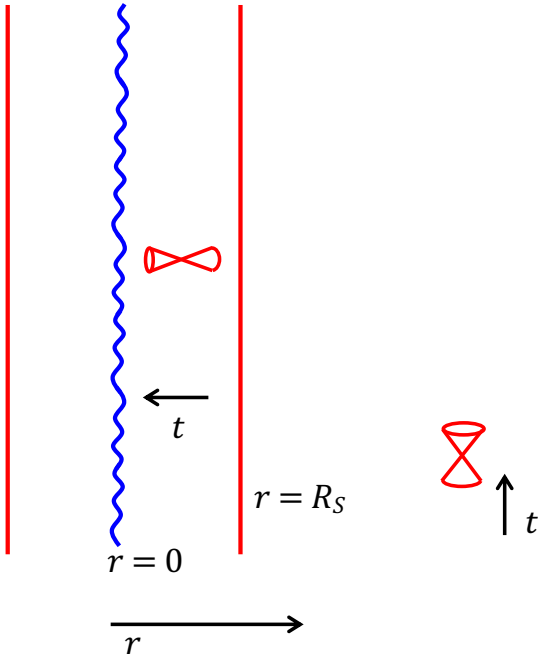
- Now perform conformal coordinate transformation $(t, r) \rightarrow (T', X')$.
- *Technical result*: (T', X') coordinates cover the original spacetime twice.

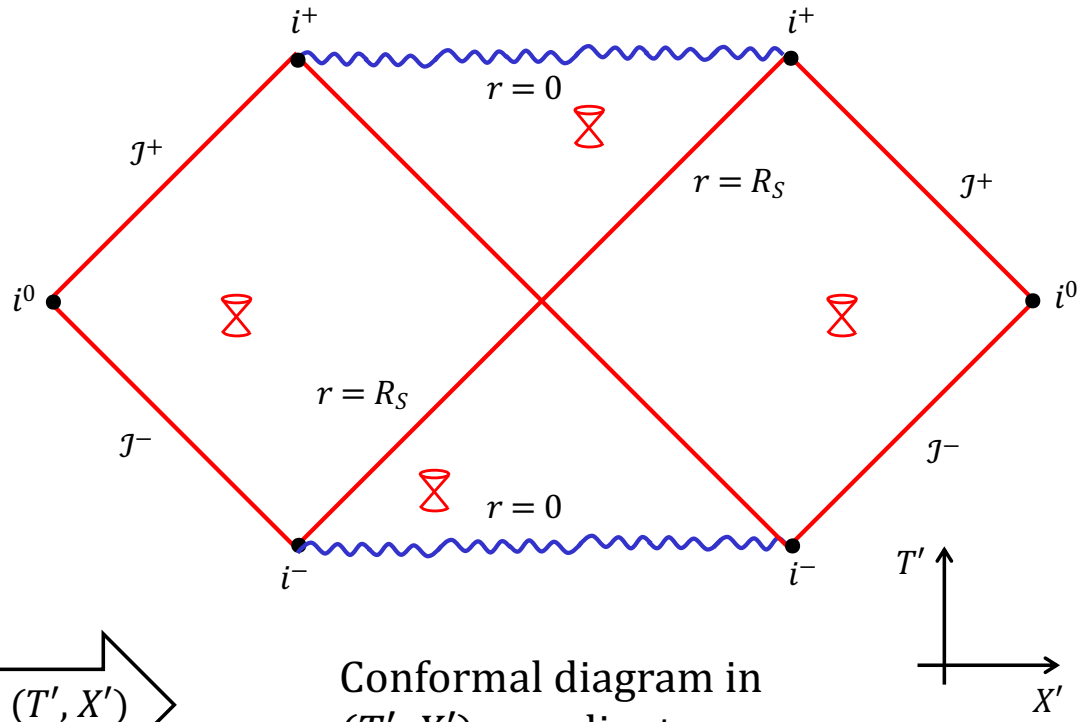
Can "extend" the diagram to represent full (T', X') coverage...

2. Schwarzschild Black Hole



Regular spacetime diagram in (t, r) coordinates

$(t, r) \to (T', X')$



Conformal diagram in (T', X') coordinates

- Singularity is a *spacelike* surface and it is the future of all timelike worldlines within the event horizon.
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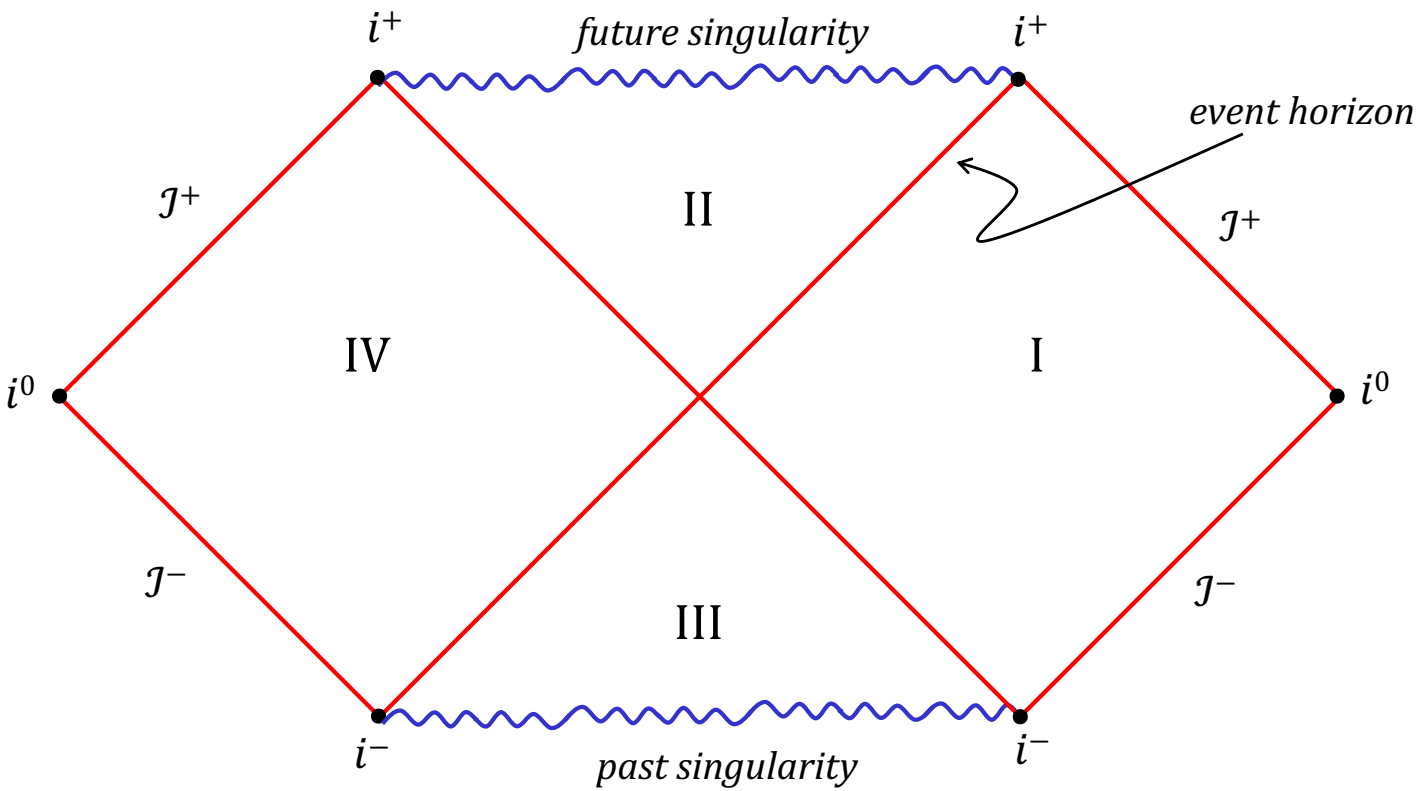
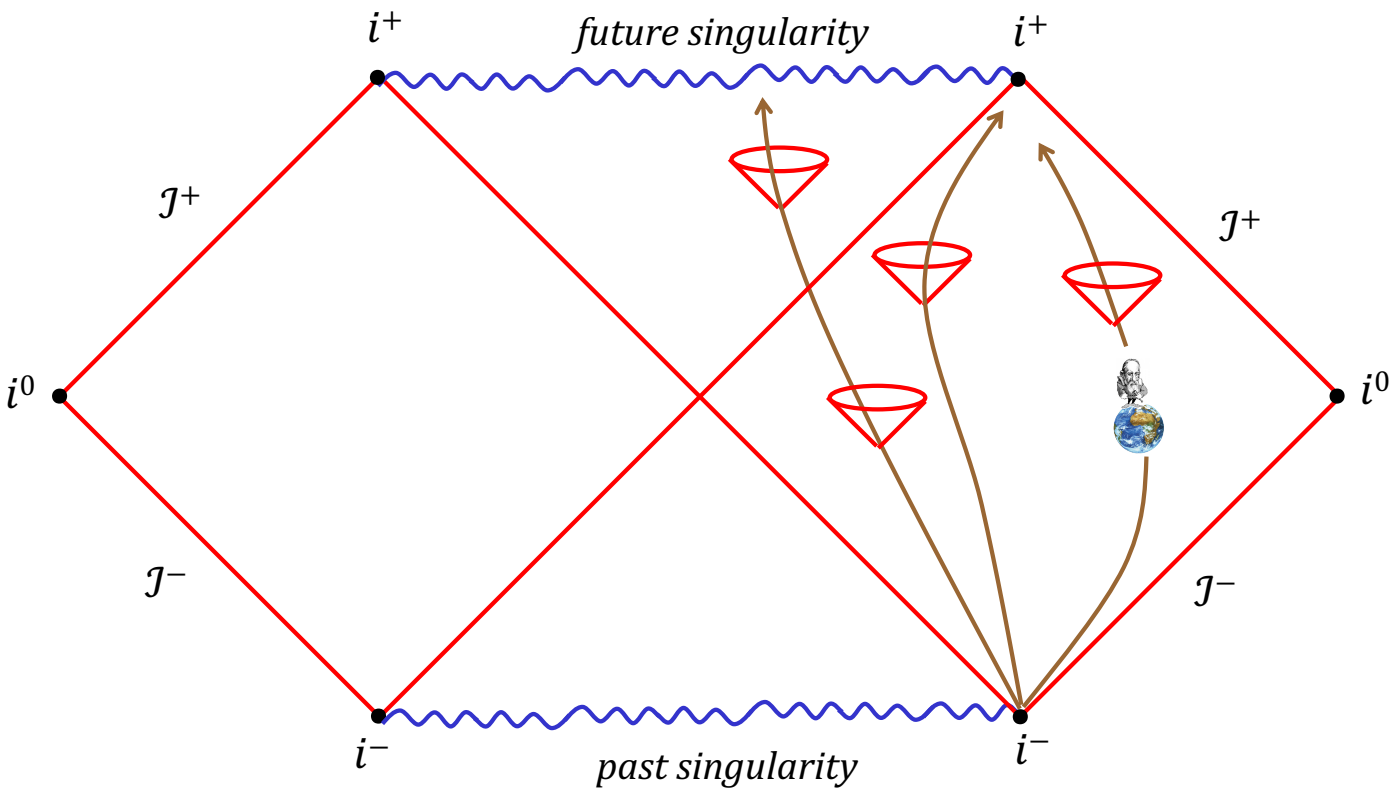
Conformal diagram of Schwarzschild Black Hole



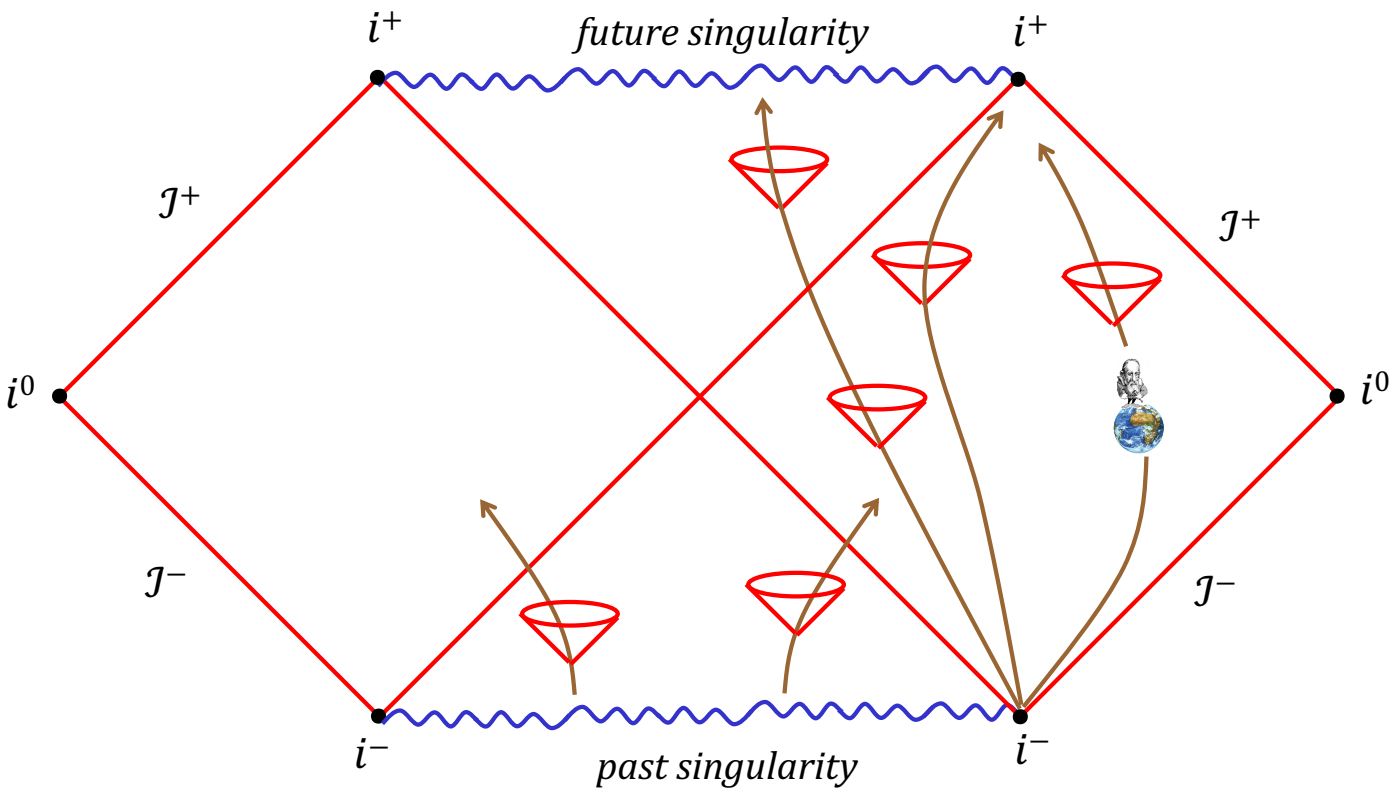
- Region I = normal spacetime outside black hole
- Region II = spacetime inside event horizon
- Regions III and IV = mathematical regions just like I and II.

Conformal diagram of Schwarzschild Black Hole



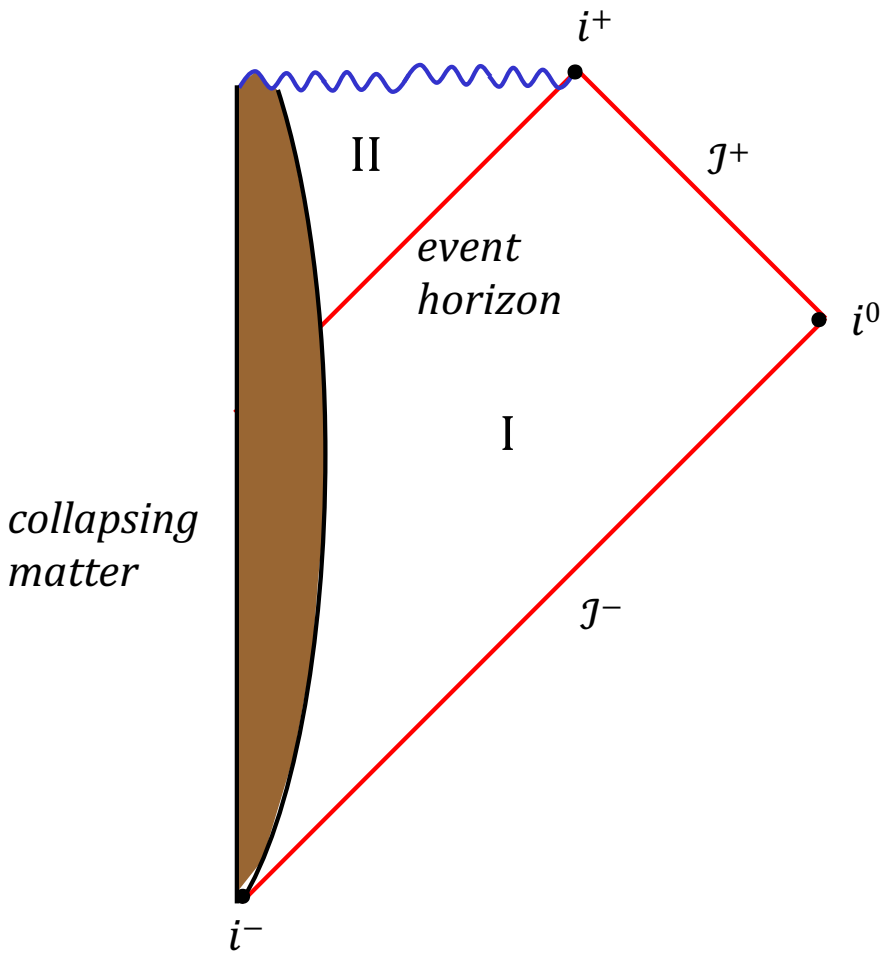
- Is swooping possible?
- No matter how closely the observer swoops to event horizon, can never receive light signals emitted from inside event horizon.
  - *Therefore, can never see inside event horizon.*

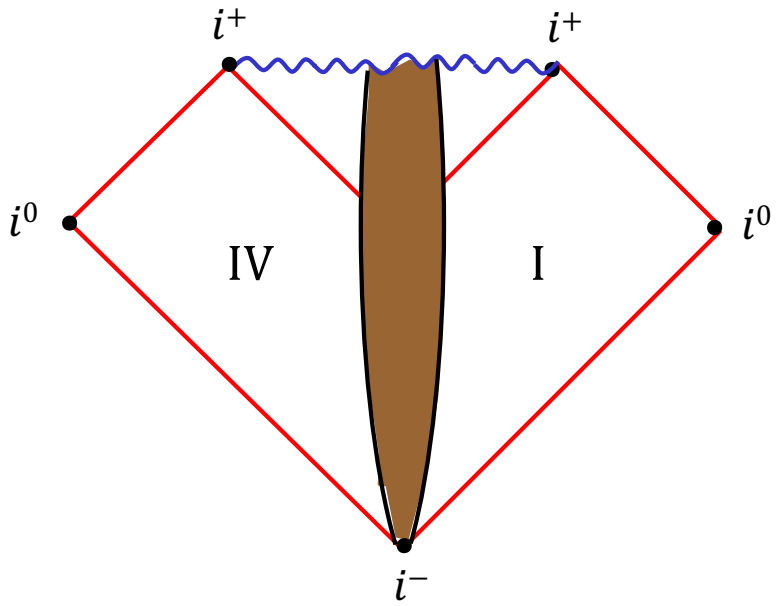
Conformal diagram of Schwarzschild Black Hole



- Only regions I and II are accessible from i^- by physical objects (speed $< c$).
- Region III acts like a black hole in reverse; a *white hole*.
  - *Potential ejection of matter out of past singularity.*
- Are regions III and IV physical?

- If a Schwarzschild black hole is formed by collapsing matter, regions III and IV are shielded from I and II.

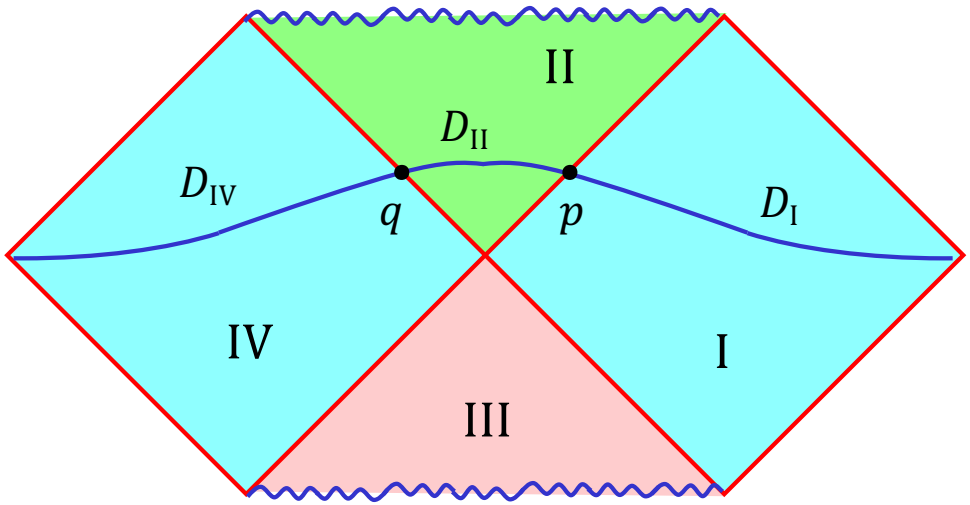


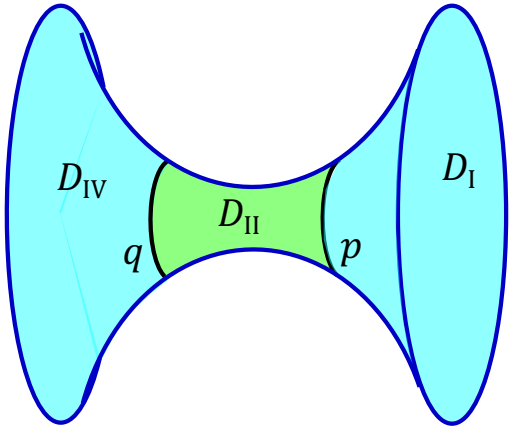


the complete picture

- Mathematically, a Schwarzschild black hole can exist independently of collapsing matter.
- If there is no shield due to collapsing matter, then regions I and IV are connected by a *wormhole*...

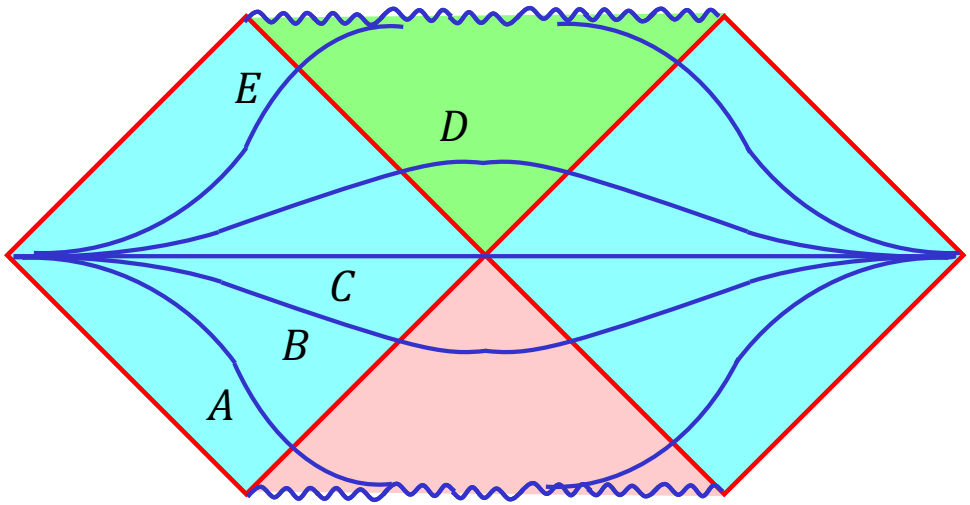
Einstein-Rosen Wormhole (1935)

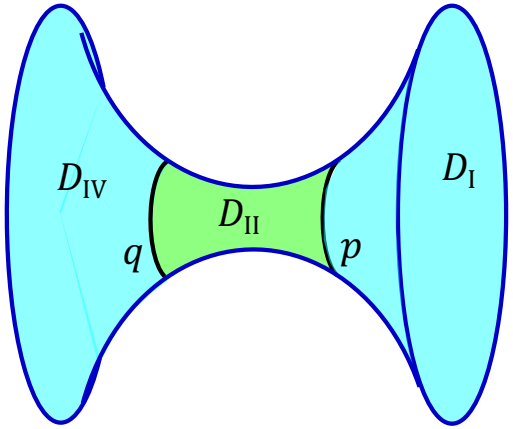




- Spacelike slice $D = D_I \cup D_{II} \cup D_{IV}$ = space at an instant.
  - *Extends from spatial infinity in region I, intersects event horizon at p, extends through region II, intersects event horizon again at q, and extends through region IV back to spatial infinity.*
- Add back a spatial dimension: Instead of a spacelike line, get a spacelike surface with the geometry of a paraboloid of revolution.
  - *Wormhole mouth at p and q; wormhole throat is region D_{II}.*
  - *<u>But</u>: The wormhole closes up too quickly for a timelike traveler to cross it!*

Einstein-Rosen Wormhole (1935)





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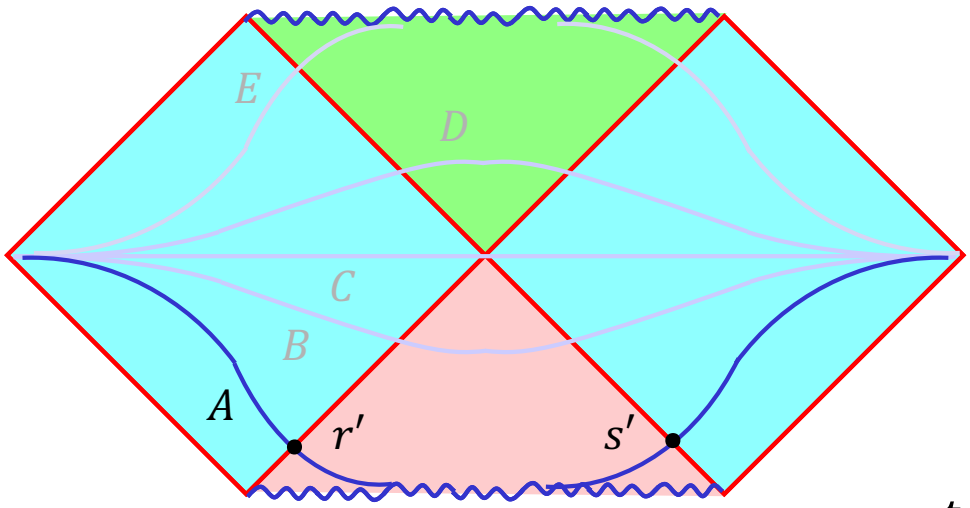
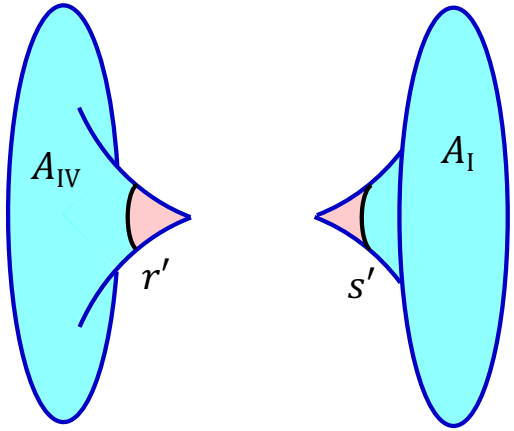
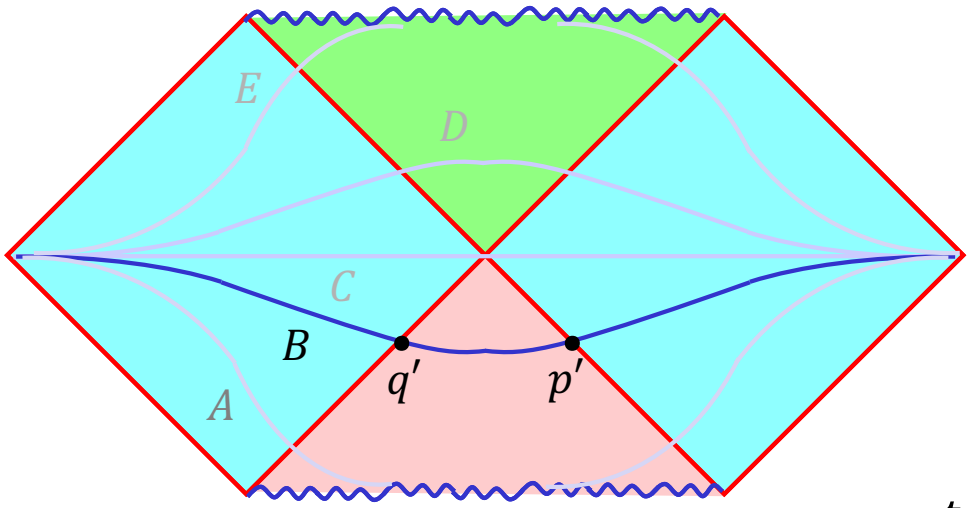
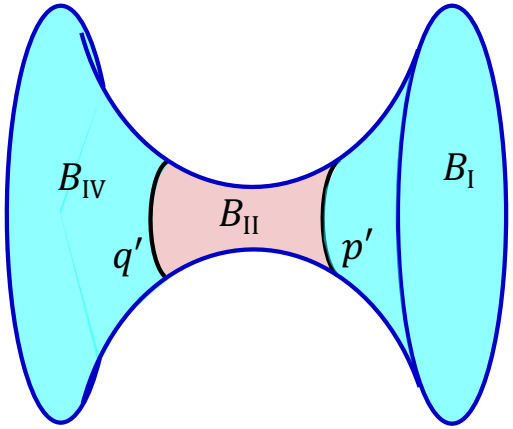




$t = t_0$

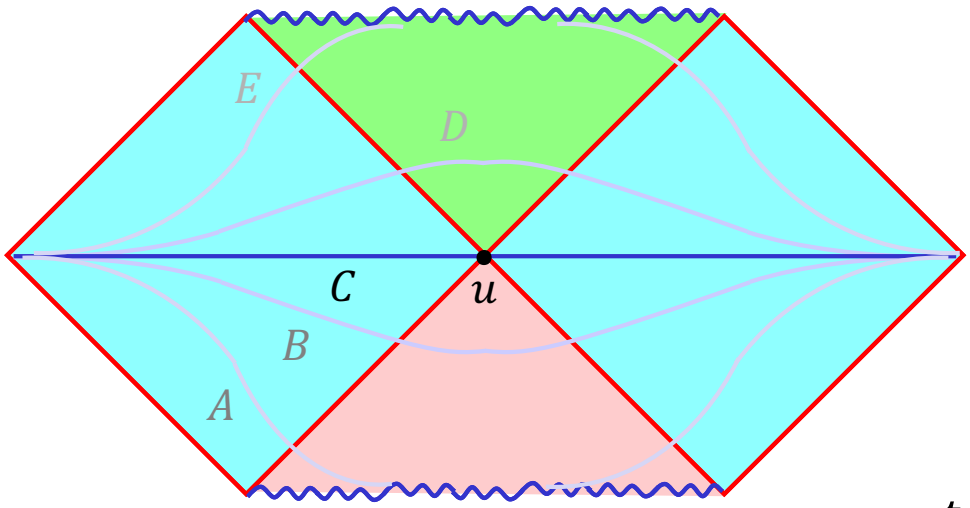
Einstein-Rosen Wormhole (1935)

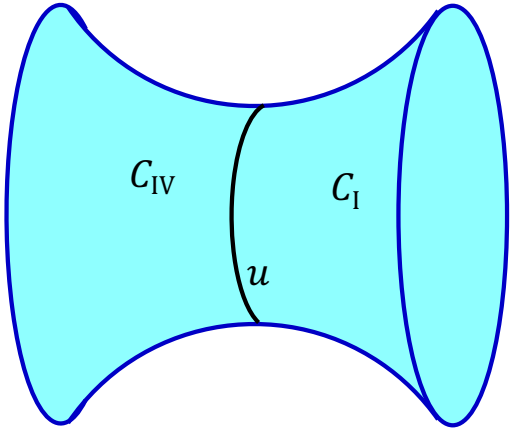




$t = t_1$

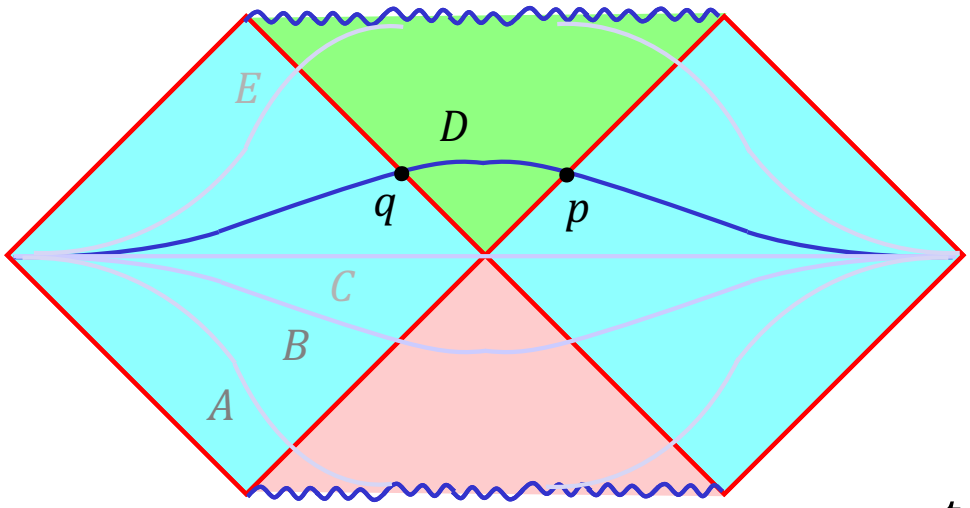
Einstein-Rosen Wormhole (1935)

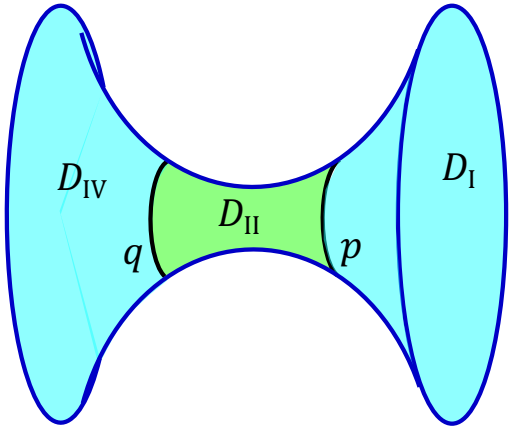




$t = t_2$

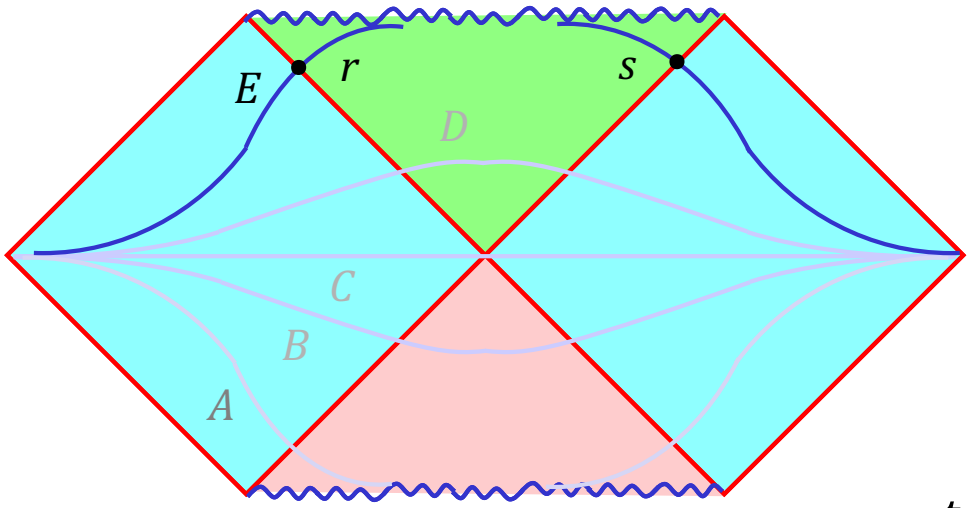
Einstein-Rosen Wormhole (1935)

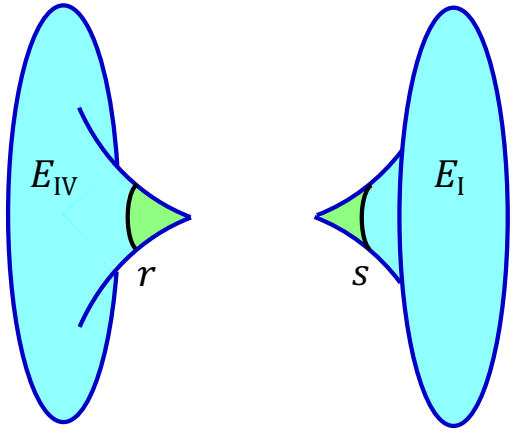




$t = t_3$

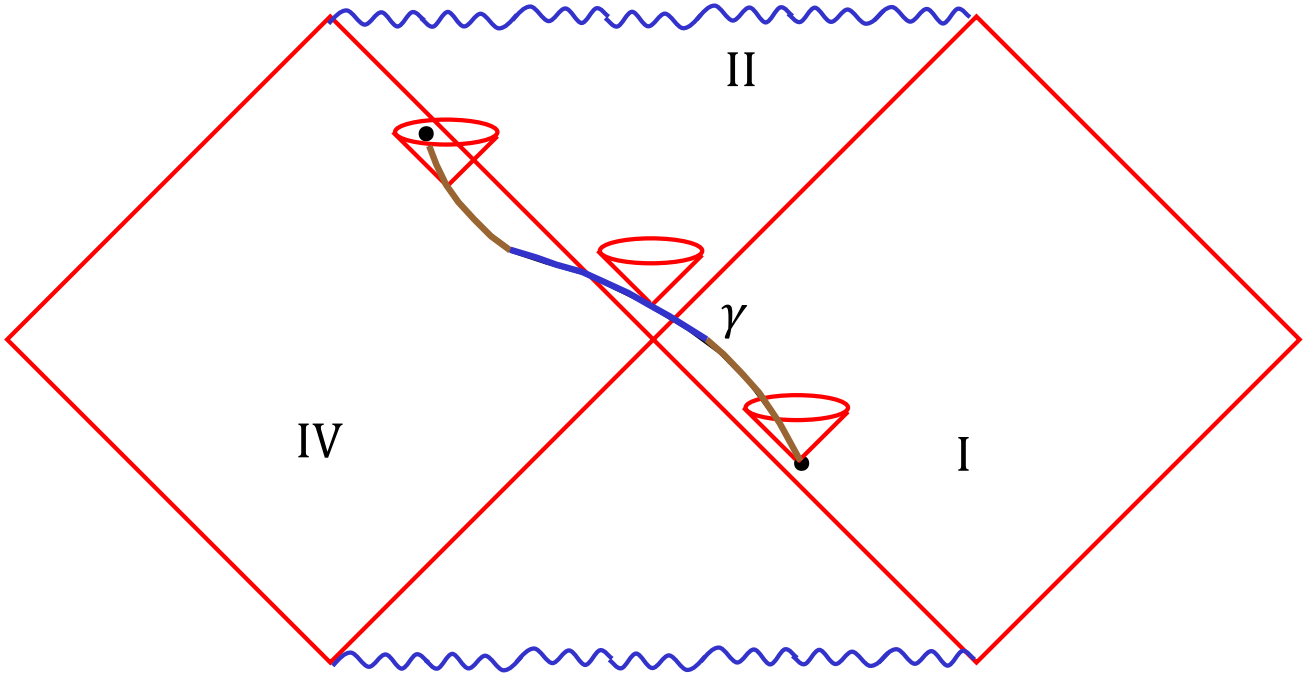
Einstein-Rosen Wormhole (1935)





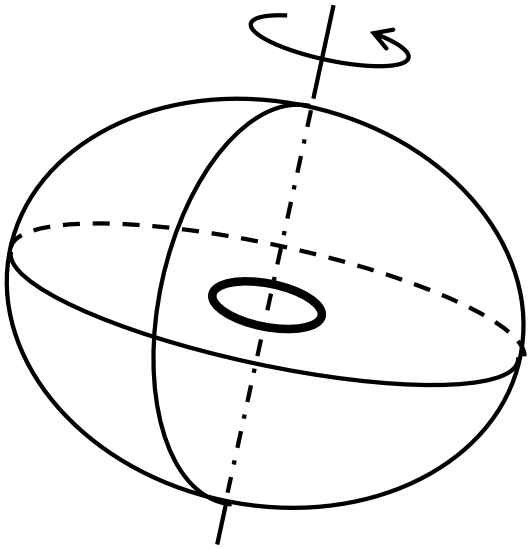
$t = t_4$

Einstein-Rosen Wormhole (1935)



- A worldline γ connecting two points in I and IV during the time the wormhole is open must, along some stretch, be spacelike (slope less that 45°).
- Can't travel from I to IV without speed exceeding *c*.
- *But*: Travel between regions *is* possible for other types of black holes...

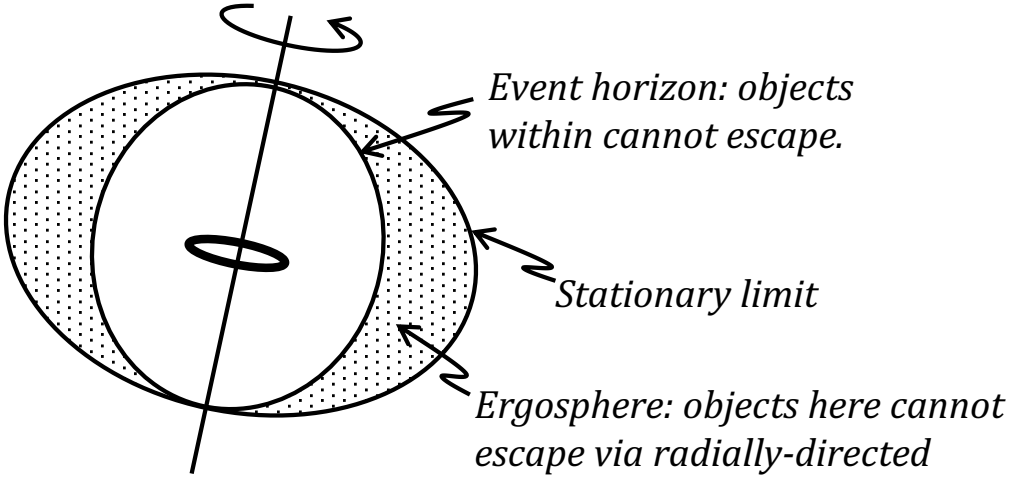
3. Charged Rotating Black Hole (Kerr-Newman)



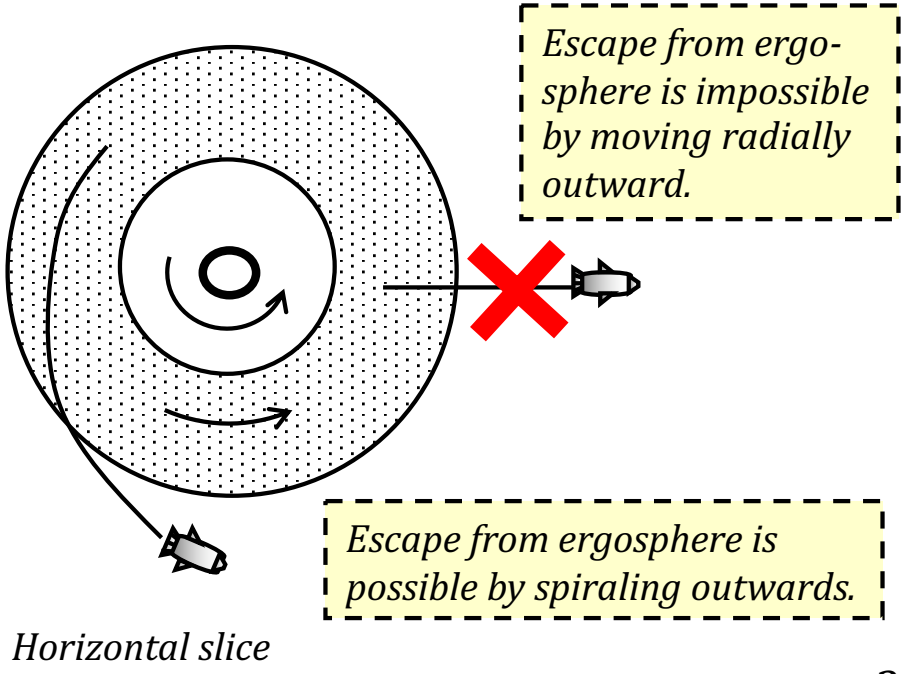
- *Consequences of rotation*:
  - Singularity is a ring, not a point.
  - Inertial frames near singularity are dragged.

Stationary limit = distance from singularity at which objects can no longer have zero angular momentum with respect to distant stars.

Ergosphere = Region between event horizon and stationary limit.



Vertical slice



Horizontal slice

Technical aside: Energy in general relativity.

$$E \equiv -\xi^a p_a$$

Time-translation 4-vector
Typically timelike: $|\xi^a| < 0$.
Encodes time-translation symmetry.

Momentum 4-vector
Always timelike: $|p_a| < 0$.
$p_a = (p_x, p_y, p_z, m_0)$

Why define energy in this way?
Because if ξ^a and p_a are *timelike*, then $\xi^a p_a < 0$, hence $E > 0$, and we *typically* want the energy to be positive.

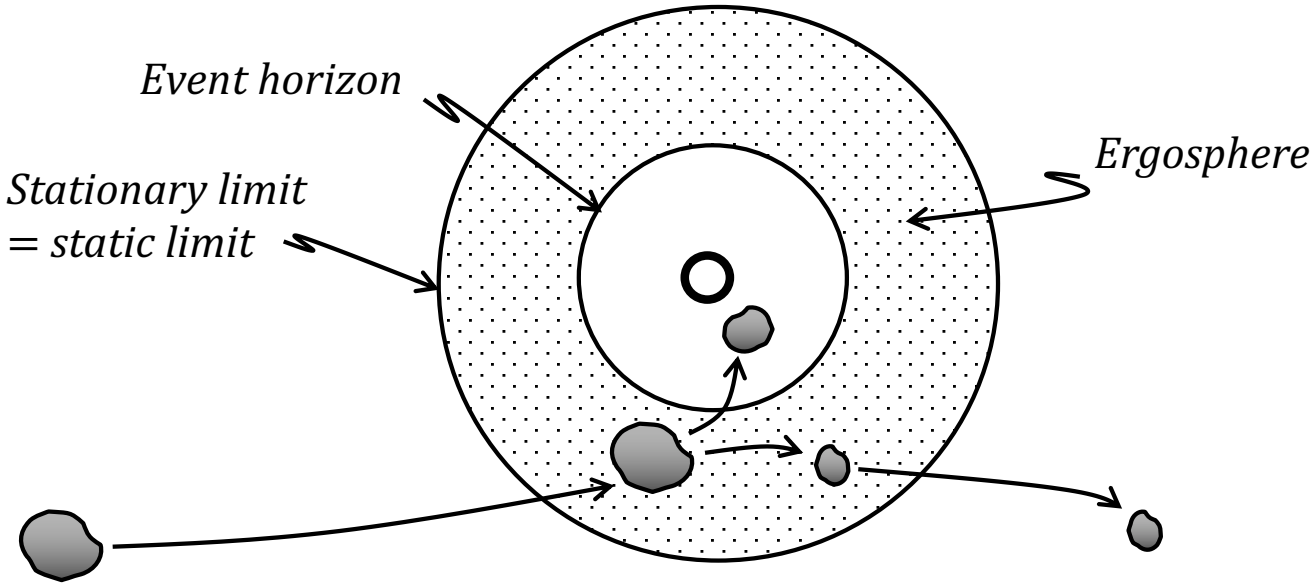
Static limit = distance from singularity at which objects can no longer escape *via* radially directed motion.

- *Technical property*: Inside static limit, ξ^a is *spacelike*: $|\xi^a| > 0$.
- *And*: If ξ^a is spacelike and p_a is timelike, then $\xi^a p_a > 0$.
- *So*: Inside static limit, $E < 0$; hence objects with *negative energy* can exist!

What does it mean to say the energy of an object is negative?
It means that it requires more energy than the object's rest mass to move it to infinity.

Claim: Inside static limit, objects with *negative energy* can exist.

- Allows the following procedure for extracting energy (Penrose 1969):



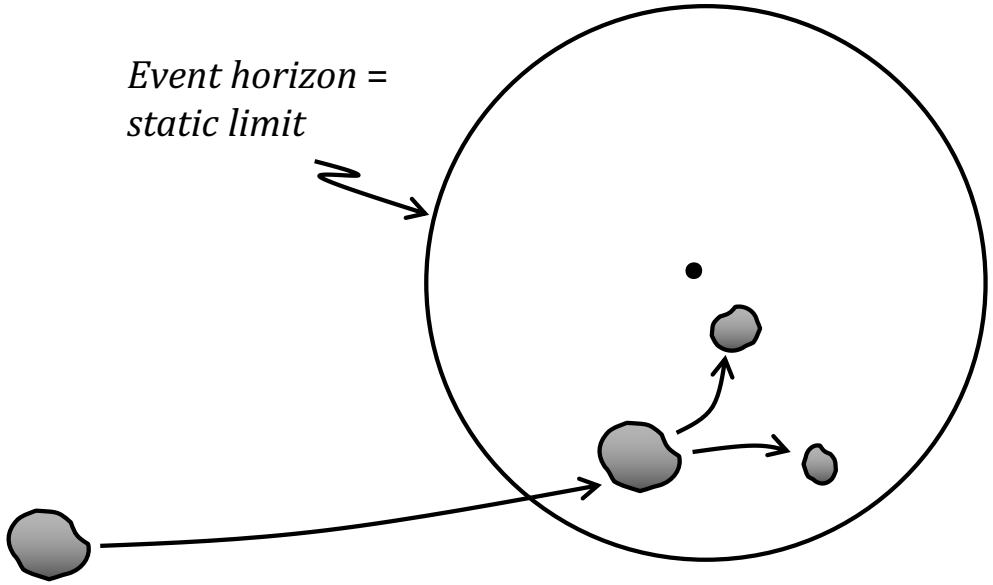
Lump of matter with energy E shot into ergosphere.

Fragments into two chunks with energies E_1, E_2 such that
(a) $E_1 + E_2 = E$.
(b) $E_1 < 0, E_2 > 0$.

Chunk with neg energy falls into singularity, chunk with positive energy emerges with greater energy than initial lump!

This doesn't work for a Schwarzschild black hole:

- Schwarzschild black hole: static limit = event horizon.
- *So*: Negative energy states are possible inside event horizon.
- *But*: To guarantee your rock is split into negative energy and positive energy pieces, you need to be inside the region of negative energy states when you split it!



No ergosphere!

Lump of matter with energy E shot into event horizon.

Fragments into two chunks with energies E_1, E_2 such that
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(b) $E_1 < 0, E_2 > 0$.

Both chunks can't escape!

Conformal diagram of Charged Rotating Black Hole

Properties

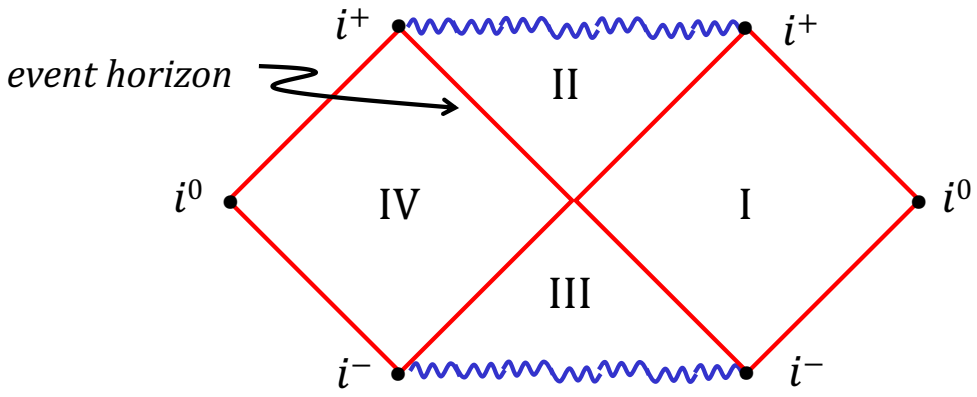
1. Singularity forms a *timelike* surface (result of charge).

   So: *Potential for escape! (No longer future for all timelike curves as in Schwarzschild black hole.)*

2. Singularity is a *ring*, not a point (result of rotation).

   So: *Can pass through it (theoretically).*

Recall Schwarzschild conformal diagram...



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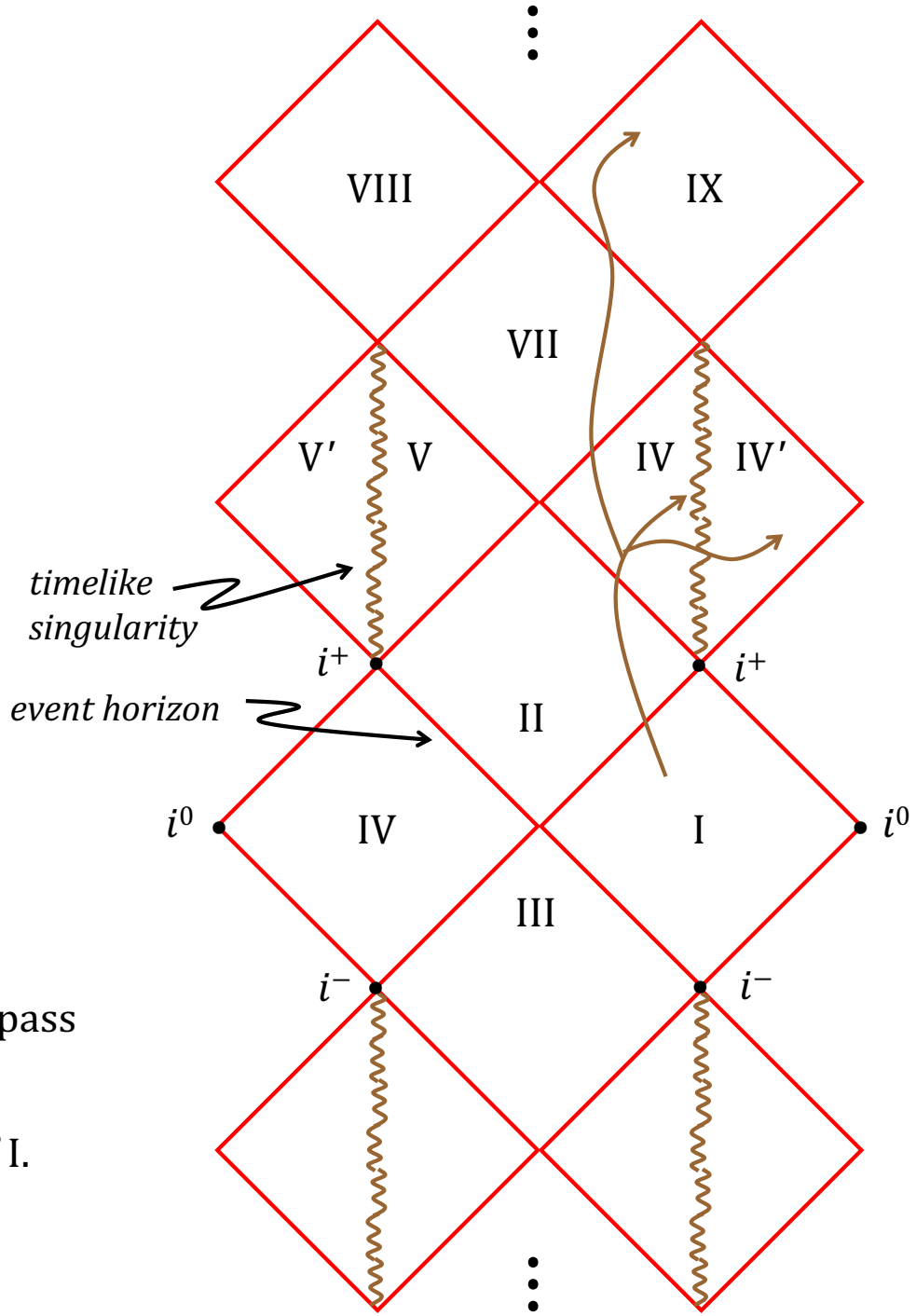
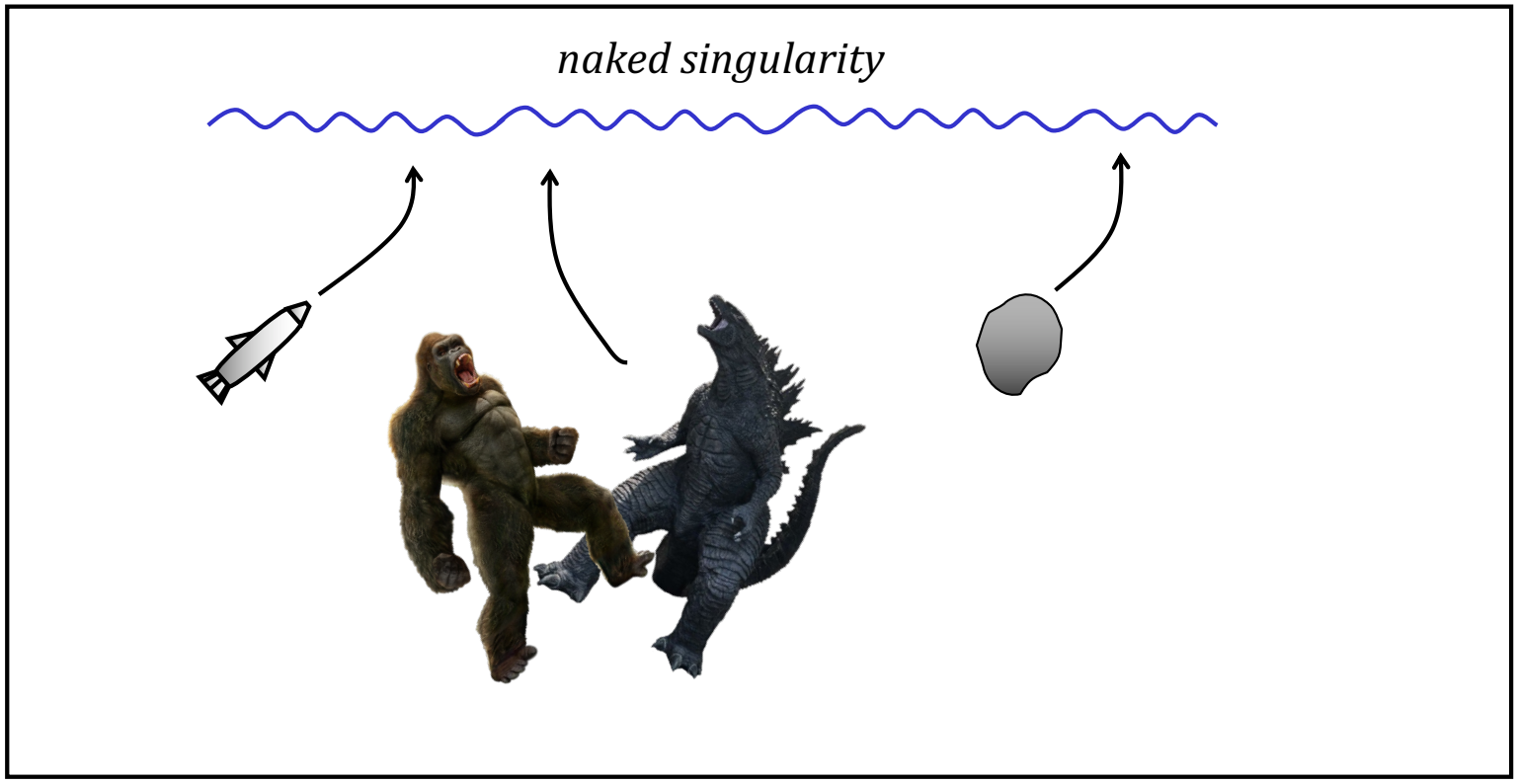


- Start in I
- Can strike ring singularity *or* pass through it to VI′.
- *Or* escape to IX, a duplicate of I.

4. Cosmic Censorship Hypothesis

Conjecture: Any singularity produced by gravitational collapse is hidden behind an event horizon.

- *Why should we hope this is correct?*
- *GR is time-reversible*: Replace t with $-t$ in any solution to the Einstein equations and the result is another solution.
- *So*: If the following is a solution to the Einstein equations:



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- *GR is time-reversible*: Replace t with $-t$ in any solution to the Einstein equations and the result is another solution.
- *Then*: It's time-reverse is *also* a solution: