12. Relativistic Cosmology

Topics:
1. Simple Solutions
2. RW Spacetimes
3. Horizon Problem

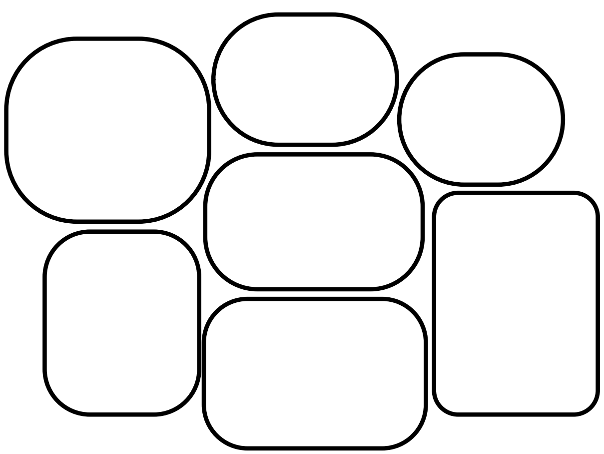
1. Simple Solutions to the Einstein Equations

1. *Minkowski spacetime*

- Initial assumptions:
  - *No matter* ($T_{\mu\nu} = 0$)
  - *No gravitation* ($R^{\sigma}_{\mu\nu\rho} = 0$; *i.e.*, zero curvature)
- Not realistic!
  - *But: In the small, all GR spacetimes look Minkowskian.*

How to mathematically construct more complex solutions

Take simple solutions as pieces and stitch them together smoothly.

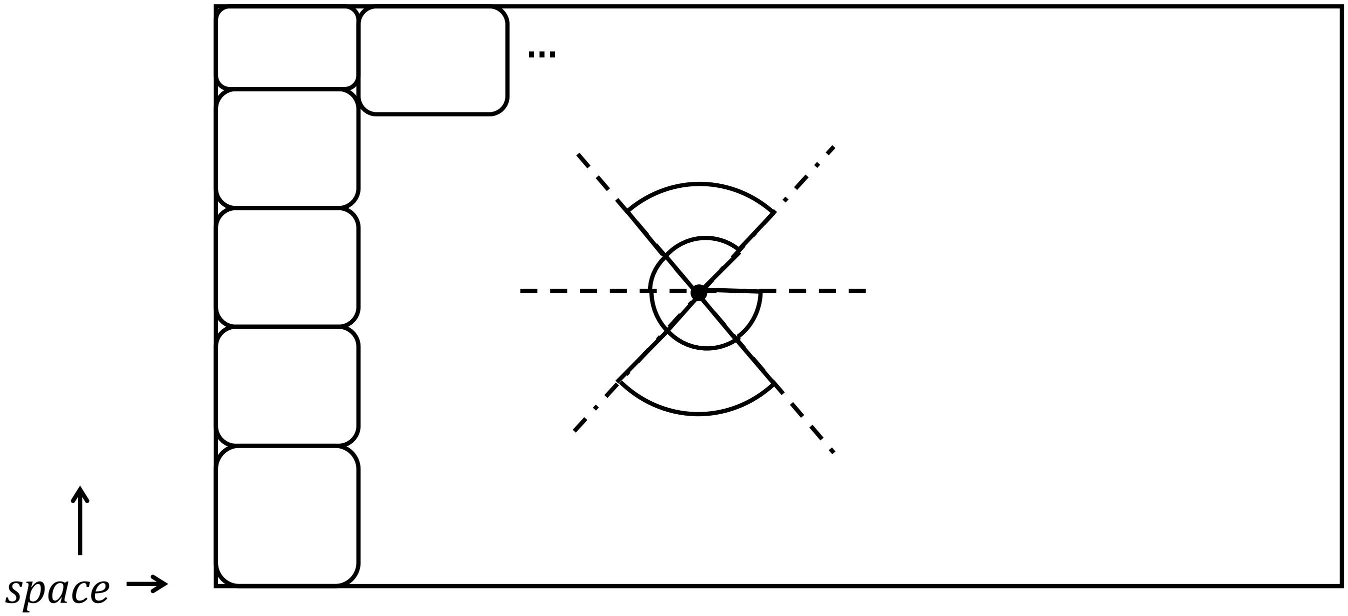


- *Problem*: How do we guarantee the stitching is "smooth"? (The geometries of pieces must smoothly blend in to each other.)
- *One Solution*: Fix geometry at infinity and work backwards

2. *Schwarzschild Solution (1915)*

Karl Schwarzschild
(1873-1916)

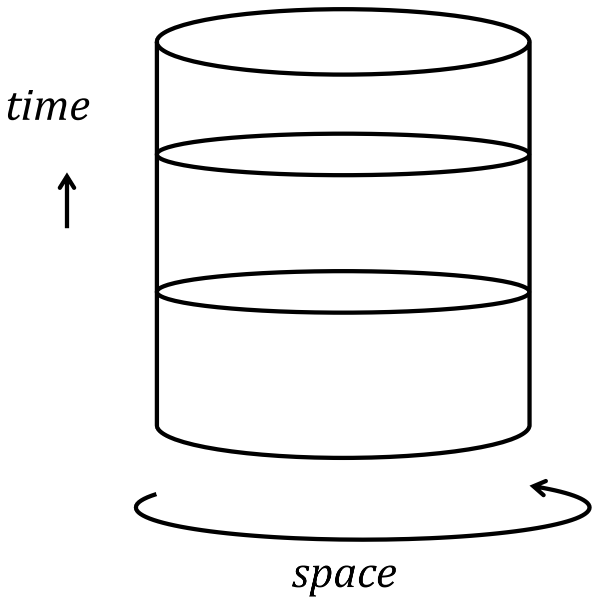
- Description of the gravitational field of the sun.
- Stitch in Minkowski (flat) pieces at spatial infinity.
- Stitch in symmetrical pieces around sun (solutions to Einstein equations that assume a single mass concentration and spherical symmetry).



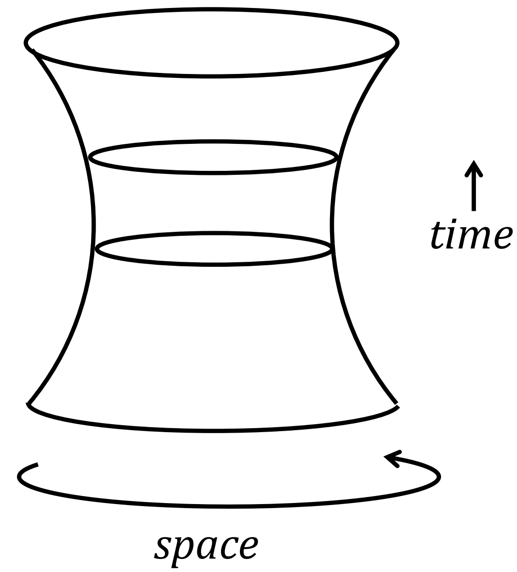
Very simplistic assumptions:

- Asymptotic spatial flatness (flatness at spatial infinity).
- Single mass concentration.
- Spherical symmetry.

3a. *Einstein Universe* (1917)

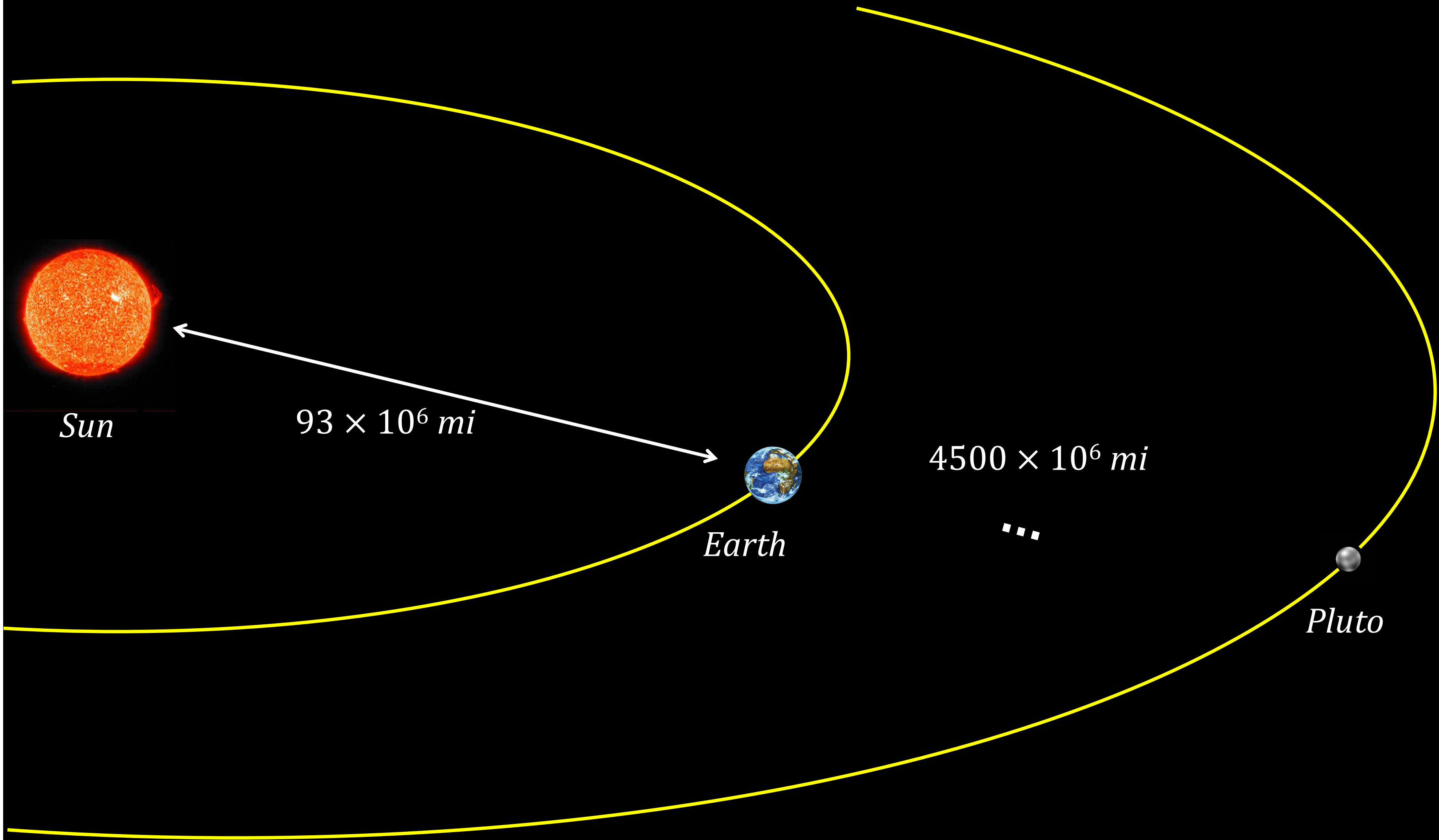


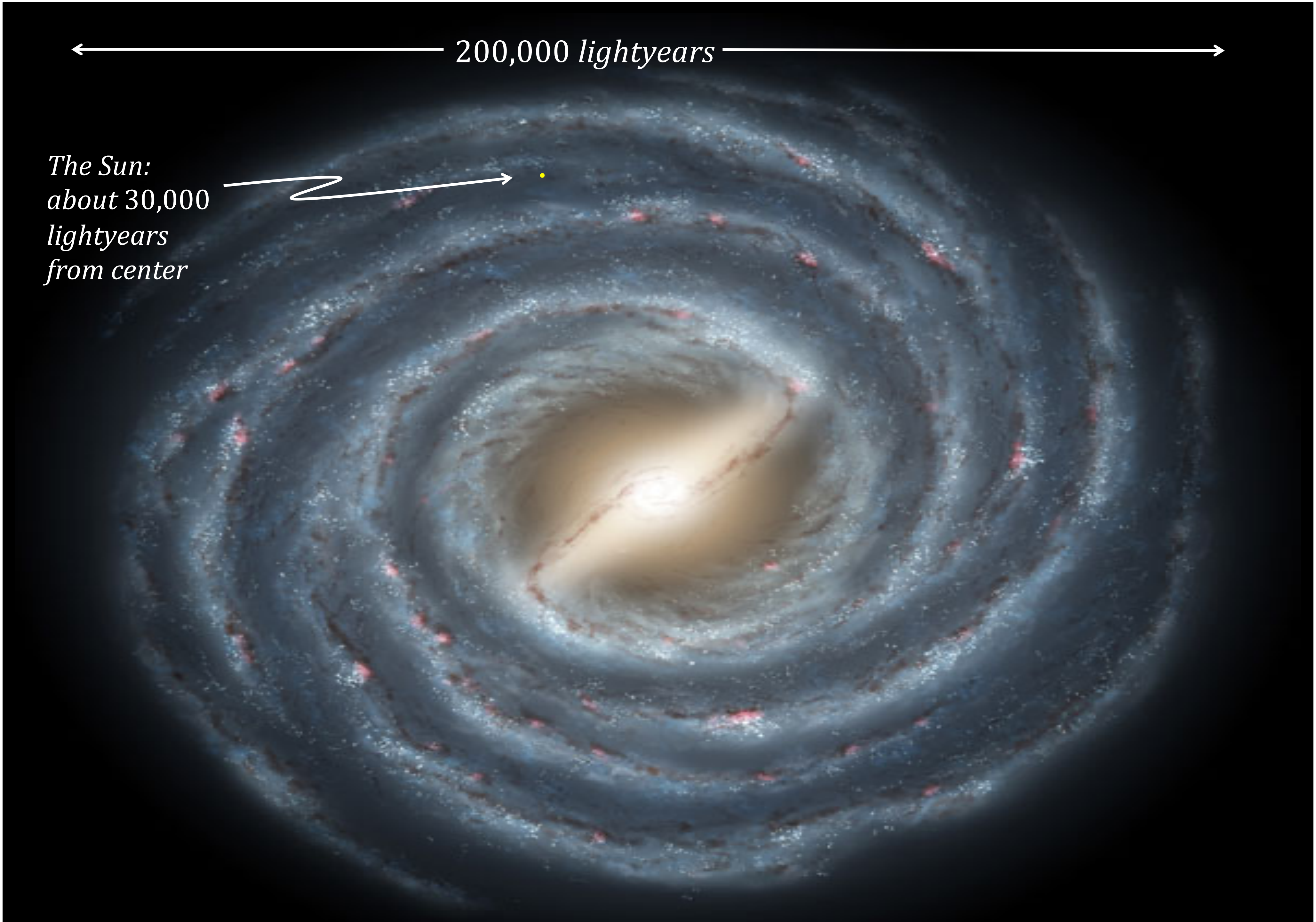
3b. *de Sitter Unvierse* (1917)



What is the large-scale structure of our universe?

Fixed by large-scale distribution of matter.





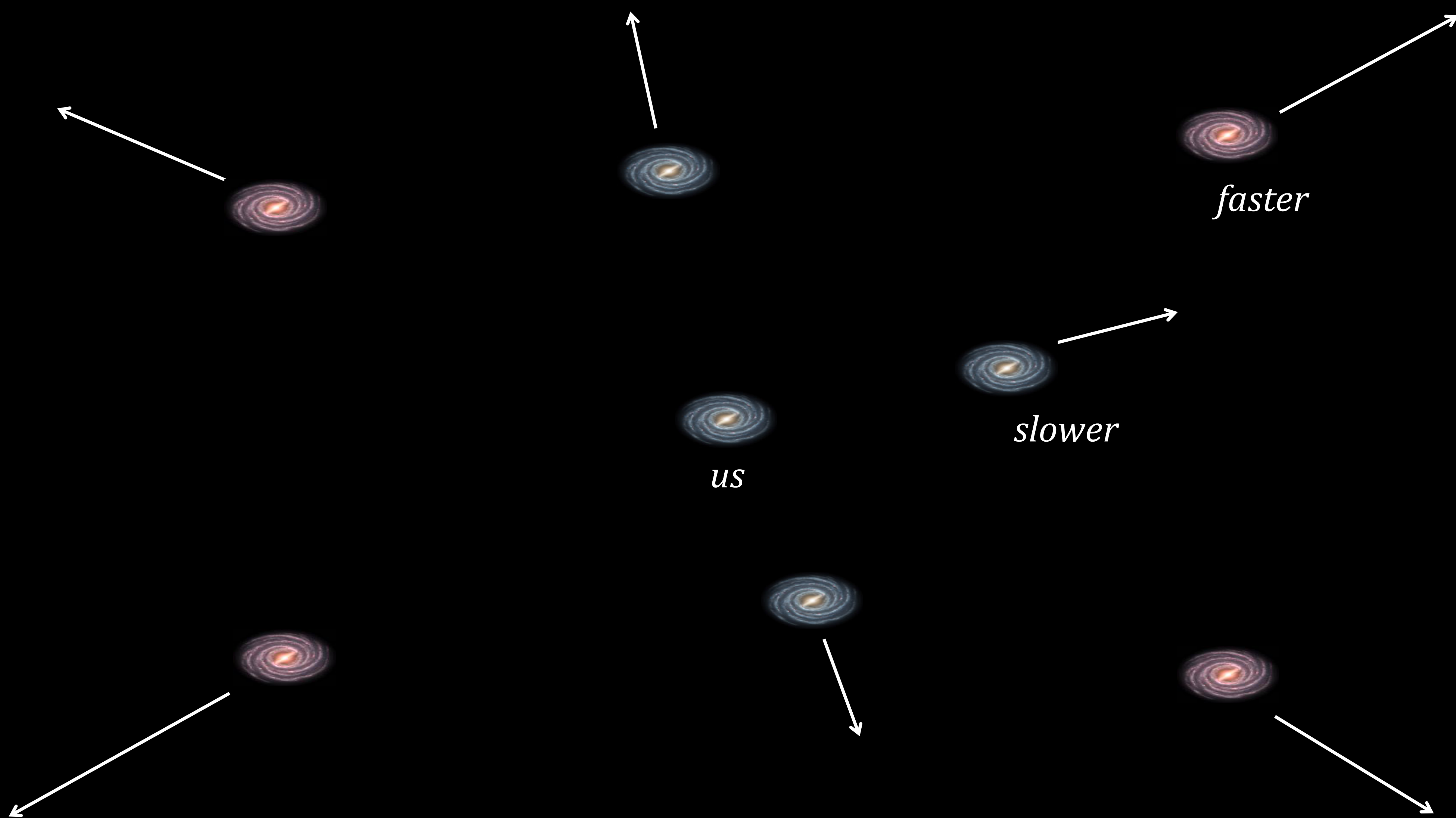
Milky Way galaxy - hundreds of billions of stars

Universe on largest scale: Galaxies form cosmic fluid (like molecules in a gas)

Important observation: The Cosmological Redshift (Hubble 1929)

Light from distant galaxies is red-shifted!

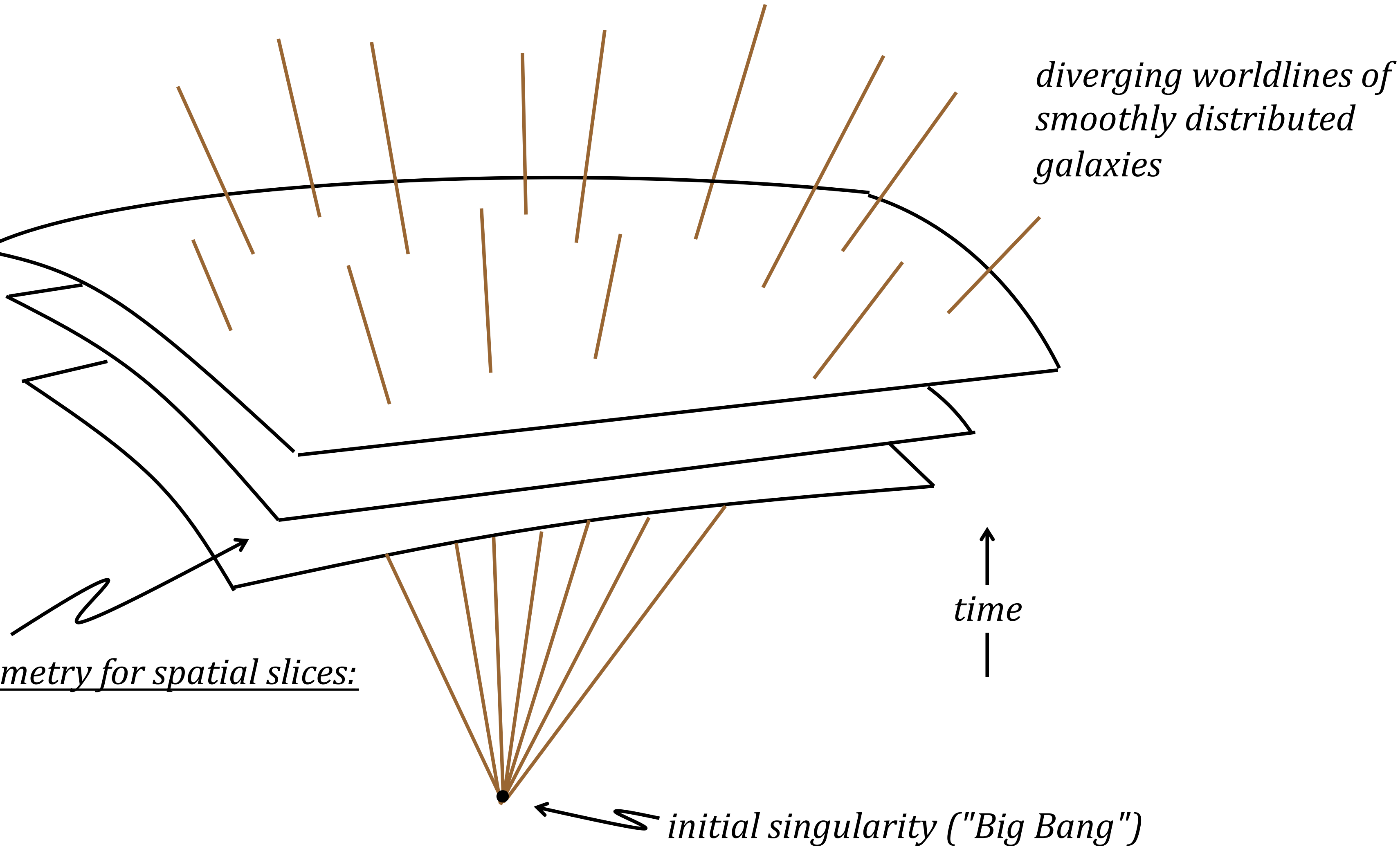
- *Hubble's Law*: The amount of a galaxy's red-shift is directly proportional to its distance from the Earth.
  - *This is what is observed.*
- *Hubble's Interpretation*: Red-shift is due to *velocity recession*.
  - *Evidence that the galaxies are receding from us!*



- Similar to the Doppler Shift: As source recedes, frequency drops = red-shifted
- *But*: Effect is really due to spacetime curvature!

2. Robertson-Walker (RW) Spacetimes

- Candidates for our universe on largest scale.
- Describe expanding universe: worldlines of galaxies converge back in time to an initial singularity (the "Big Bang").



<u>*3 choices of geometry for spatial slices:*</u>

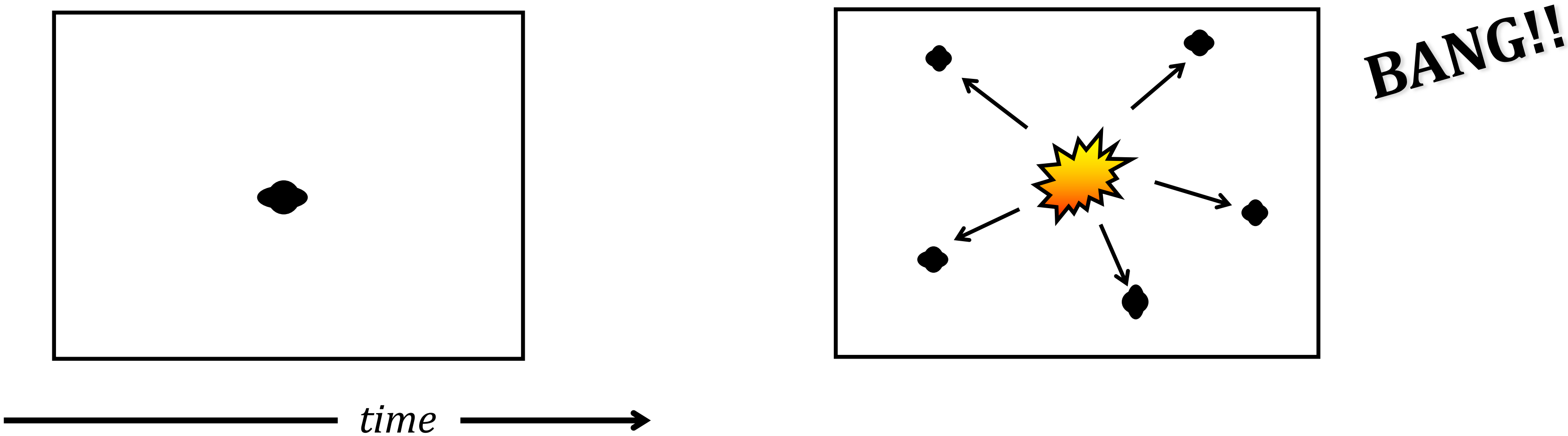
(1) Spherical

(2) Euclidean

(3) Hyperbolic

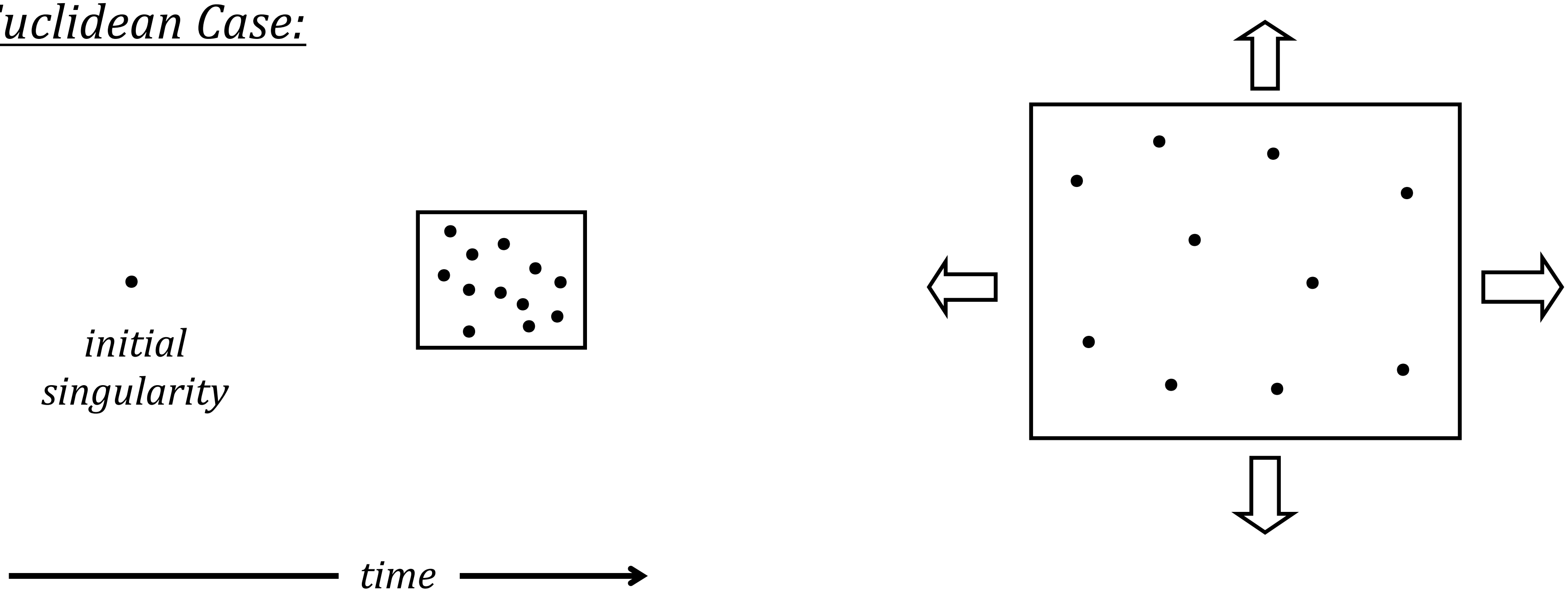
Expansion of Universe

Wrong Picture: Explosion in pre-existing spacetime



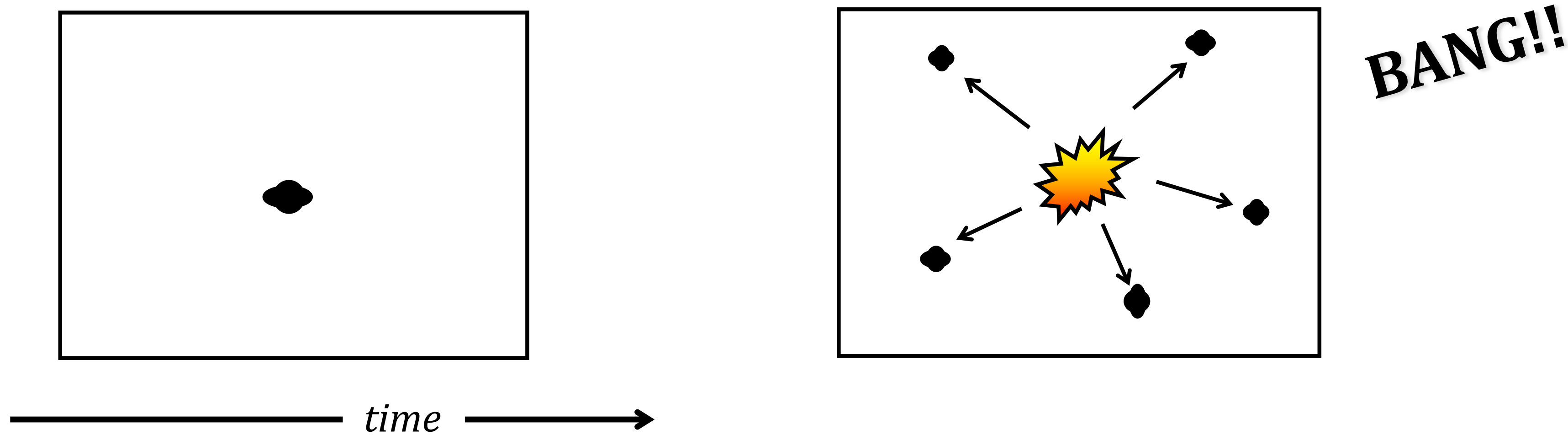
Right Picture: Expansion of spacetime

Euclidean Case:



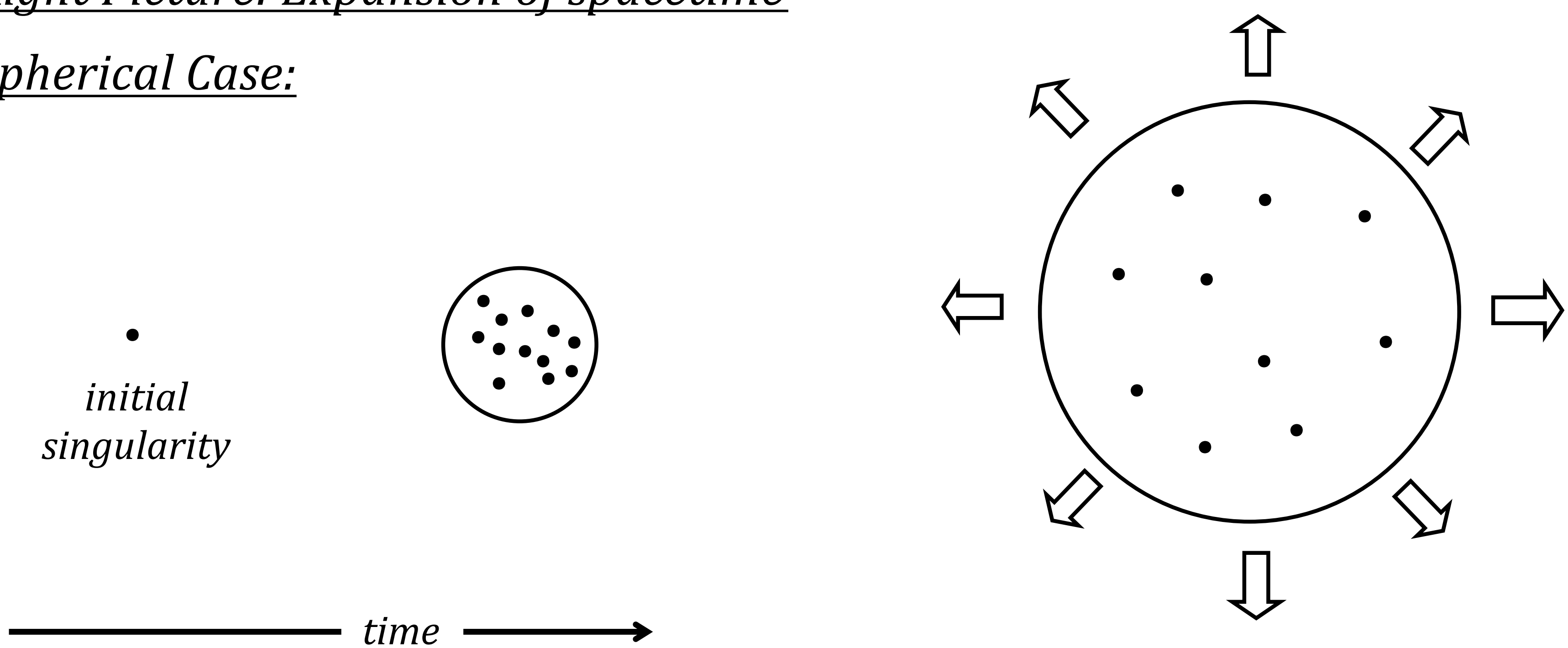
Expansion of Universe

Wrong Picture: Explosion in pre-existing spacetime



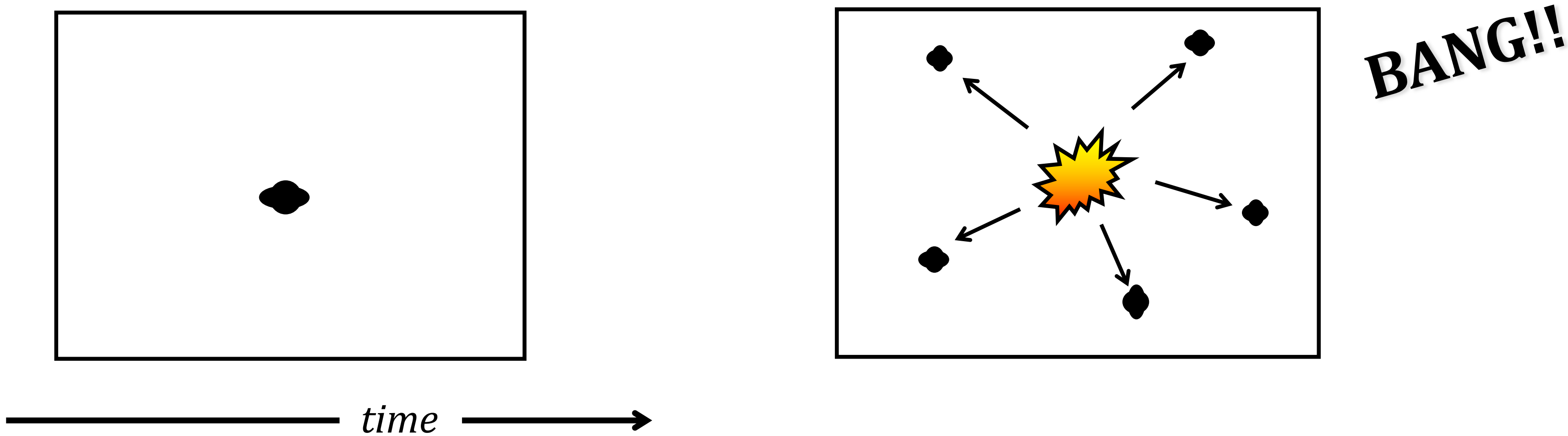
Right Picture: Expansion of spacetime

Spherical Case:



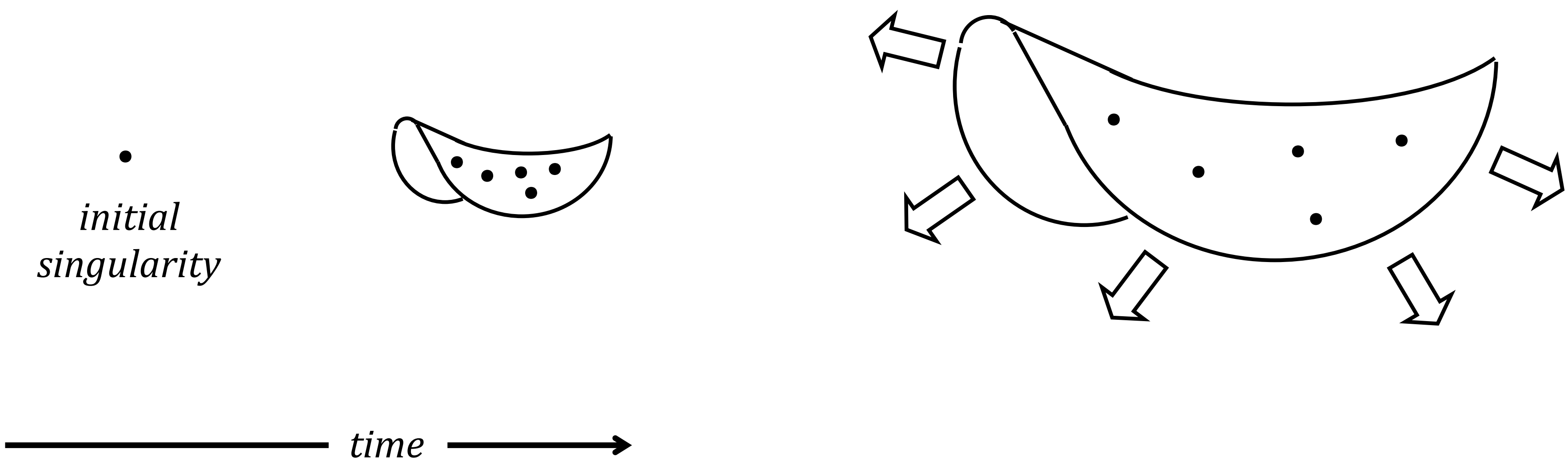
Expansion of Universe

Wrong Picture: Explosion in pre-existing spacetime



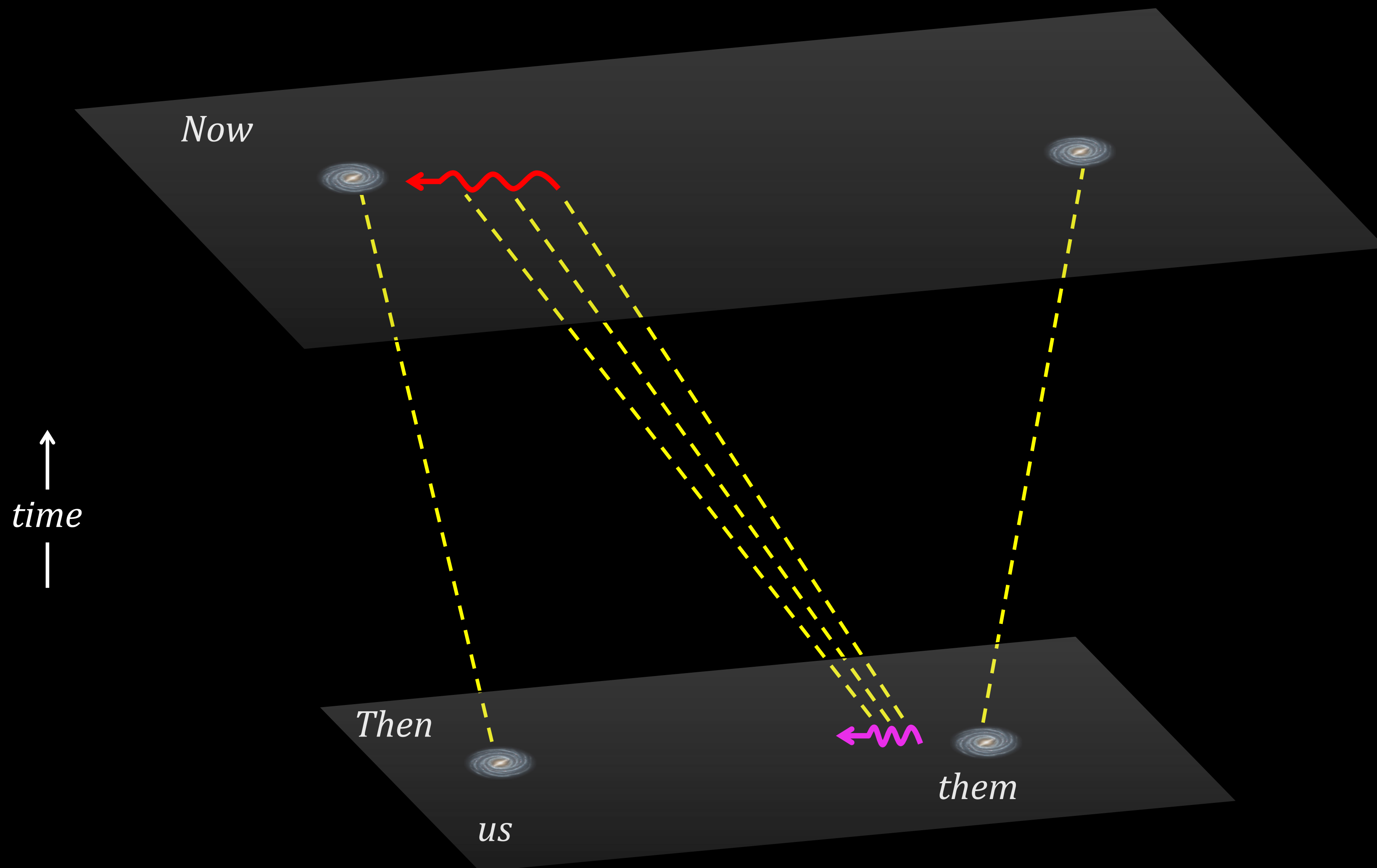
Right Picture: Expansion of spacetime

Hyperbolic Case:



RW explanation of Cosmological Redshift:

Expansion of RW spacetimes explains galaxy recession.



- Light signal is stretched as space stretches.
- Wavelength increases ⇒ frequency gets red-shifted.

Which is the correct geometry for spatial sections?

- Spherical, Euclidean, Hyperbolic?
- Depends on the average density ρ of matter in the universe.
- Prediction of RW solutions:

$$\textit{average density } \rho \begin{cases} = \rho_{crit} & \Rightarrow \text{ Euclidean} \\ > \rho_{crit} & \Rightarrow \text{ spherical} \\ < \rho_{crit} & \Rightarrow \text{ hyperbolic} \end{cases}$$

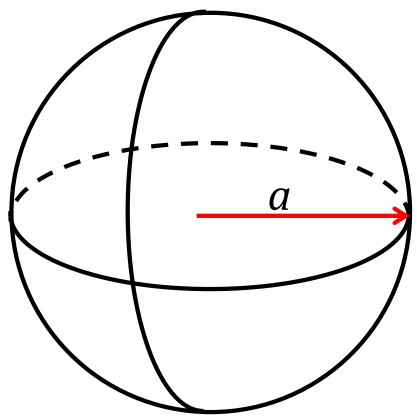
$$\textit{critical density } \rho_{crit} \approx 9 \times 10^{-30}\, g/cm^3$$
$$\approx 6 \text{ protons}/m^3$$

Take 1/100 *gram and spread it over volume of Earth!*

Unanswered Question: What is the average density ρ of the universe?

- *Observed matter in universe gives* $\rho < \rho_{crit}$.
- *Add dark matter and get* $\rho \approx \rho_{crit}$.

Case 1: $\rho > \rho_{crit}$ (*spherical, closed geometry*)



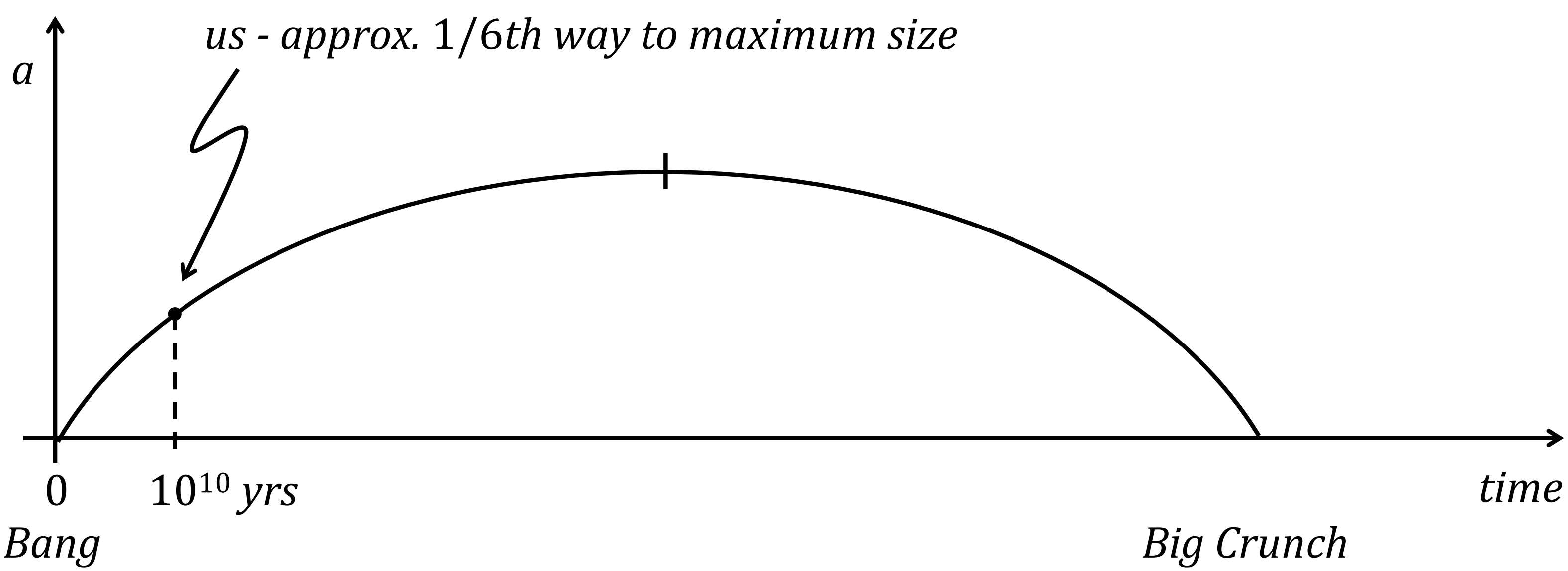
finite volume, no edge

a = scale factor (measure of curvature)

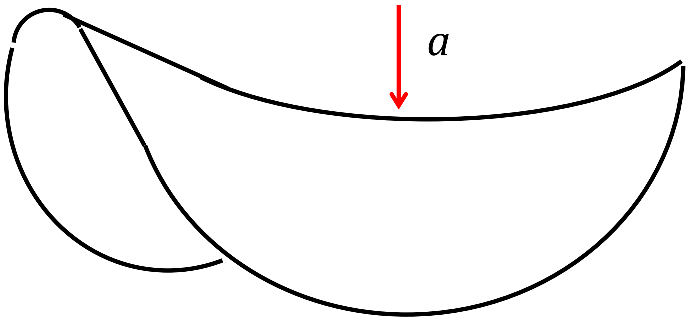
= (*speed of light*) × (*age of universe*)

= 1.36×10^{16} *lightyears*

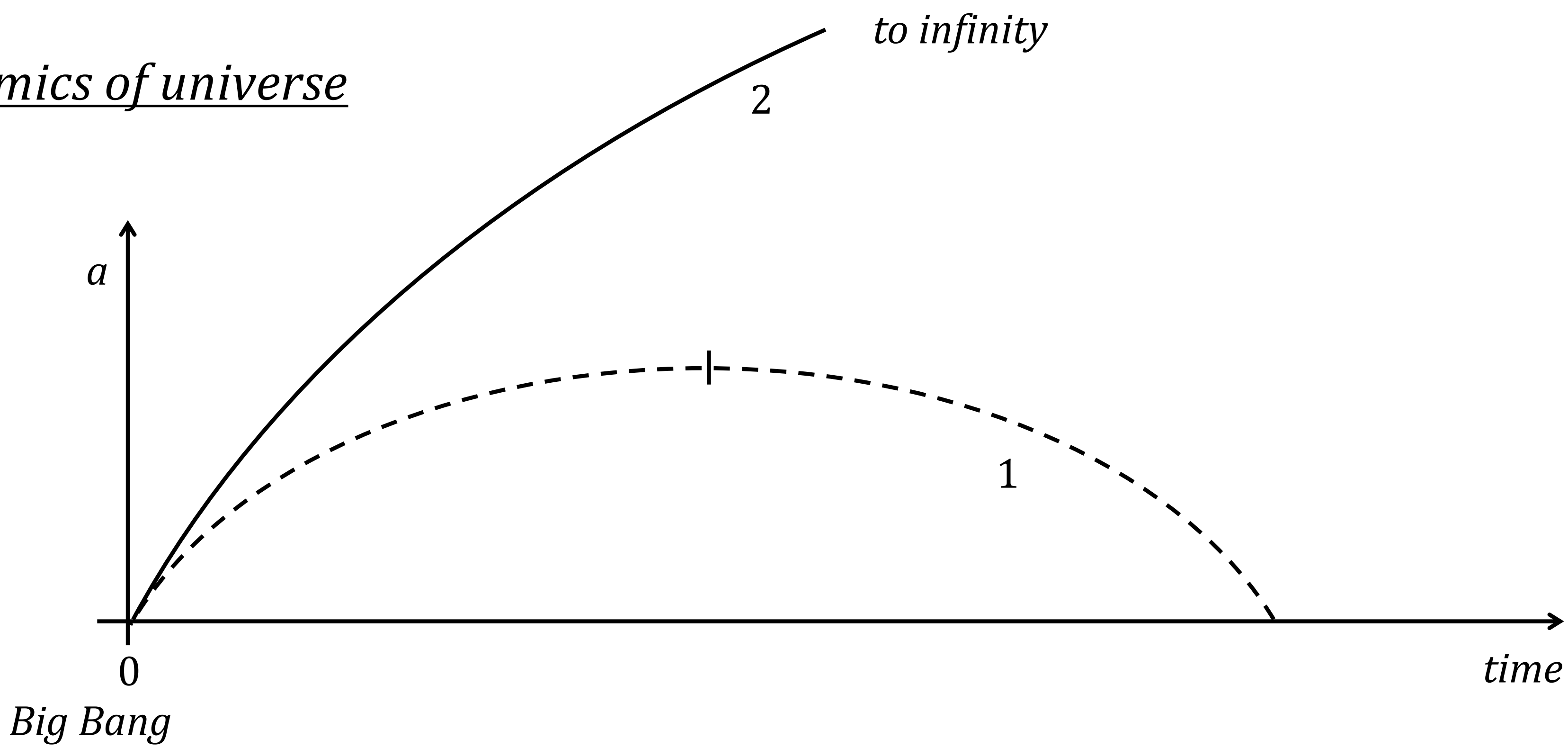
<u>*Dynamics of universe*</u>



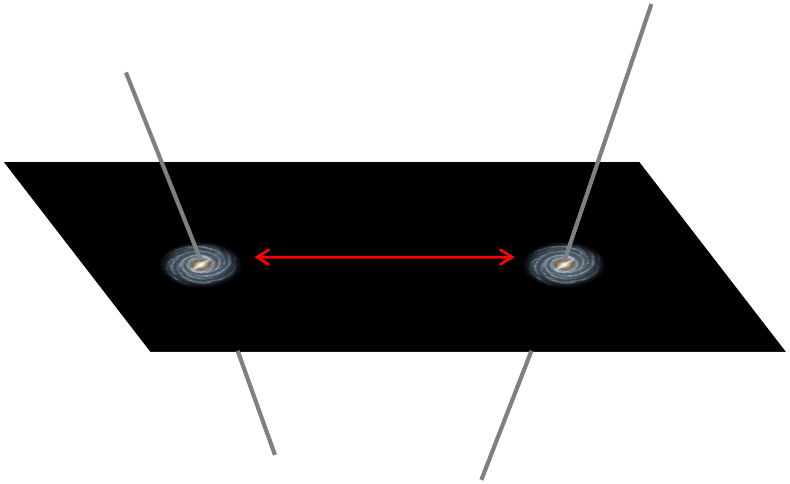
Case 2: $\rho < \rho_{crit}$ (*hyperbolic, infinite, open geometry*)



<u>*Dynamics of universe*</u>

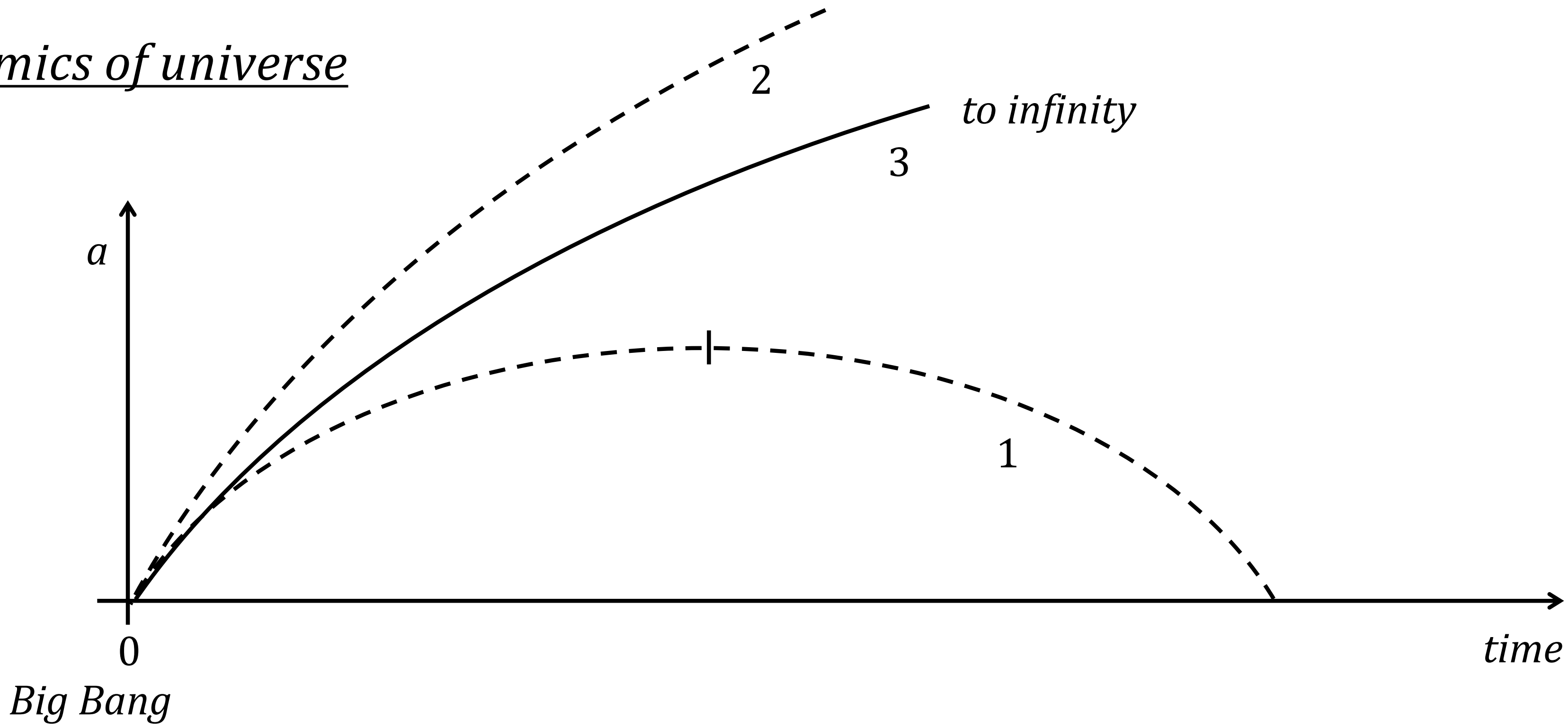


Case 3: $\rho = \rho_{crit}$ (*Euclidean, infinite, open, flat geometry*)



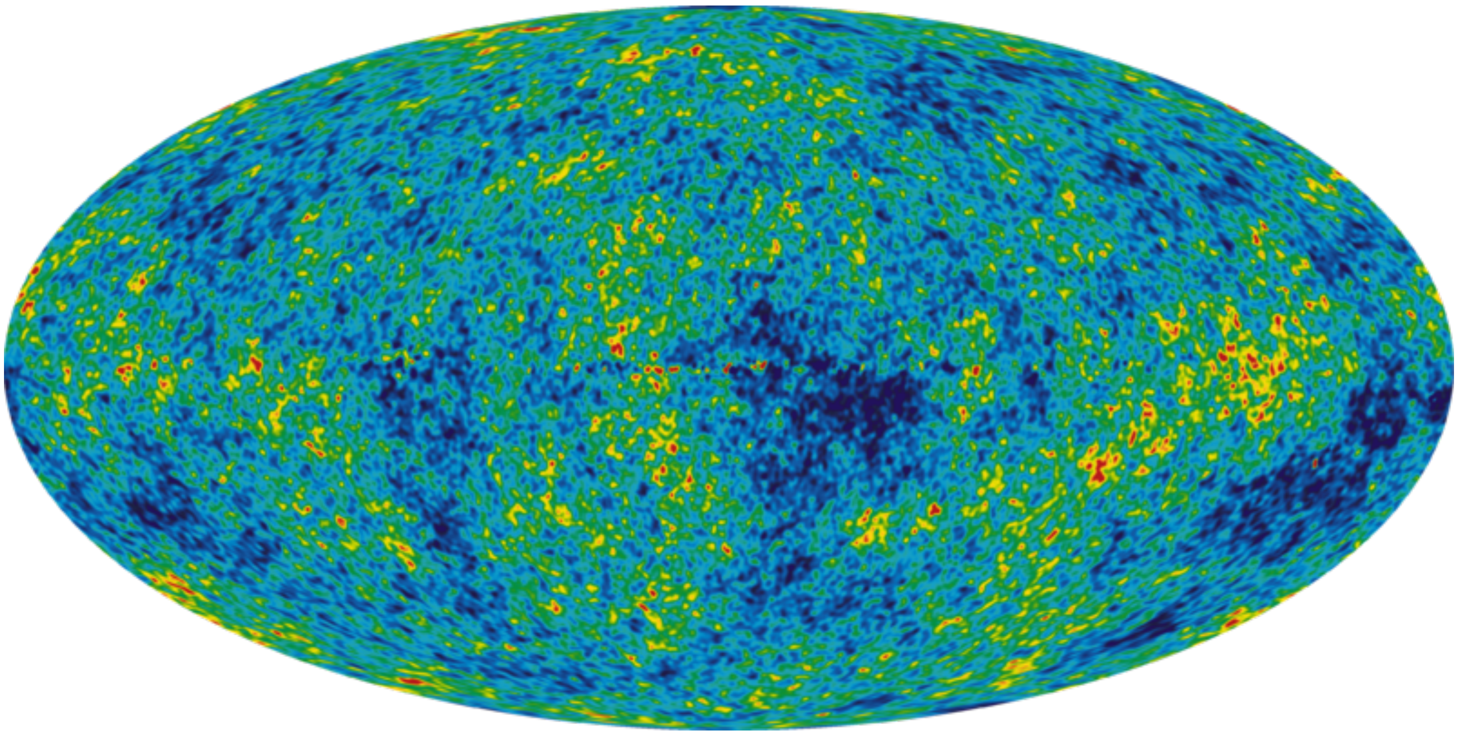
scale factor measures how far galaxies have diverged

Dynamics of universe



3. The Horizon Problem

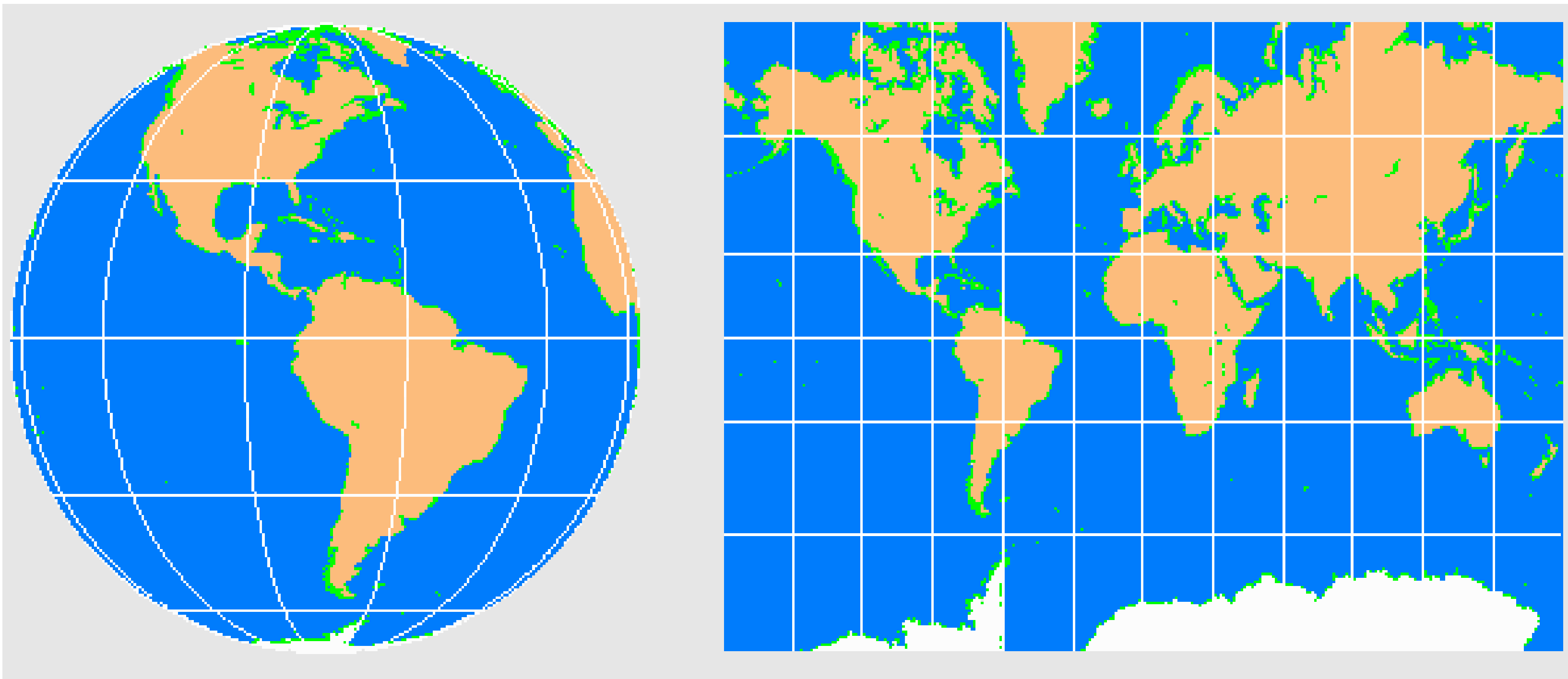
- Current universe is uniform (isotropic and homogeneous) and has been since shortly after the Big Bang.
- *Evidence*: The Cosmic Microwave Background radiation (CMB) left over from the Big Bang is uniformly spread over the entire universe.



- *But*: Robertson-Walker spacetimes do not expand *fast enough* to have allowed all parts to causally interact and reach equilibrium by the time the universe became uniform!

Mercator map of the earth "flattens" out the curvature at the expense of perspective.

- *It does this by representing longitude lines as straight lines.*

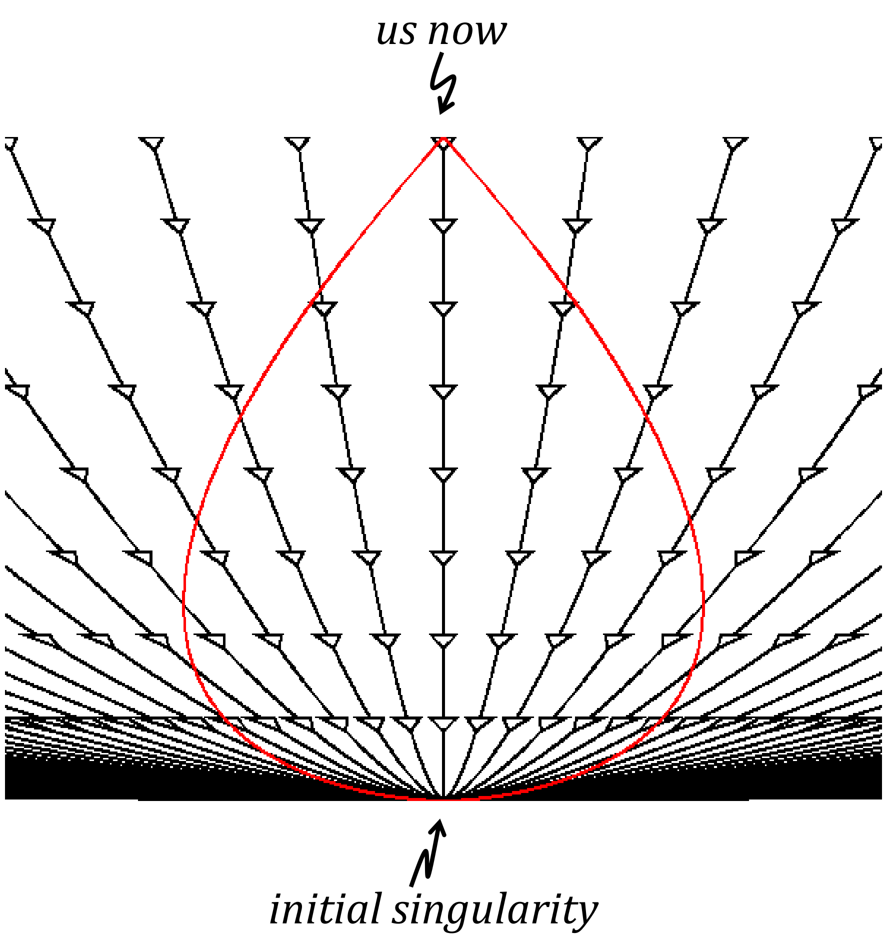


Normal map *showing curvature.*

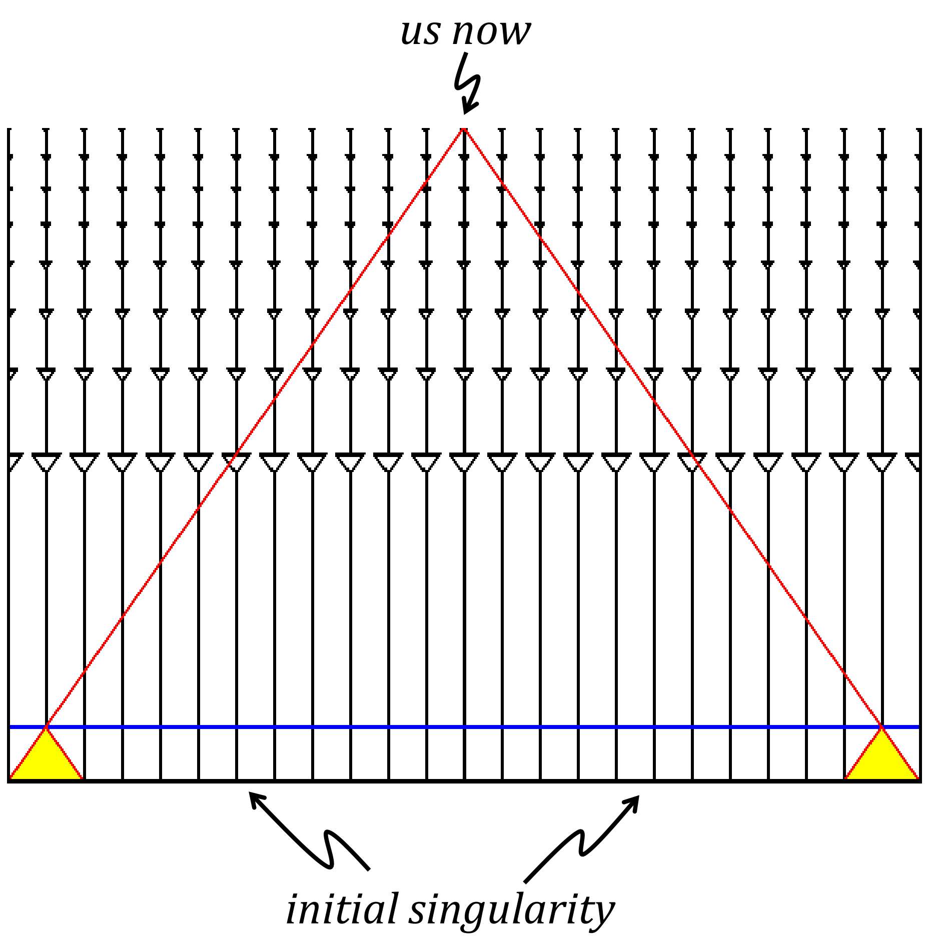
Mercator map *without curvature.*
Antarctica covers the entire bottom edge!

A *conformal spacetime diagram* does the same thing.

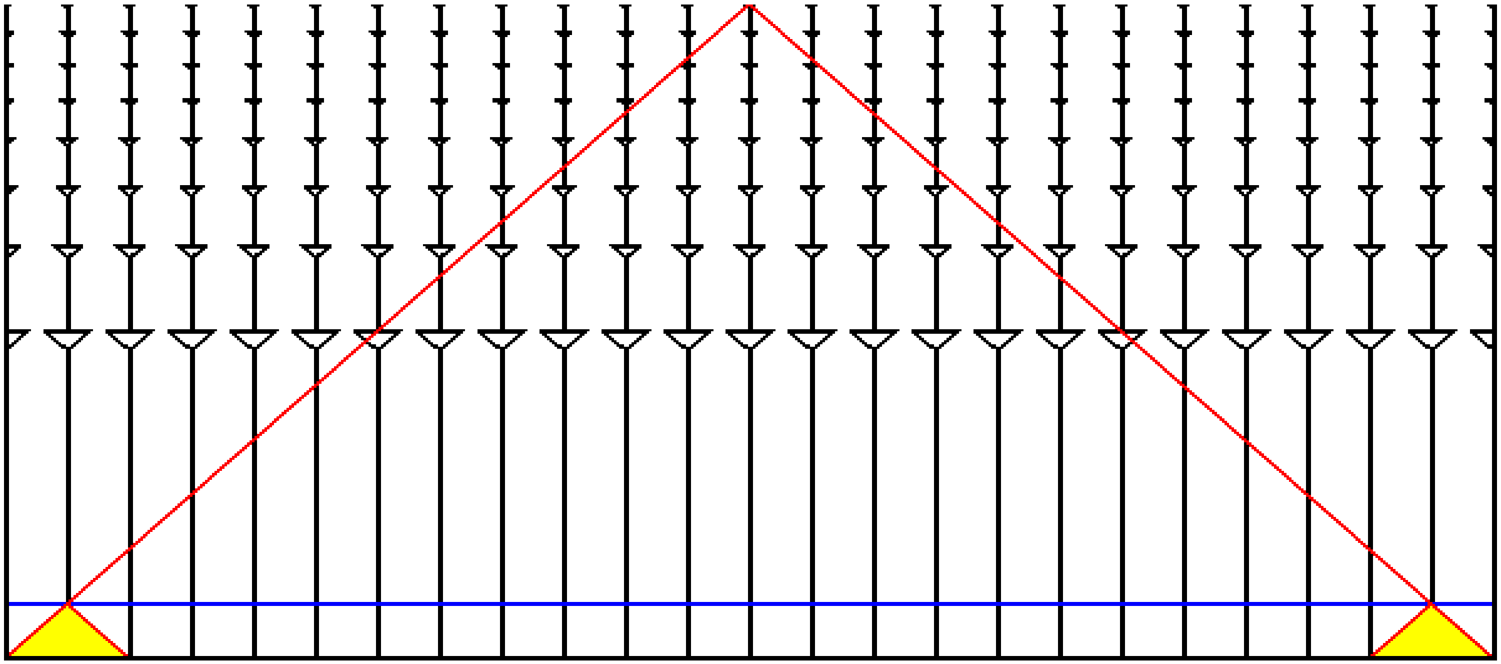
- *It flattens out curvature by representing lightlike trajectories as straight lines.*



A ***normal spacetime diagram*** *of a Robertson-Walker spacetime showing curvature.*



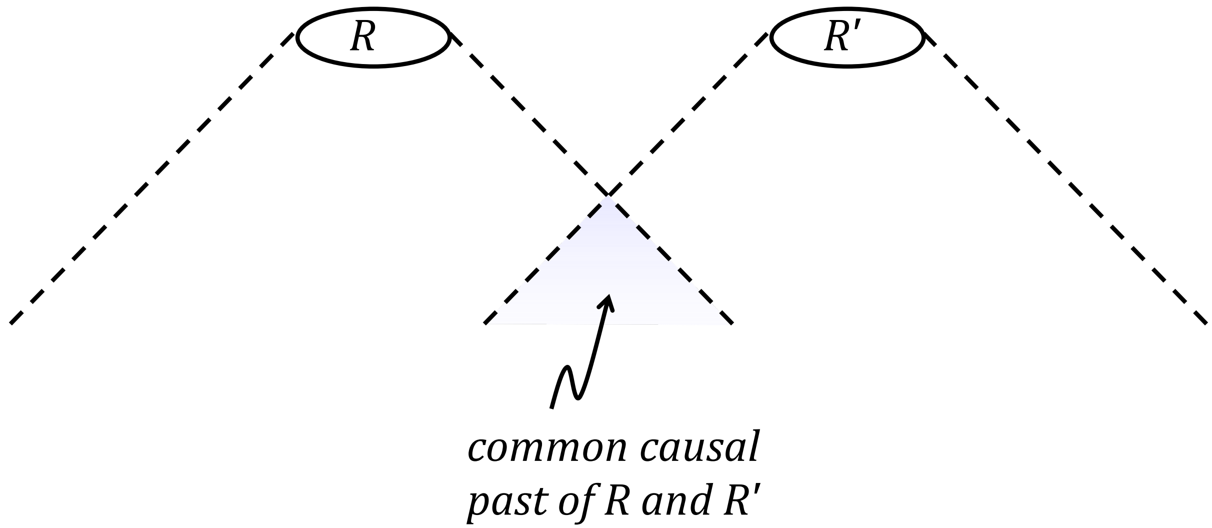
A ***conformal spacetime diagram*** *of the same spacetime. The initial singularity covers the entire bottom edge!*



- Yellow triangles are the past lightcones of two events that occur shortly after the Big Bang (at the time matter decouples from radiation).
  - *They do not intersect! (The initial singularity cuts them off).*
- <u>*So*</u>: The events cannot interact with each other.
  - *No causal signal travelling ≤ c can pass between them.*
- <u>*So*</u>: How can they both be in equilibrium?
  - *Current popular explanation: Inflation!*

Is this a Problem?

Consider: Two spacelike-separated regions of spacetime R and R'.



- Distinguish two types of explanations of the states of R and R':

Common Cause Explanation of states of R and R′

An explanation of the states R, R' in terms of their *common causal past*.

Deductive-Nomological (DN) Explanation of states of R and R′

An explanation of the states R, R' in terms of how these states can be derived from a *deterministic dynamical law*.

Claim: The Horizon "Problem" is a problem only if we require Common Cause explanations of the uniformity of the universe.

Why prefer Common Cause explanations?

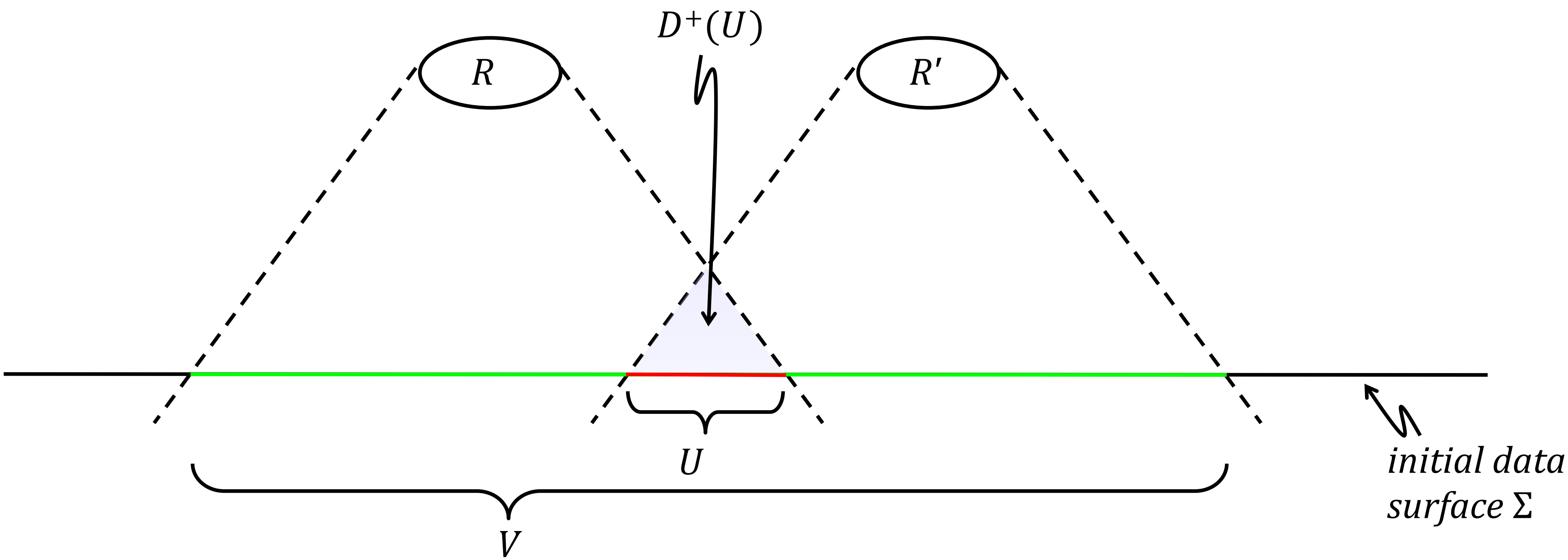
If you think only *material bodies* are real and that they can only influence each other with signals traveling at or less than *c*.

Why allow DN explanations?

If you think *fields* are just as real as bodies.

- *If fields are real, then the state of a body O at some time t will depend not only on everything the body has interacted with in its causal past, but also the state of the field it is embedded in at time t.*
- *And the state of this field may depend on objects or fields outside the casual past of O.*

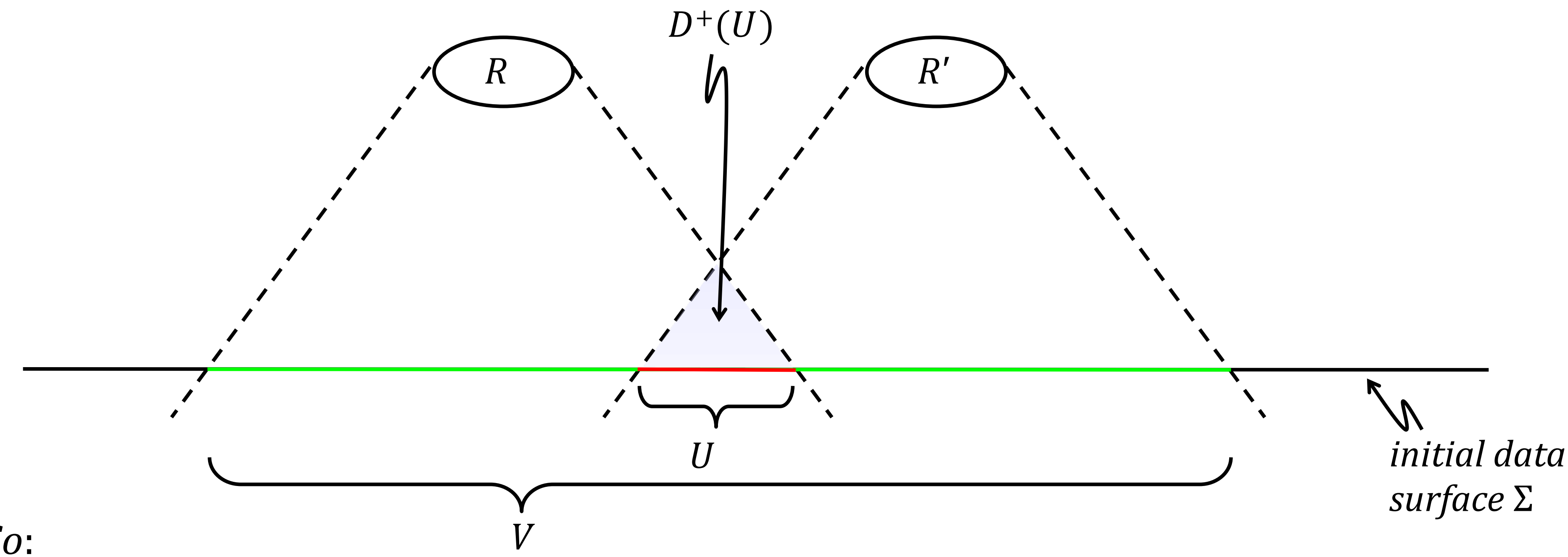
Ex: If states at R and R' are governed by a *deterministic dynamical law*, then to fully specify them, need to specify initial data at some past (or future) *instant of time*; *i.e.*, need to specify initial data on a *spacelike* surface Σ, called a *Cauchy* ("KO-shee") surface:



- $U =$ That part of Σ that determines events in the *common causal past* of R, R'.
- $V =$ (green + red lines) $=$ That part of Σ that determines events *including* R, R'.
- $D^+(U) =$ All events that are determined by U.

A surface *determines* a set of events just when every causal (timelike/lightlike) past-directed path through any of those events intersects the surface.

Ex: If states at R and R' are governed by a *deterministic dynamical law*, then to fully specify them, need to specify initial data at some past (or future) *instant of time*; *i.e.*, need to specify initial data on a *spacelike* surface Σ, called a *Cauchy* ("KO-shee") surface:



So:

(1) The initial data (on U) that determines the common causal past of R, R' will in general fail to completely determine R, R'. (R and R' lie outside $D^+(U)$.)

(2) The initial data (on V) that *does* completely determine R, R' extends beyond the common causal past of R, R'.

Upshot: Common Cause Explanations are limited in the context of deterministic dynamical laws.