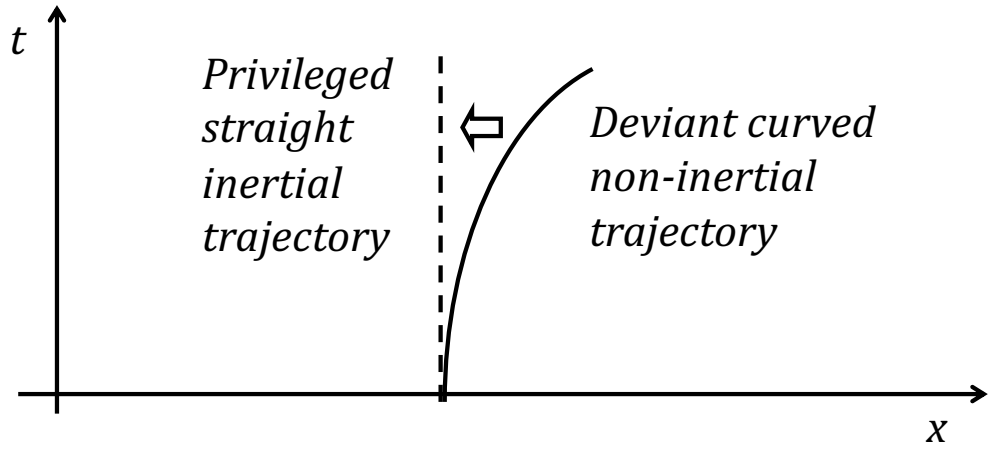
11. Interpreting General Relativity

Topics:
1. Mach's Principle
2. Cosmological Constant
3. Hole Argument

Substantivalism

(S1) Spacetime exists as substance independently of physical objects.

(S2) Inertial effects are due to an object's motion with respect to spacetime.



Spacetime "resists" deviant non-inertial trajectories, pushing them back toward inertial norm.

Relationalism

(R1) Spacetime consists in the relations between physical objects.

(R2) Inertial effects can only be due to an object's motion with respect to other objects.

Question: How should we interpret general relativity?

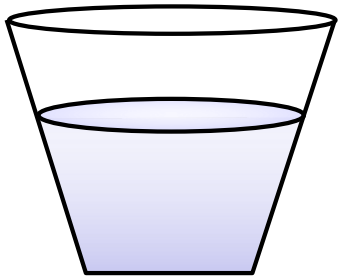
1. Mach's Principle and GR: S2 *vs* R2

Against R2: Newton's Bucket Thought Experiment

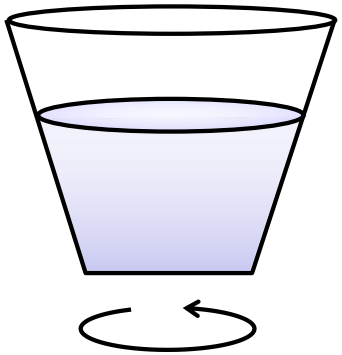
Claim: *Rotational motion* is best explained as motion with respect to substantival space, and not as motion with respect to other physical objects.

Set Up: Consider a water-filled bucket suspended from a rope. Twist up the rope and release the bucket. Observe its motion at three stages:

- Stage 1 right before release.
- Stage 2 right after release (water not yet rotating).
- Stage 3 at which water and bucket are rotating at the same rate.

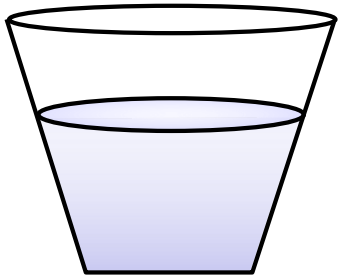


Stage 1
water at rest
bucket at rest

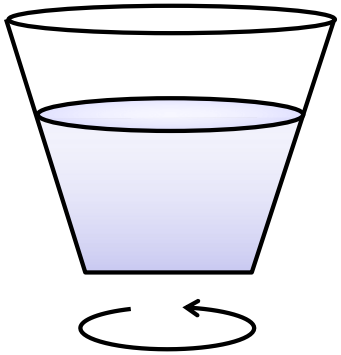


Stage 2
water at rest
bucket rotating

Stage 3
water rotating
bucket rotating, at same rate.



Stage 1
water at rest
bucket at rest



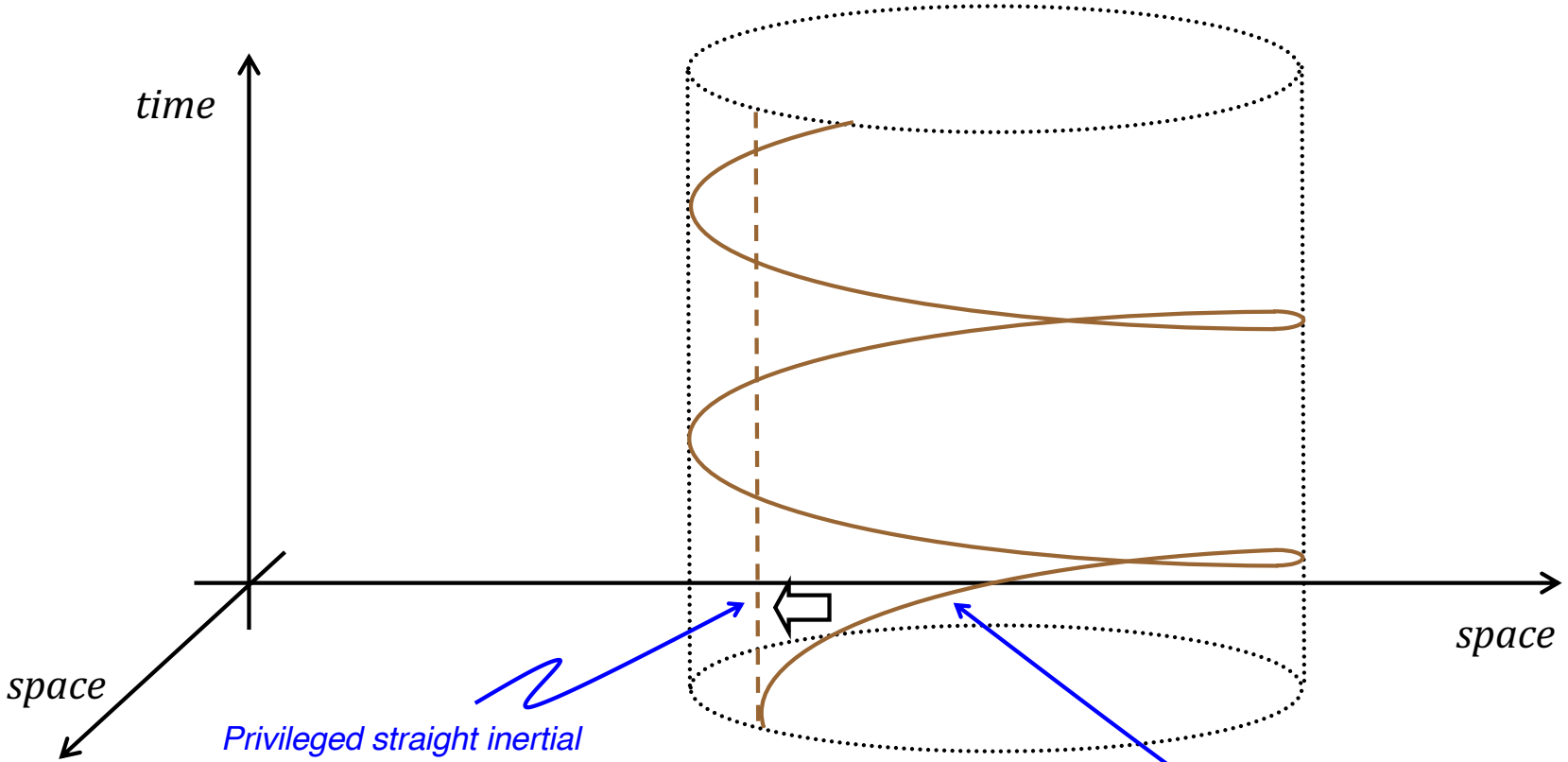
Stage 2
water at rest
bucket rotating

Stage 3
water rotating
bucket rotating, at same rate.

Question: At what stage can we say the water is in rotation?

Substantivalist (Newton): Stage 3. Presence of inertial force is explained in terms of motion with respect to substantival space.

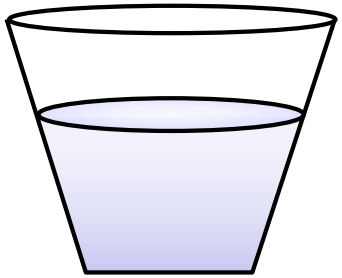
Einstein's Newton: Substantival space is the *cause* of the inertial force experienced by the water: It "resists" the non-inertial (rotational) motion of the water and "pushes" it back toward inertial motion.



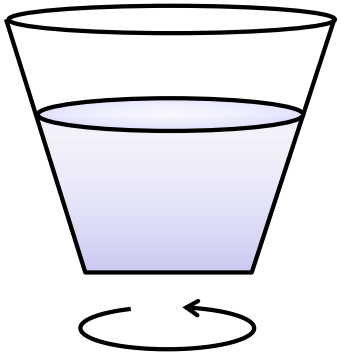
Privileged straight inertial trajectory of a water molecule at rest.

Deviant curved (helical) non-inertial trajectory of a rotating water molecule.

Spacetime "resists" deviant non-inertial rotational trajectories of water molecules, pushing them back toward inertial norm.



Stage 1
water at rest
bucket at rest



Stage 2
water at rest
bucket rotating

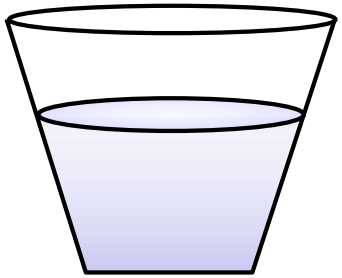
Stage 3
water rotating
bucket rotating, at same rate.

Question: At what stage can we say the water is in rotation?

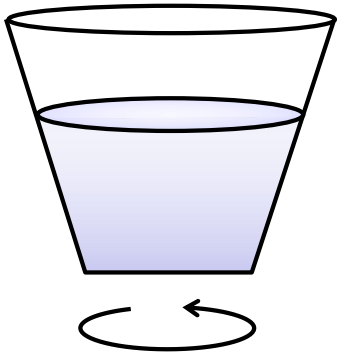
Relationalist Options:

(1) (Descartes): "True" motion is motion with respect to physical objects in immediate contact with the moving object and considered to be at rest.

- *So*: Water must be truely rotating at Stage 2, and truely at rest at Stages 1 and 3.
- *But*: Stages 1 and 3 are physically distinct! Water experiences a force in stage 3 but not in stage 1.



Stage 1
water at rest
bucket at rest



Stage 2
water at rest
bucket rotating

Stage 3
water rotating
bucket rotating, at same rate.

Question: At what stage can we say the water is in rotation?

Relationalist Options:

(2) Stage 3.

But then:

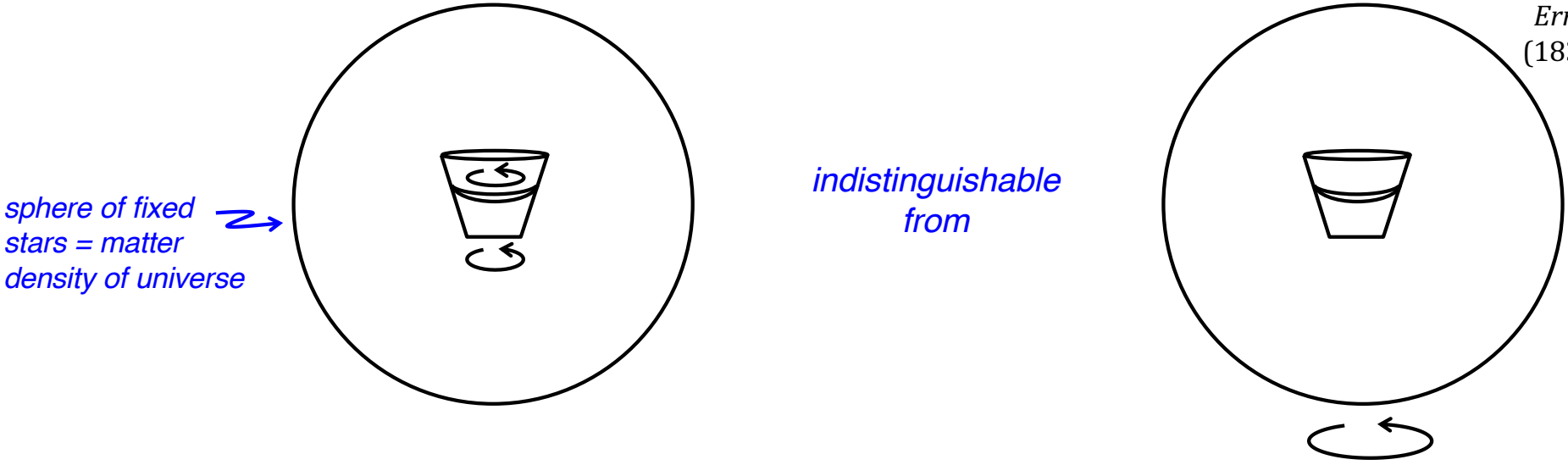
(a) With respect to what physical object is the water in stage 3 rotating?

(b) How does motion with respect to this object *cause* the inertial force experienced by the water?

Mach's Responses (*Science of Mechanics* 1893)

Ernst Mach
(1836-1916)

(a) Water is rotating with respect to the "fixed stars" in stage 3.



Water rotating.
Fixed stars at rest.

Water at rest.
Fixed stars rotating.

(b) *Mach's Principle*: The *matter density* in the universe is the cause of inertial forces on objects undergoing non-inertial motion

- *Details? How* does the matter density of the universe cause inertial forces?
  - *Mach provides no explanation.*
  - *Einstein thinks general relativity supplies the explanation!*

In GR: The structure of spacetime...
...*determines* the inertial frames of reference (*i.e.*, the families of straights).
...*is determined by* the matter density.

- *Newton's substantivalist (Einstein's interpretation)*:

The structure of spacetime is the cause of inertial forces on accelerating objects.

- *Mach's relationalist*:

The matter density in the universe is the cause of inertial forces on accelerating objects.

- *In GR*: The matter density in the universe determines the structure of spacetime, which then determines the inertial frames of reference.
- Is "determining the inertial frames of reference" the same as "being the cause of inertial forces"?

Does GR agree with Newton's substantivalist or Mach's relationalist?

Three Questions of Interpretation

(1) Does the GR account support substantivalism or relationalism?

Depends on how you interpret the "structure of spacetime":

(a) A *substantivalist* may say: "The structure of spacetime is given by properties of real spacetime points."

- *Take all physical fields out of the universe and real spacetime would be left.*

(b) A *relationalist* may say: "The structure of spacetime is given by properties of the metric field, which is a real physical field."

- *Take all physical fields out of the universe and nothing would be left.*

$$G_{\mu\nu}(g_{\mu\nu}) = \kappa T_{\mu\nu}$$

metric field *matter fields*

Should the metric field also be considered a matter field?

Substantivalist: No!

Relationalist: Yes!

(2) Does the GR account support Mach's Principle?

Depends on how you interpret what matter is!

- In GR, there are *"vacuum" solutions* to the Einstein equations.
  - *Non-flat solutions in which the matter density is zero ($T_{\mu\nu} = 0$)!*
  - *"Gravitational waves" with no sources.*

$$G_{\mu\nu} = \kappa T_{\mu\nu} = 0$$

Doesn't necessarily mean zero curvature!

(a) *A substanativalist may say*: "In GR, there can be inertial forces (as experienced by gravitational waves) in a universe *devoid of matter!"*
  - *So Mach's Principle does not hold in general.*

(b) *A relationalist may respond*: "In vacuum solutions, the inertial forces are still determined by a matter field; namely, the metric field!"
  - *Moreover, such 'vacuum' solutions don't really describe universes devoid of matter; what they describe are universes in which the only matter field is the metric field!*
  - *So Mach's Principle does hold in general!"*

(3) Do vacuum solutions support substantivalism or relationalism?

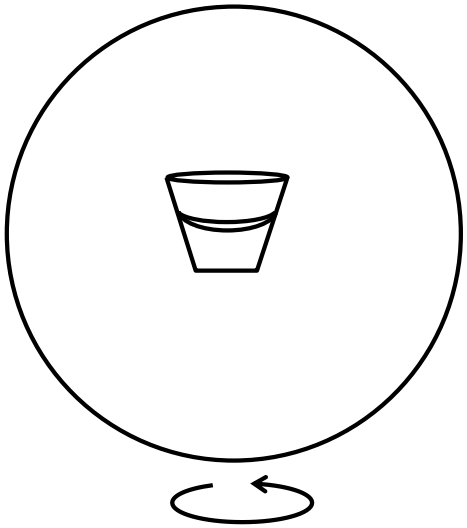
(a) *A substantivalist may say*: "This supports my view: Gravitational waves are propagations of spacetime itself."

(b) *A relationalist may respond*: "This supports my view: Gravitational waves are propagations in the metric field."

"One hundred years after Albert Einstein predicted the existence of gravitational waves, scientists have finally spotted these elusive ripples in space-time. In a highly anticipated announcement, physicists with the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) revealed on 11 February that their twin detectors have heard the gravitational 'ringing' produced by the collision of two black holes about 400 megaparsects (1.3 billion light-years) from Earth." Casteivecchi & Witze (2016) 'Eintein's Gravitational Waves Found at Last', *Nature News*.

2. The Cosmological Constant

Einstein's (1916) "Machian" solution to Newton's bucket:



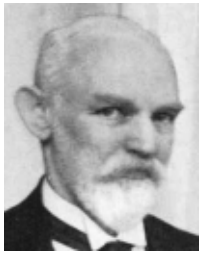
The rotating shell of the fixed stars determines the metric field in the region of the bucket.

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

encodes metric field of region of bucket

encodes rotating sphere of fixed stars

- de Sitter (1916) points out that Einstein's Machian solution requires *boundary conditions* at spatial infinity. So it's not fully Machian!



Willem de Sitter (1872-1934)

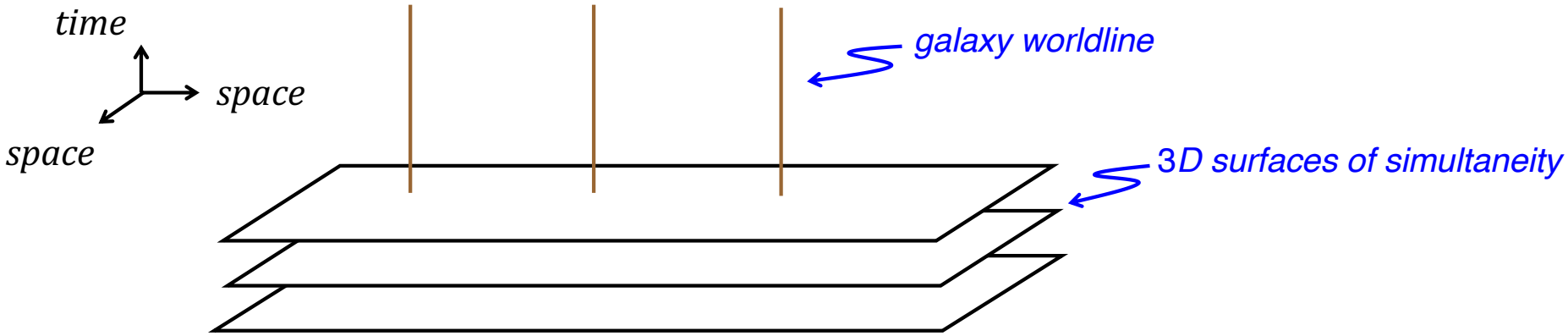
$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

encodes metric field of region of bucket, provided one specifies boundary conditions at spatial infinity

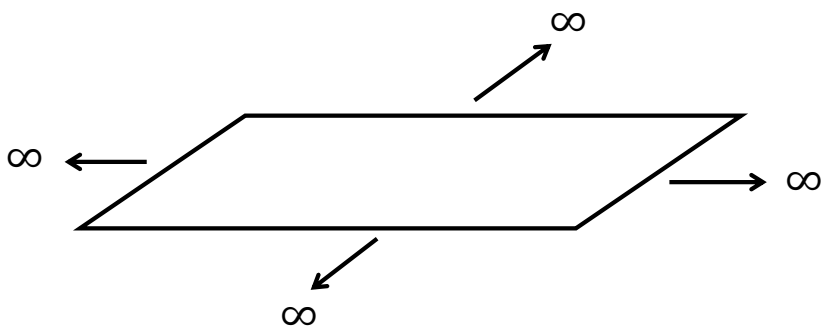
encodes rotating sphere of fixed stars

Einstein's Response: Do away with spatial infinity! Assume a *spatially closed* universe.

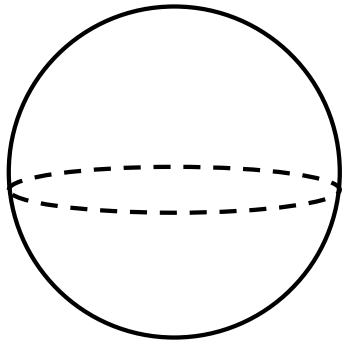
- Traditional Picture of Universe (1 spatial dim suppressed):



- 3-dim surfaces of simultaneity (3-dim space) assumed to be Euclidean, hence are "open" (infinite, unbounded) and extend to spatial infinity.
- Einstein replaces these with closed (finite, bounded) spherical surfaces.



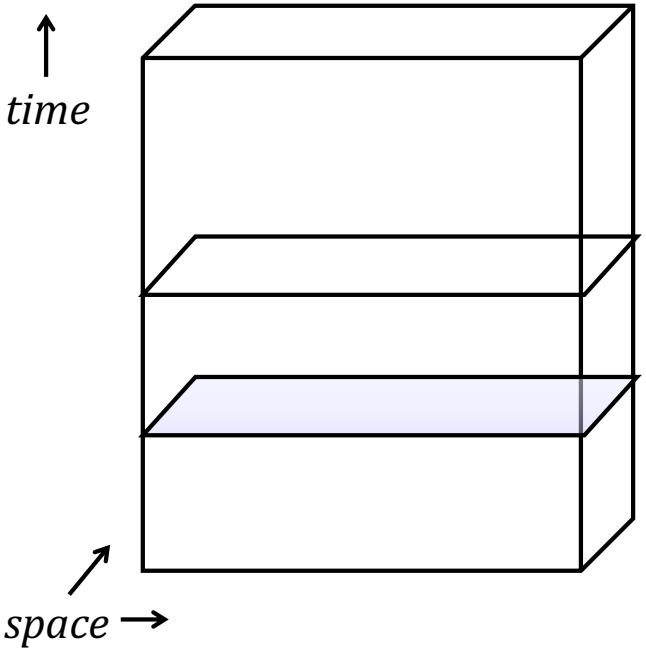
infinite, unbounded



finite, unbounded

Traditional Picture

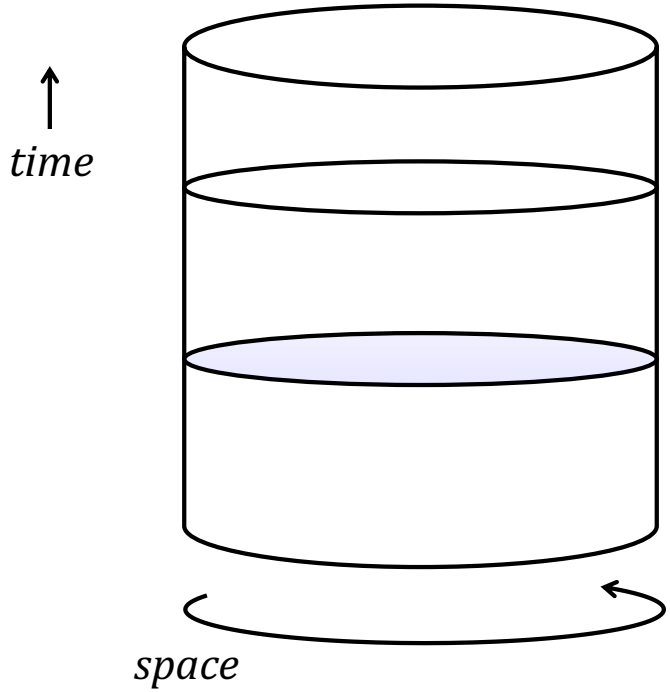
In 2 spatial dim and 1 time dim.



Spatial slice = open 2D Euclidean plane. Add back other spatial dimension and get open 3D Euclidean space.

The Einstein Cylinder Universe (1917)

In 1 spatial dim and 1 time dim

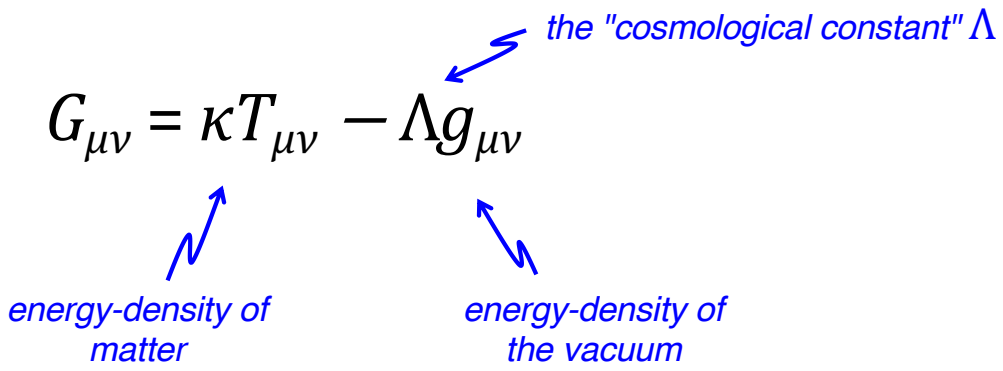


Spatial slice = closed 1D circle. Add back 2 other spatial dimensions and get closed 3D spherical space!

Big Problem: The Einstein Universe is not a solution to the Einstein equations!

Einstein's (1917) Response: *Force* it to be a solution!

- Modifies the Einstein equations with a "*cosmological constant*" term:



- Allows solutions with *static* distribution of matter, like the Einstein Universe.

How? The cosmological constant term can be interpreted as representing the energy-density of the vacuum ($T_{\mu\nu} = 0$). If it's taken to be negative, it mathematically counteracts the positive (attractive) gravitational field due to the energy-density of matter (represented by a positive term in the Einstein equations).

According to Einstein, Λ plays two roles:

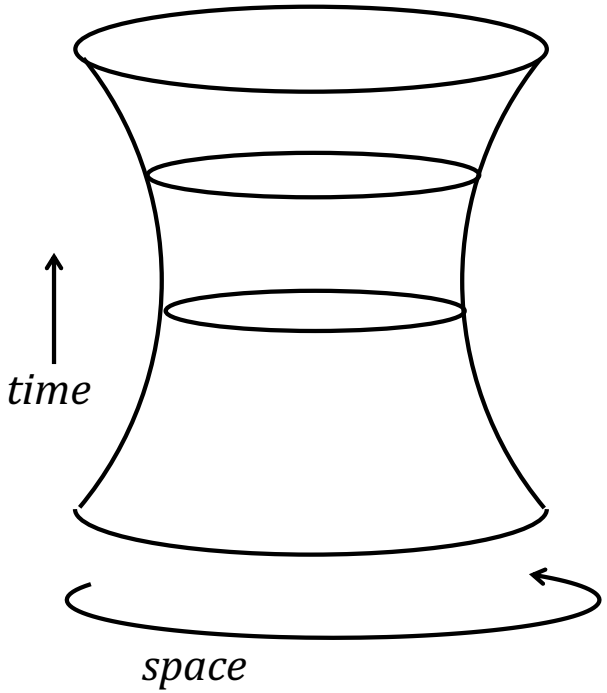
(1) *It vindicates Mach's Principle in GR*. (It allows pure Machian solutions to Newton's Bucket that do not rely on boundary conditions at spatial infinity.)

(2) *It guarantees static solutions to the Einstein equations*.

- De Sitter responds with...

The de Sitter Hyperboloid Universe (1917)

In 1 spatial dim and 1 time dim

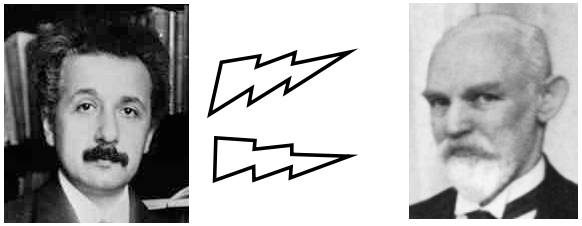


2 *Important Characteristics*:

(a) A solution to the Einstein equations with cosmological constant *and* with $T_{\mu\nu} = 0$. (A *vacuum solution with cosmological constant.*)

(b) A *non-static* solution: the spatial slices vary in size.

- *Consequence of (a)*: *If* you think vacuum solutions count against Mach's Principle, then the cosmological constant does *not* guarantee Mach's Principle!



- *Einstein's Response*:

The De Sitter Universe is not static, so we can ignore it. (Implication: only static solutions are physical.)

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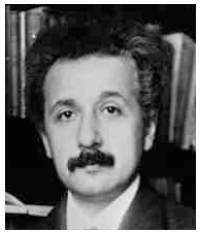


- *But*: 1920's - Discovery of *dynamic expansion* of the universe. Entails that non-static solutions to field equations *are* realistic.

- *Famous Einstein comment*: Cosmological constant was "greatest blunder of my life!"

- *Einstein's Last laugh:* 1990's - Evidence suggests expansion of the universe is accelerating at a rate that cannot be explained by the Einstein equations in their original form. Suggests a new role for the cosmological constant term!

3. The Hole Argument: S1 *vs*. R1

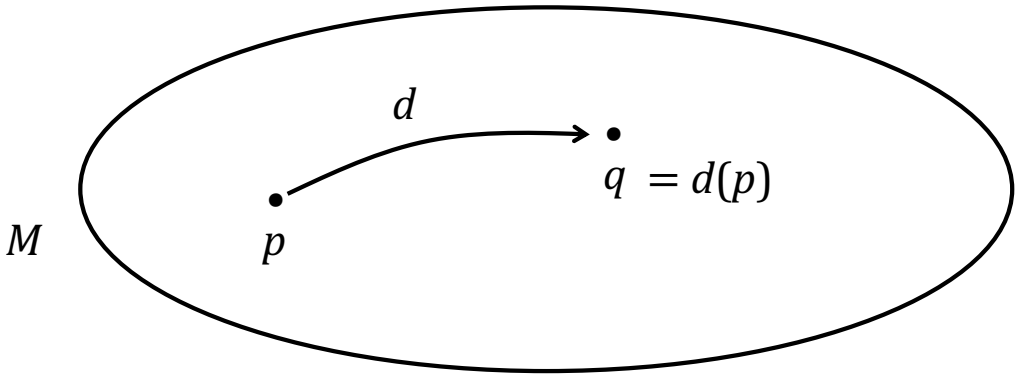
- *Manifold Substantivalism*: The 4-dim collection of points (a manifold) of a general relativistic spacetime represents real substantival spacetime points.

Claim (Hole Argument): If we adopt a manifold substantivalist interpretation of GR, then we have to conclude that GR is indeterministic!

Symmetries and Equations of Motion

Symmetries = transformations that leave equations of motion unchanged.

- Newton's equations: Symmetries = Galilean transformations
- Maxwell's equations: Symmetries = Lorentz transformations
- Einstein equations: Symmetries = "diffeomorphisms"

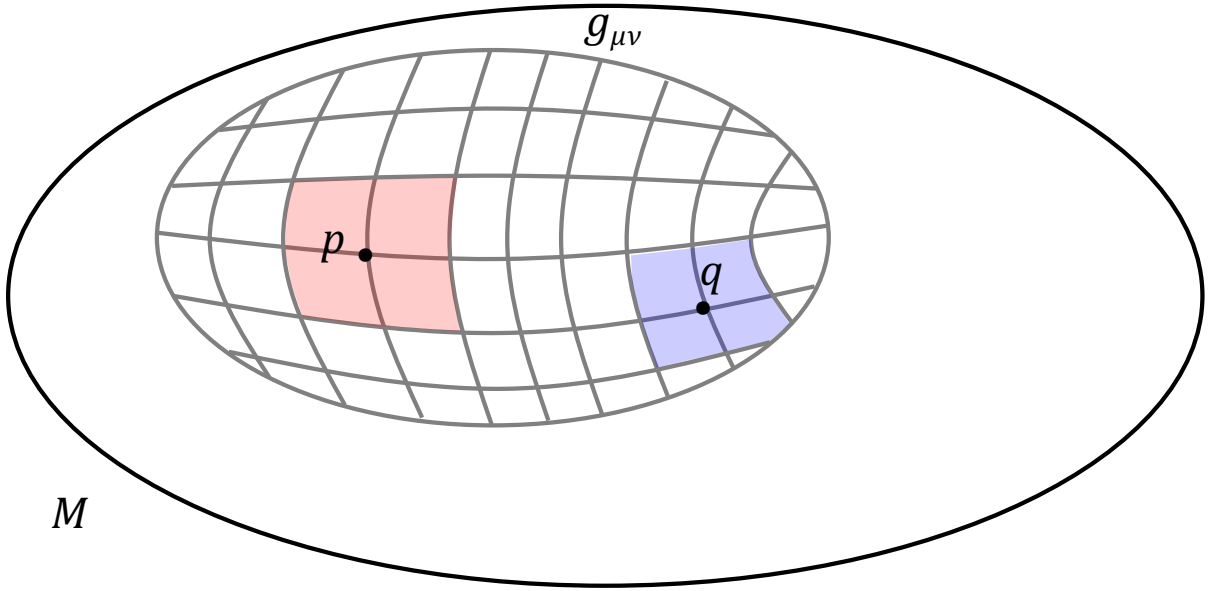


diffeomorphism = transformation between points on a manifold
= arbitrary coordinate transformation

What this means

- Let $d{*}g_{\mu\nu}$ be what you get when you act with d on $g_{\mu\nu}$.
- *Then*: If $G_{\mu\nu}(g_{\mu\nu}) = \kappa T_{\mu\nu}$, then $G_{\mu\nu}(d{*}g_{\mu\nu}) = \kappa T_{\mu\nu}$.
  - *If $g_{\mu\nu}$ is a solution to the Einstein equations with matter distribution $T_{\mu\nu}$, then so is $d{*}g_{\mu\nu}$.*

- $d{*}g_{\mu\nu}$ is obtained from $g_{\mu\nu}$ by "shifting" it to a different set of points.



What this means

- Let $d*g_{\mu\nu}$ be what you get when you act with d on $g_{\mu\nu}$.
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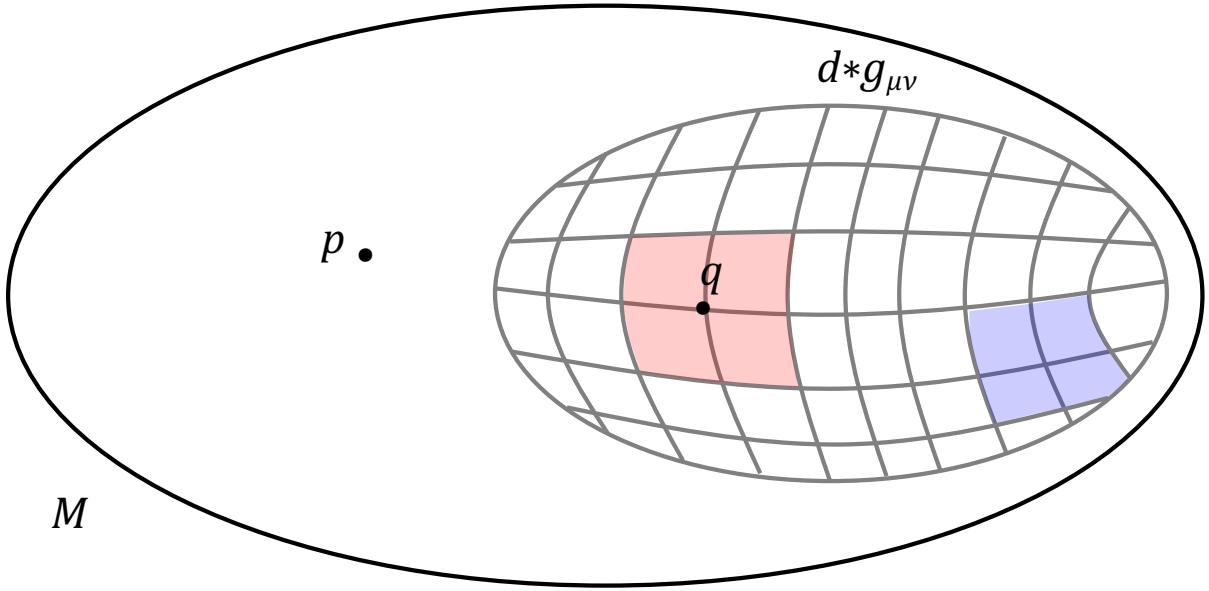
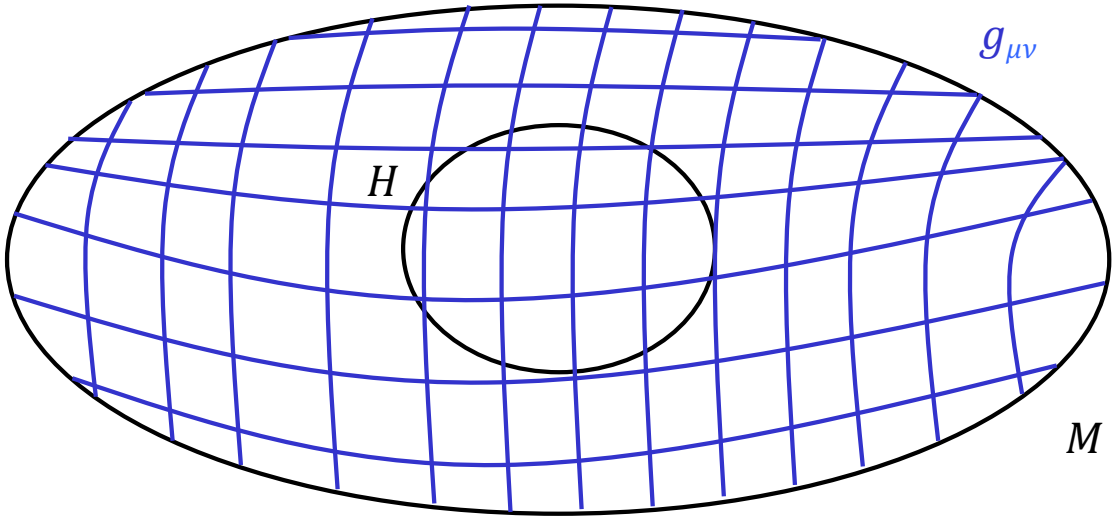


Now: Construct a "hole" diffeomorphism h such that:

(1) h = identity outside a region H (the "hole") of M.

(2) $h \neq$ identity inside H.



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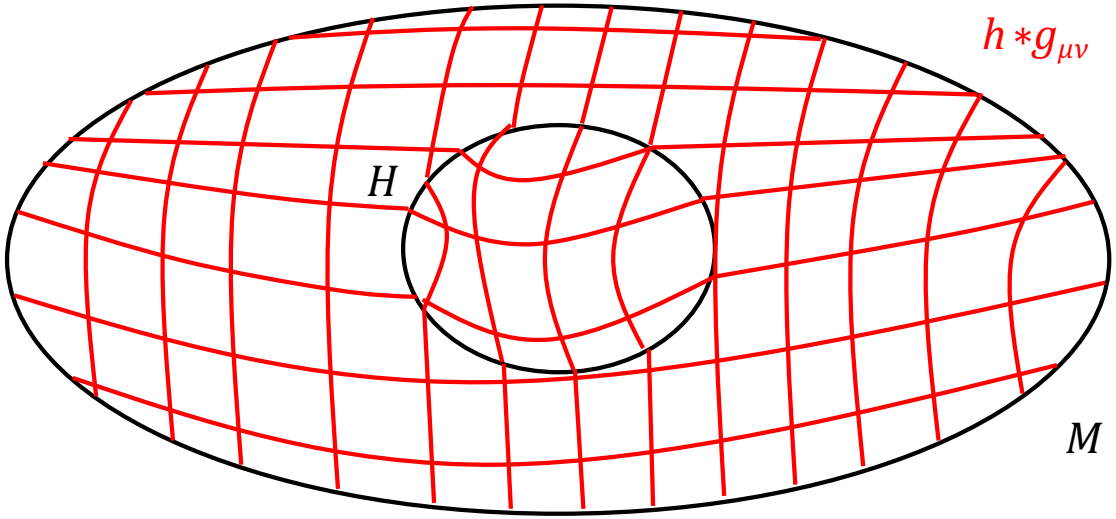

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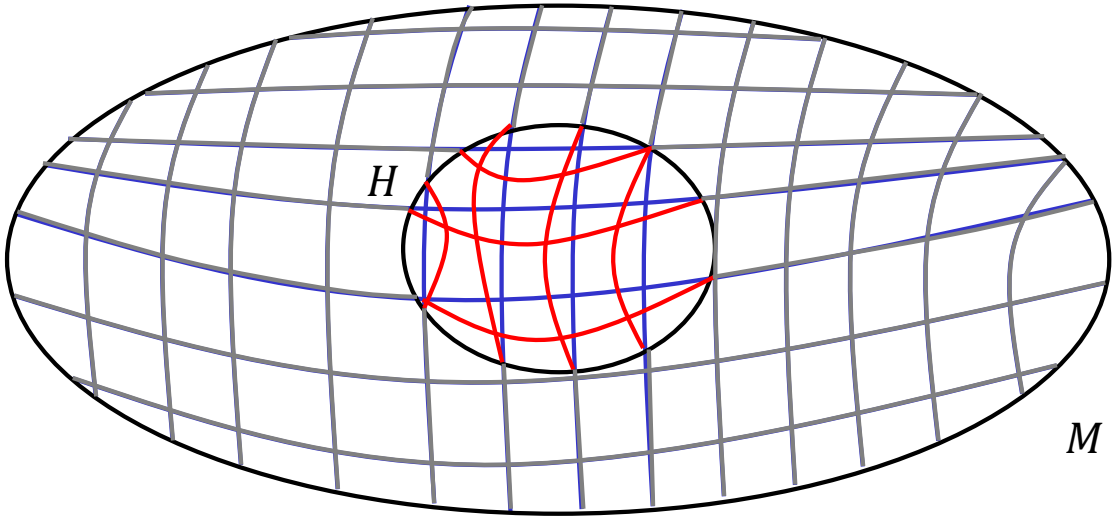

- $g_{\mu\nu}$ and $h*g_{\mu\nu}$ disagree inside H.
- $g_{\mu\nu}$ and $h*g_{\mu\nu}$ agree outside H.
- h shifts the metric $g_{\mu\nu}$ only in the hole.

- Manifold substantivalists must claim that $g_{\mu\nu}$ and $h*g_{\mu\nu}$ describe different states of affairs.
- *But*: $g_{\mu\nu}$ and $h*g_{\mu\nu}$ are *physically indistinguishable* (both are solutions for the same matter distribution).
- *So:* Manifold substantivalists must conclude that the Einstein equations are *indeterministic*.

A complete specification of the matter distribution outside the hole fails to uniquely determine the metric inside the hole.

Some Options

1. Adopt a relationalist interpretation of GR.
   - *Since $g_{\mu\nu}$ and $h^*g_{\mu\nu}$ describe the same spacetime relations between objects, and differ only on what points they are spread over, a relationalist will claim they are not distinct: they represent the same state of affairs.*

2. Modify your spacetime substantivalism.
   - *Claim that spacetime points (or regions) are real, but this doesn't necessarily mean $g_{\mu\nu}$ and $h^*g_{\mu\nu}$ describe distinct states of affairs.*
   - *Maybe spacetime points obtain their "identities" in strange ways.*
   - *Maybe they obtain them only after a field has been "spread" over them, and not before.*

3. Modify your spacetime realism.
   - *Claim that spacetime structure can be thought of as real without having to additionally claim that spacetime points are real (or that regions of a manifold are real).*

Non-Trivial options: they influence how you might attempt to reconcile GR with quantum theory!