09. Geometrization of Gravity

1. Motivation

Principle of Relativity = The *laws of physics* are the same in all *inertial* reference frames.

laws of physics → *1916: Newton's Laws of Motion and Maxwell's Laws*

inertial → *non-accelerating*

- *Recall*: In special relativity, as in Newton's theory of motion, velocity is *relative*, and acceleration is *absolute*.

(a) Can we extend the P. of R. to *accelerated* reference frames?
- *Can acceleration be made relative?*

(b) Can we add *gravity* to SR?
- *Can the "laws of physics" be extended to include Newton's Law of Gravity?*

Einstein's proposal: *Geometricize* the gravitational force; turn it into a manifestation of spacetime curvature.
- *Solves (b) and partially solves (a).*
- *Gravitationally-induced accelerations become relative.*

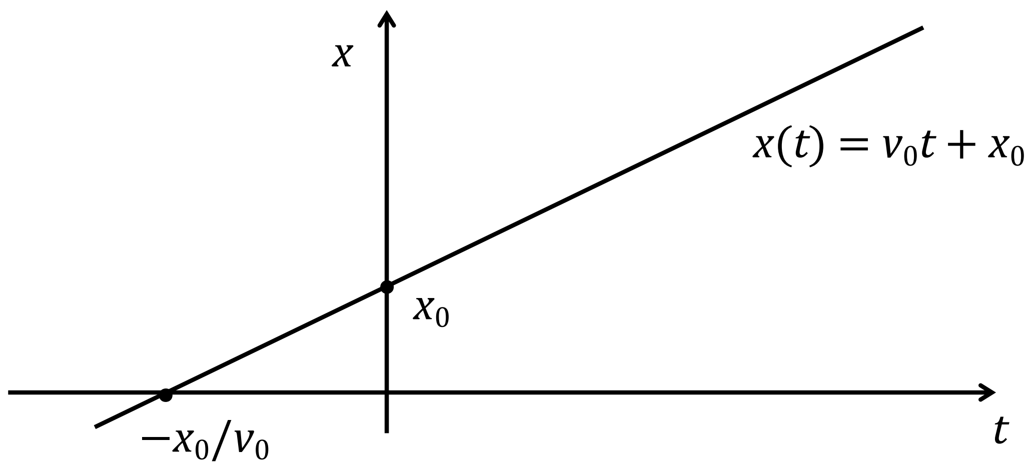
2. Geometrization of Gravity

Two key observations:

(i) *Geometry*

Equation for a straight line:

$$\frac{d^2x}{dt^2} = 0, \quad \text{or} \quad x(t) = v_0 t + x_0, \quad \text{where } v_0, x_0 = \text{constants}$$



- In inertial frames, Newton's 2nd Law is $F = ma = m\dfrac{d^2x}{dt^2}$.
- In the absence of external forces ($F = 0$) an object's position $x(t)$ as a function of time is the equation of a straight line! (Newton's 1st Law.)

(ii) *Physics*

- Consider when the external force an object experiences is due to gravity:

$F = \dfrac{GMm_g}{r^2}$	The Newtonian gravitational force on an object of mass m_g due to another object of mass M a distance r away.
$= -m_g \nabla \Phi$	$\Phi = -GM/r$ is the Newtonian gravitational potential field (describes the particular gravitational field produced by mass M).

- Newton's 2nd Law becomes: $-m_g \nabla \Phi = m_i a$

gravitational mass (pointing to m_g)

⇓

measure of degree to which an object experiences the Newtonian gravitational force

inertial mass (pointing to m_i)

⇓

measure of inertia of an object -- tendency of object to obey Newton's 1st Law

- Is m_g the same as m_i?
  - *Conceptually and mathematically, no!*
  - *Physically, yes! All known experiments indicate that* $m_g = m_i$.

Consequence of $m_g = m_i$:

> Universality of Gravitational Force
>
> In any given gravitational field (described by some Φ), all objects fall with the same acceleration $a = -\nabla\Phi$.

- This is regardless of the object's internal properties (it's mass, charge, etc.).

The gravitational force is universal: it affects all objects in the same way.

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Constrast with the electromagnetic force:

$$\vec{F}_{EM} = q(\vec{E} + \vec{v}\times\vec{B})$$

Electromagnetic force experienced by an object with electric charge q moving at speed v in the presence of electric E and magnetic B fields.

- Newton's 2nd Law becomes:

$$q(\vec{E} + \vec{v}\times\vec{B}) = m_i\vec{a} \quad \text{or} \quad \vec{a} = \frac{q}{m_i}(\vec{E} + \vec{v}\times\vec{B})$$

- Not universal! How much an object accelerates in given E- and B-fields depends on its charge and its inertial mass in the ratio q/m_i.

Different objects will experience different electromagnetically-induced accelerations.

Since gravity is universal, let's incorporate it into the structure of spacetime!

- Let's "geometrize" it.

- The motion of an object in a gravitational field is given by

$$\frac{d^2x}{dt^2} = a = -\nabla\Phi$$

← *A curved line in a flat space.*

- We can rewrite it as:

$$\left(\frac{d^2x}{dt^2} + \nabla\Phi\right) = 0$$

← *A straight line in a curved space!*

- We can view these particular "$\nabla\Phi$"-straight lines in the curved space as the paths of objects that are undergoing gravitationally-induced acceleration.

- The "extra" term $\nabla\Phi$ can be encoded into a "non-flat" metric.

In *flat* Galilean and Minkowski spacetimes, there is a distinction between:

straight trajectories;
no forces present

$$\frac{d^2x}{dt^2} = 0$$

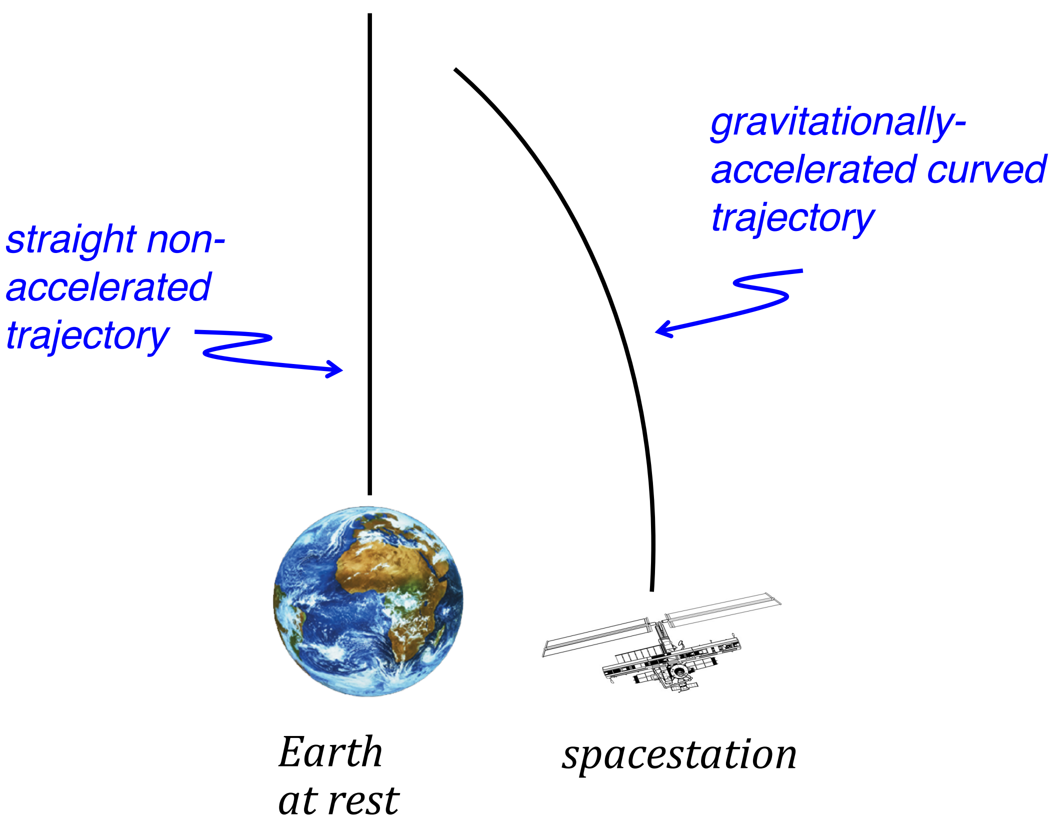
$$\mathrm{ma} = 0$$

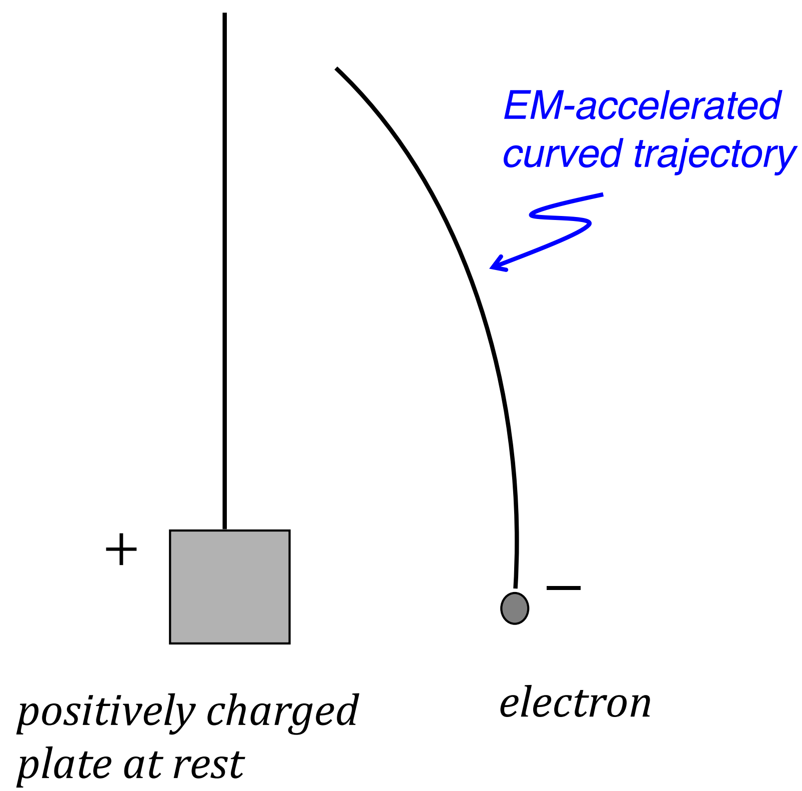
vs.

curved trajectories;
forces present

$$\frac{d^2x}{dt^2} \neq 0$$

$$\mathrm{ma} = \mathrm{F}$$





In *curved* general relativistic spacetimes:

- *No* distinction between straights and grav.-accelerated trajectories.
- *Still* a distinction between straights/grav.-accelerated trajectories, and *all other* force-induced accelerated trajectories.

straight trajectories;
no forces present

$$\left(\frac{d^2x}{dt^2} + \nabla\Phi\right) = 0$$

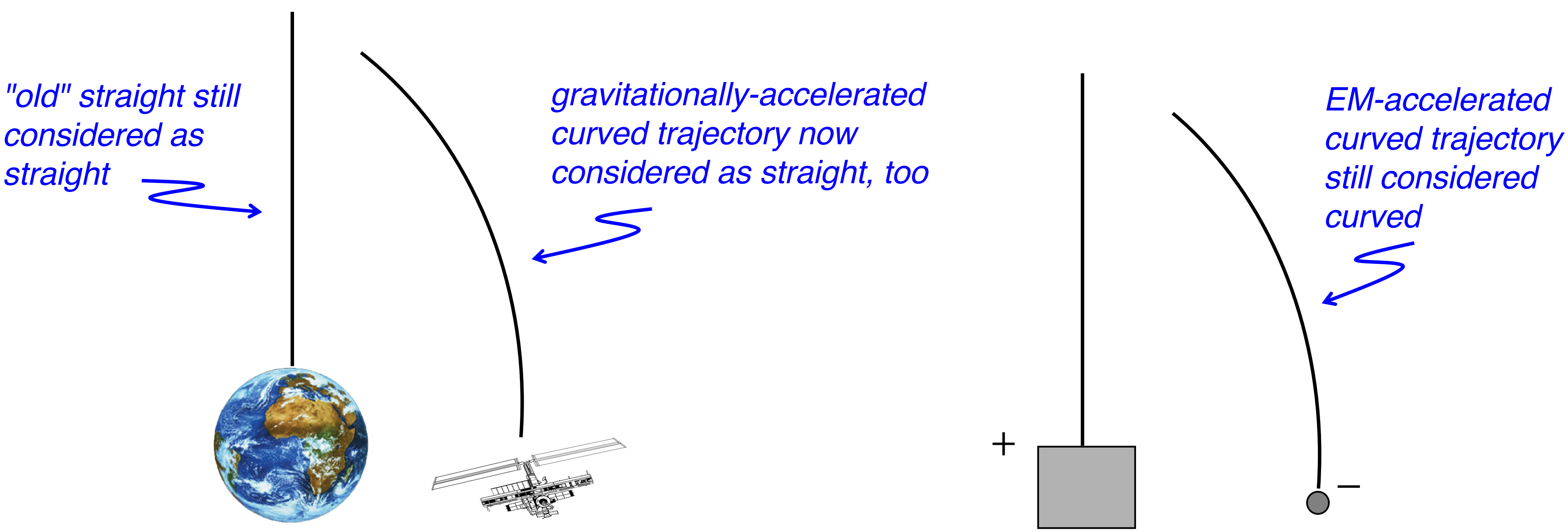
$$ma = 0$$

vs.

curved trajectories;
forces present

$$\left(\frac{d^2x}{dt^2} + \nabla\Phi\right) \neq 0$$

$$ma = F$$



Consequences of Geometrizing Gravity

1. Inertial reference frames (defined by the families of straight trajectories in spacetime) now include objects at rest, in constant motion, *or* gravitationally accelerating.

2. *Gravitationally-induced* acceleration is thus relative (in exactly the same way that position and velocity are relative).
   - *Whether or not you are gravitationally accelerating depends on your frame of reference.*

3. All *other* types of acceleration are still absolute.
   - *Whether or not you are non-gravitationally-accelerating is independent of your frame of reference (such accelerations always come "packaged" with attendant forces).*

Interpretions of the geometrization of gravity

(a) *Substantivalist interpretation*: The gravitational field is no longer a physical field that exists *in* spacetime; rather it is now part of the curvature of spacetime itself.

- *We've demoted the status of the gravitational field from physics to geometry.*

(b) *Relationalist interpretation*: The metric field is physically real and just is what was previously called the gravitational field.

- *We've promoted the status of the metric field from geometry to physics.*

- Both interpretations *agree* that the *structure* of spacetime is no longer flat, as in Special Relativity and Newtonian dynamics.
- They *disagree* over how spacetime structure manifests itself.
  - Substantivalist: it's the structure of a real spacetime.
  - Relationalist: it's the structure of a real physical field (the metric field).

3. The Principle of Equivalence

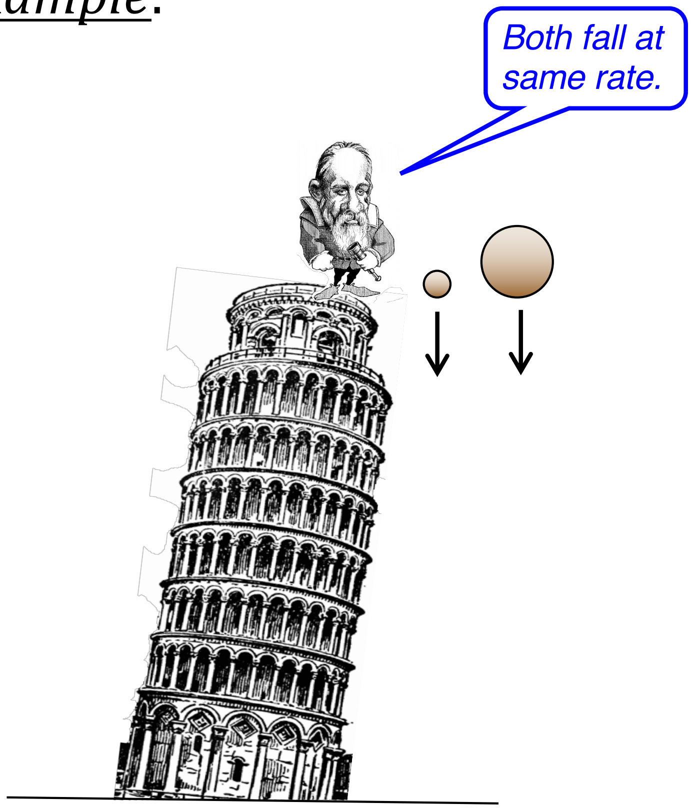
- The geometrization procedure assumes $m_g = m_i$.
  - *All empirical evidence supports this assumption.*
  - *Einstein elevates it to a principle:*

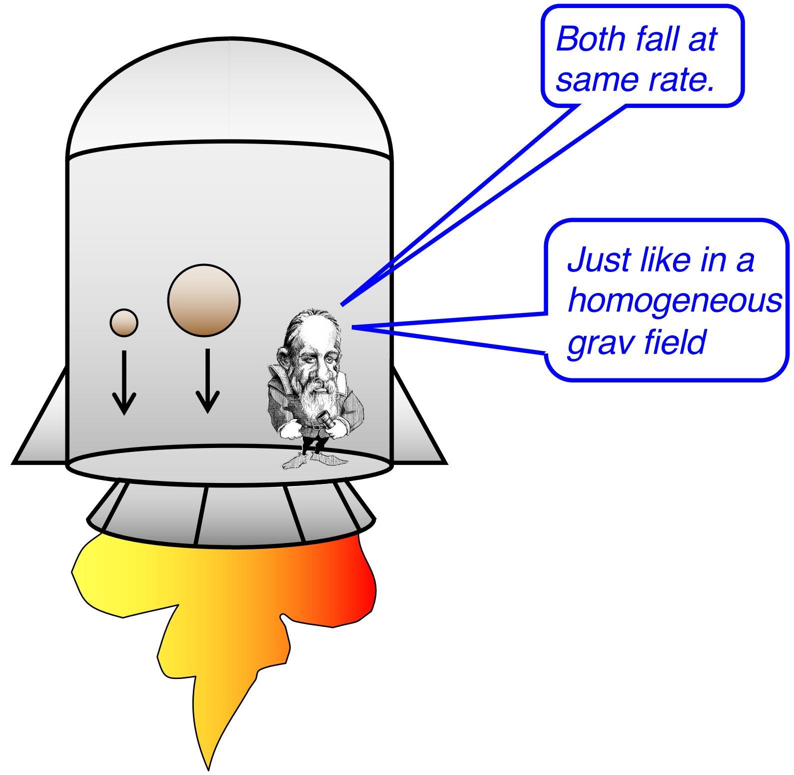
> *Principle of Equivalence (Version 1.0)*
> The inertial mass and gravitational mass of any object are equal.

or equivalently:

> *Principle of Equivalence (Version 2.0)*
> The effects due to a (homogeneous) gravitational field are indistinguishable from the effects due to uniform acceleration.

Example:





Homogeneous grav field (on surface of Earth). $\Leftrightarrow$ Rocket accelerating uniformly at 9.8 m/s^2.

- The Principle of Equivalence says these are indistinguishable reference frames:
  - *Any experiments (involving Newton's Laws of Motion, Maxwell's Laws, or Newton's Law of Gravity) cannot distinguish one from the other.*

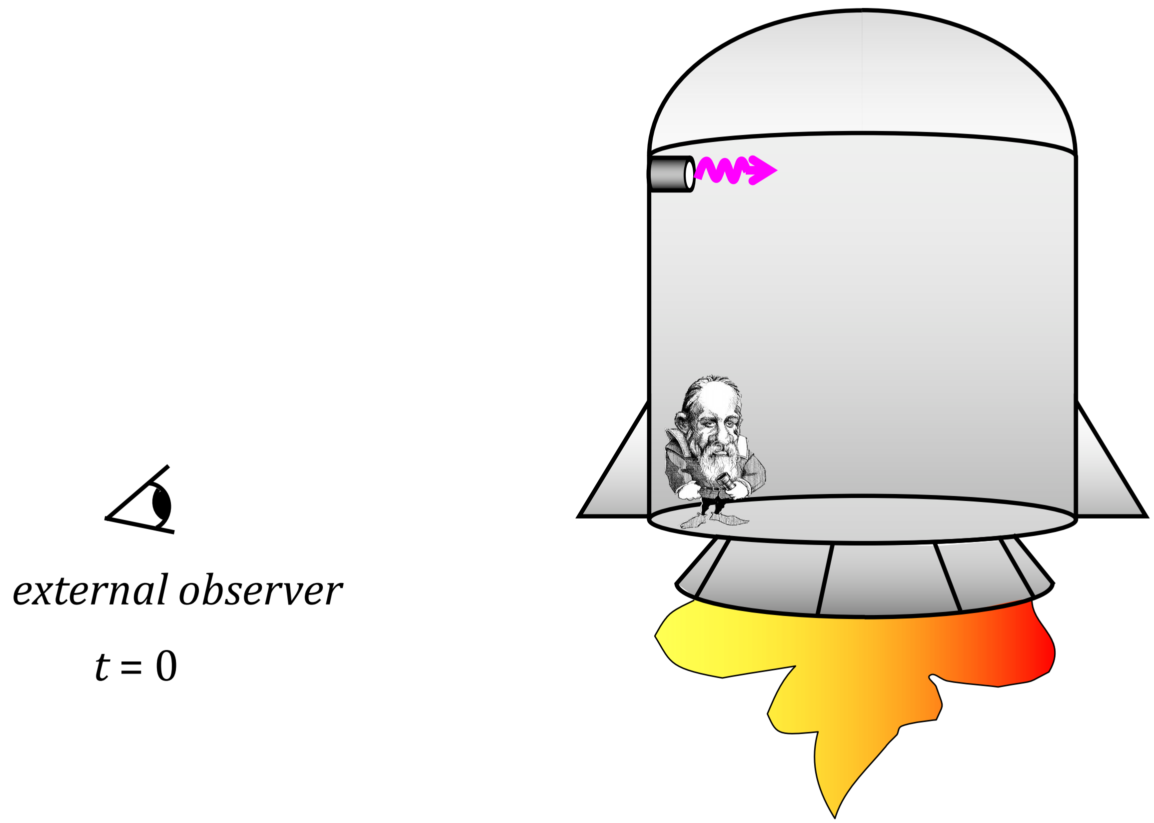
How to make predictions using the Principle of Equivalence (PE)

(1) Observe phenomena in uniformly accelerating frames.

(2) Apply the PE to predict that the same phenomena will be observed in homogeneous gravitational fields.

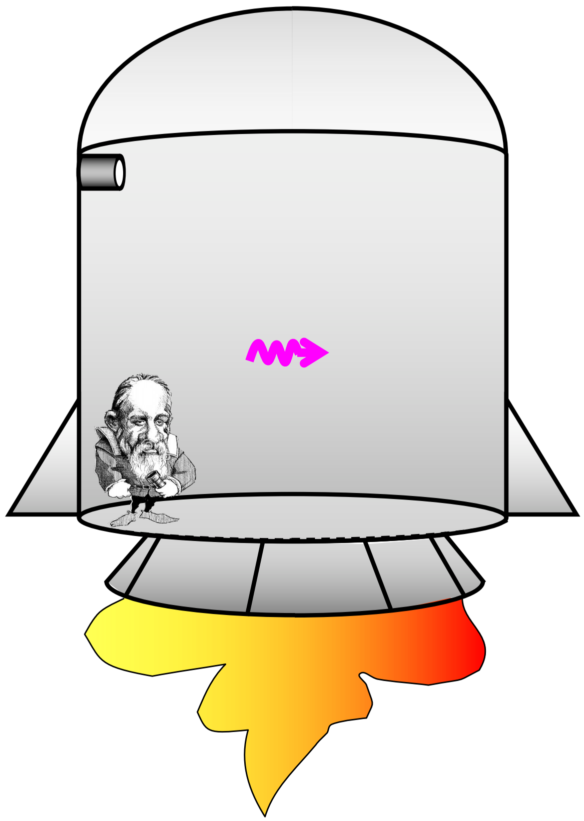
Example 1

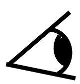
Observations of light in uniformly accelerating frame:



Example 1

Observations of light in uniformly accelerating frame:





external observer

$t = 1$

Example 1

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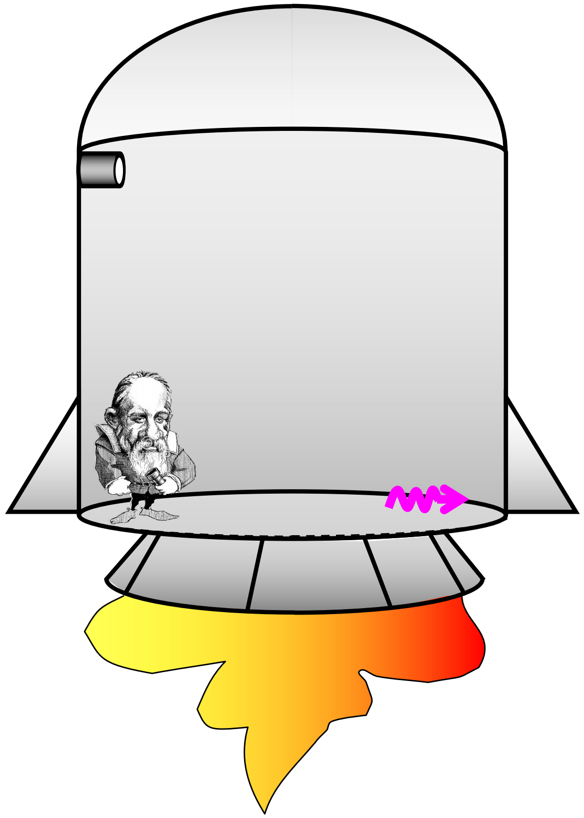

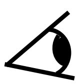

external observer

$t = 2$

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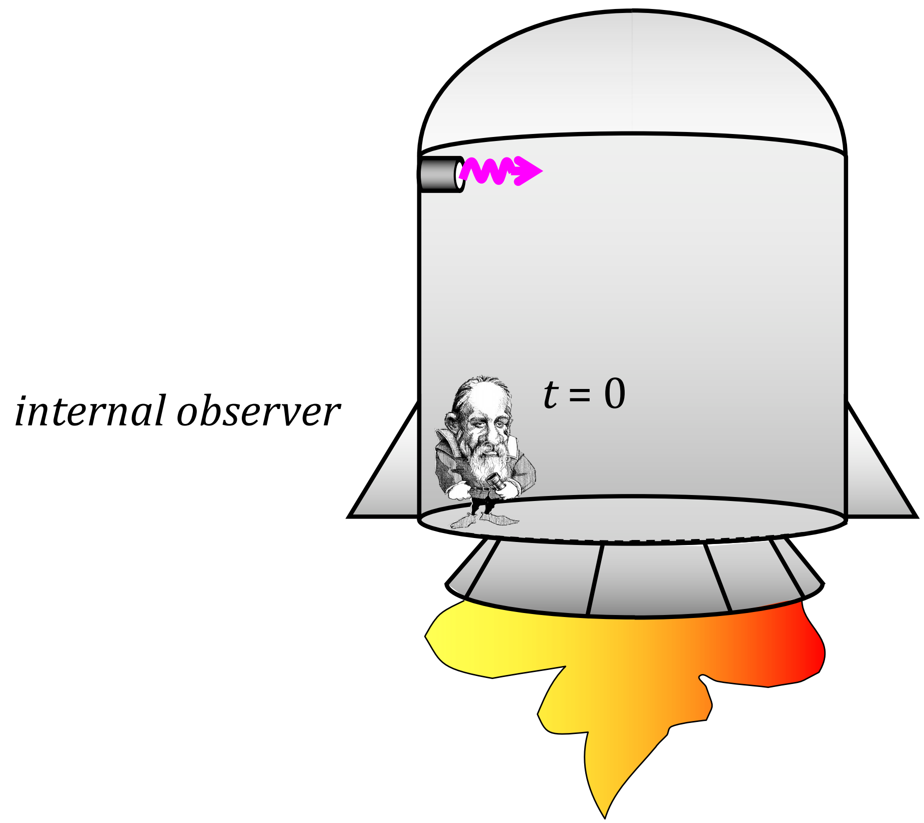
Observations of light in uniformly accelerating frame:



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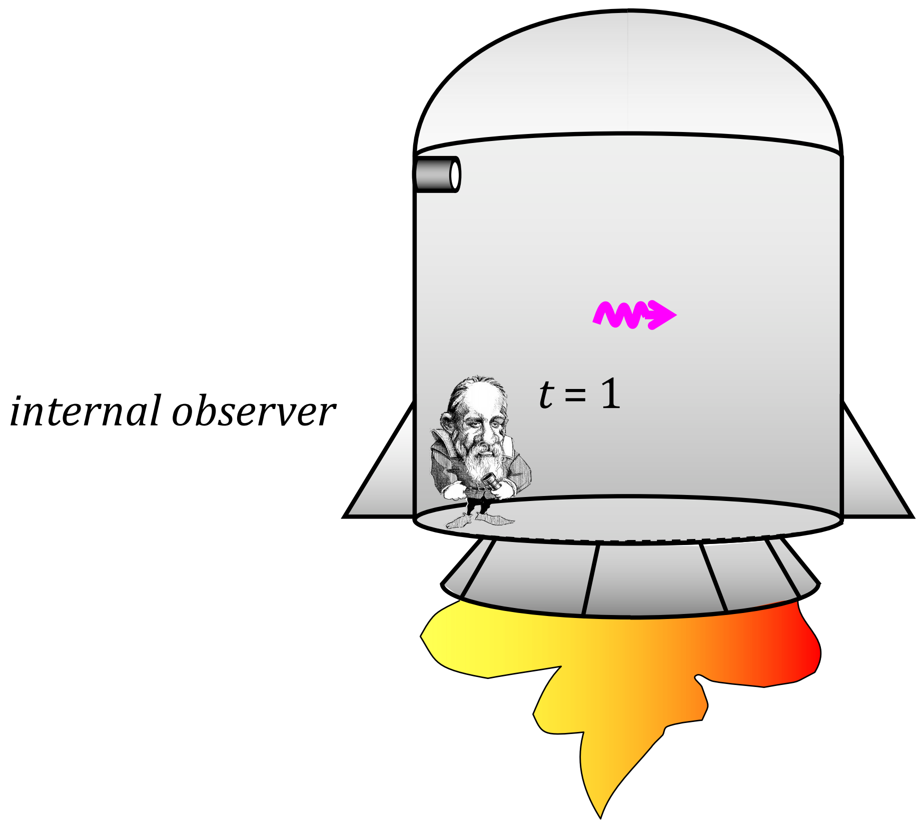

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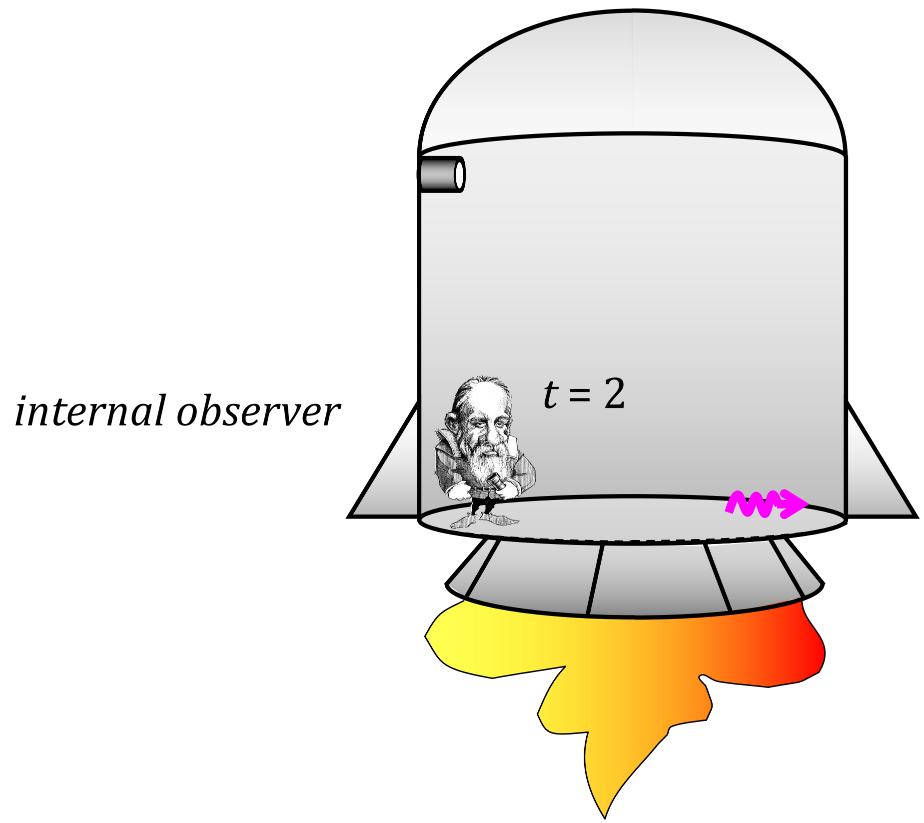
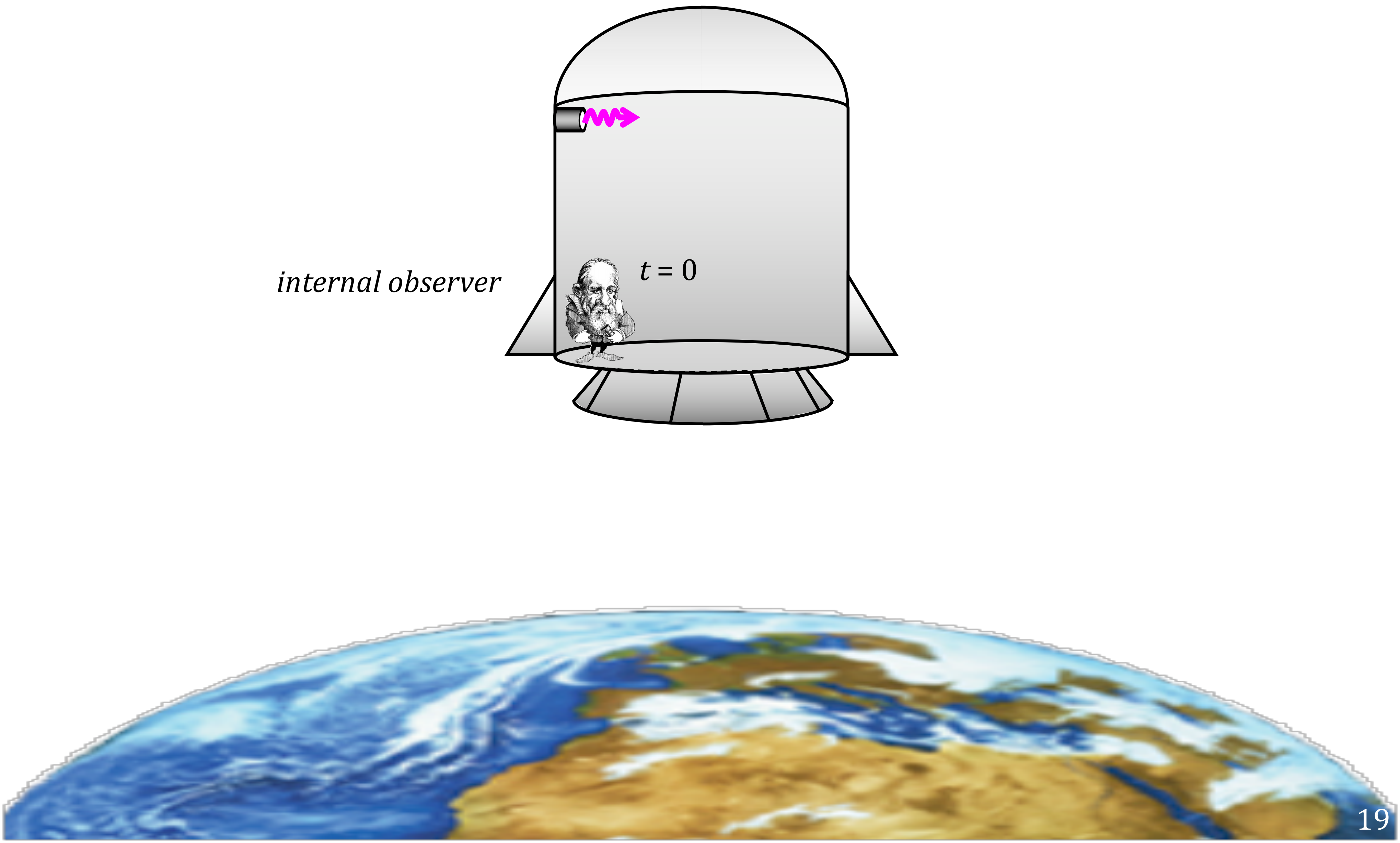


Example 1

Indistinguishable frame in presence of homogeneous gravitaitonal field:



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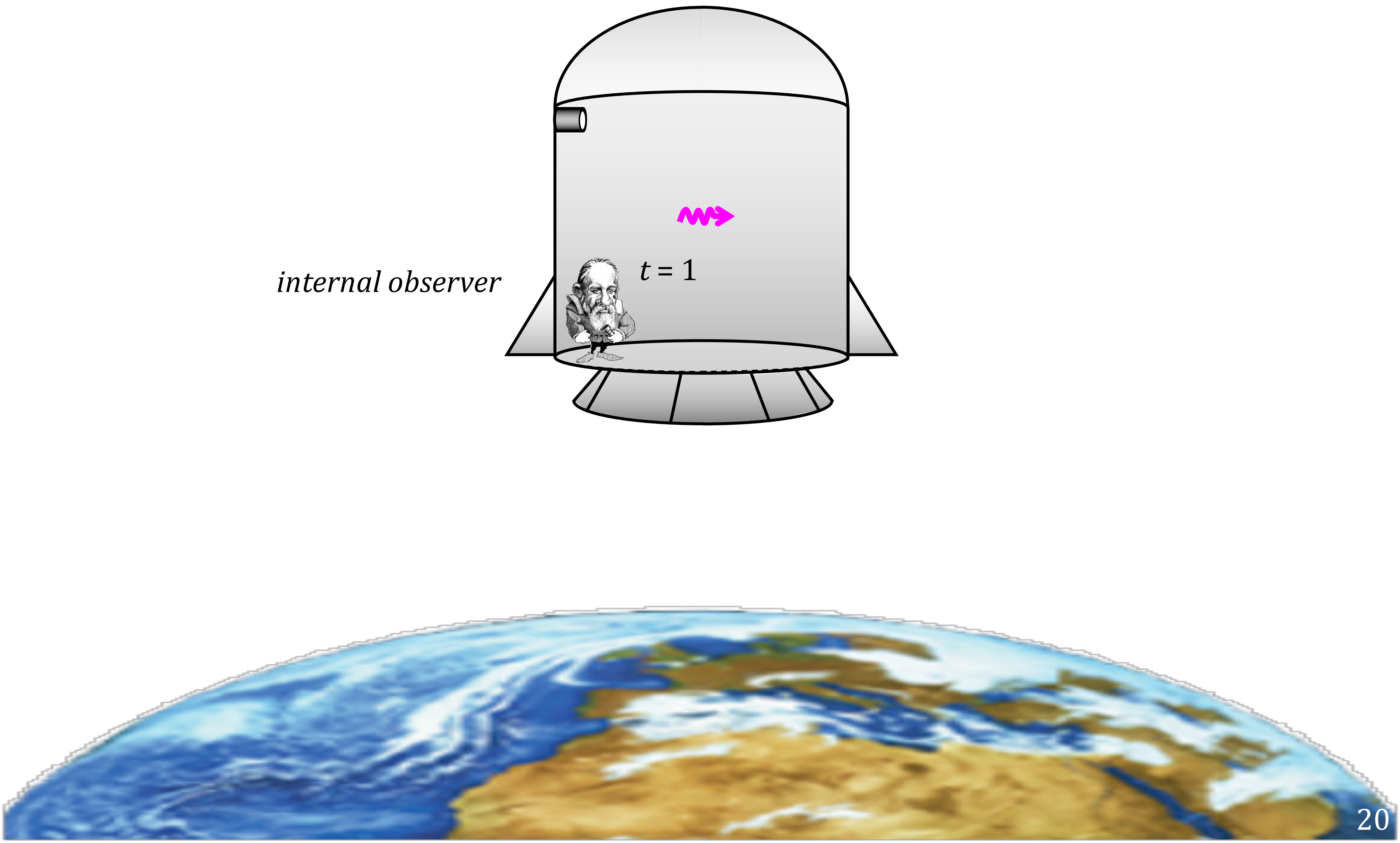

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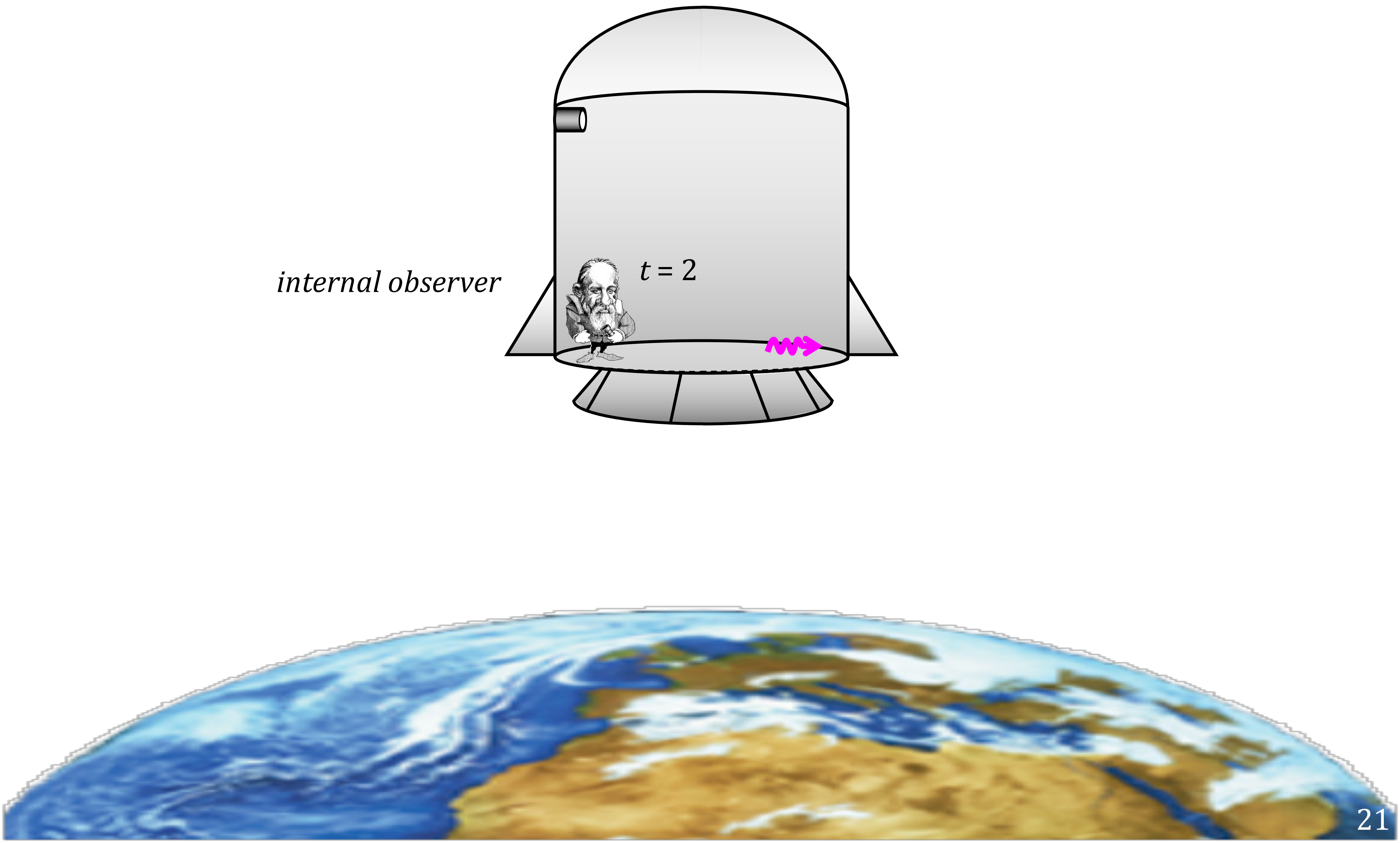
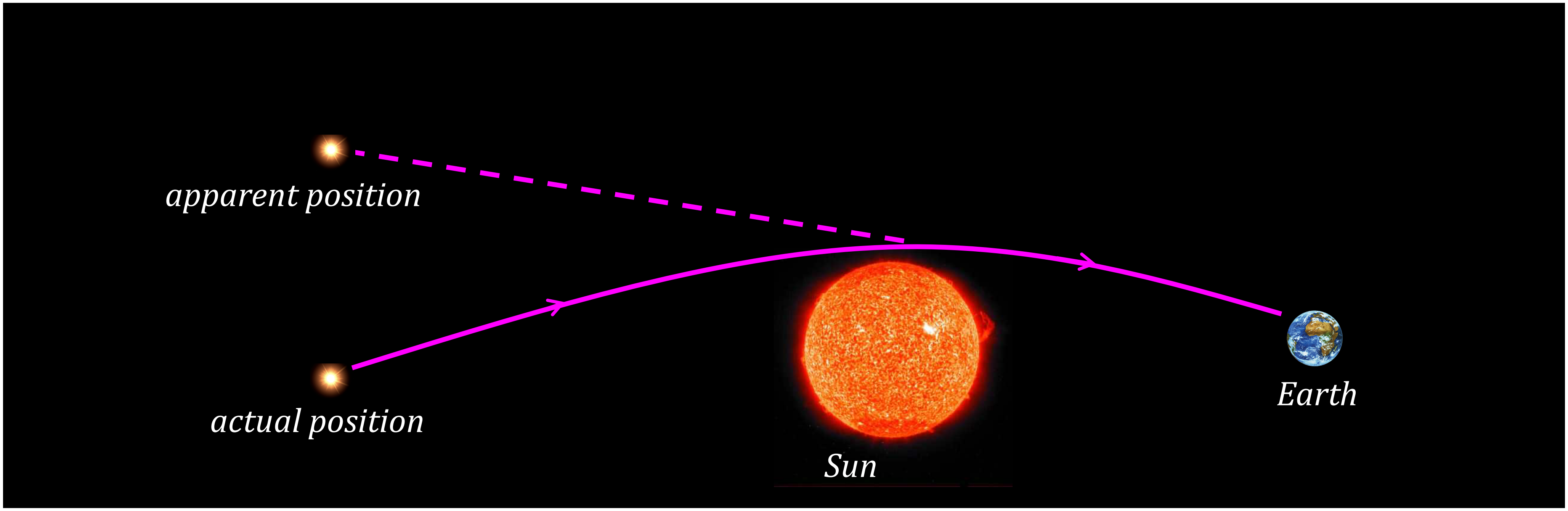


Example 1

Prediction: Light falls (is bent) in a gravitational field.

Experimental Evidence: Light rays bend in the vicinity of the sun.



- Photograph star field at different times of year and see which stars are shifted. Expect deflection of 1.75 *sec of arc.*
- To correct for sun's glare, take photos during solar eclipse.
- 1919: Eclipse Expedition led by Sir Arthur Eddington to S. America and S. Africa.
- Reported as success. Einstein becomes famous (cover of Life Magazine)

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

Special Cable to THE NEW YORK TIMES.

LONDON, Nov. 9.—Efforts made to put in words intelligible to the non-scientific public the Einstein theory of light proved by the eclipse expedition so far have not been very successful. The new theory was discussed at a recent meeting of the Royal Society and Royal Astronomical Society, Sir Joseph Thomson, President of the Royal Society, declares it is not possible to put Einstein's theory into really intelligible words, yet at the same time Thomson adds:

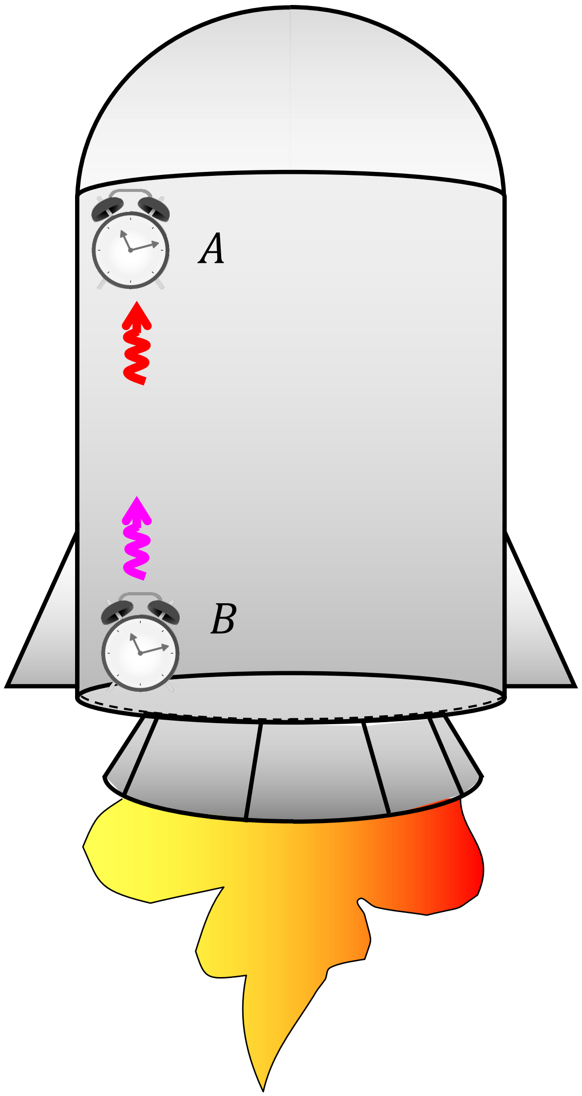
"The results of the eclipse expedition demonstrating that the rays of light from the stars are bent or deflected from their normal course by other aerial bodies acting upon them and consequently the inference that light has weight form a most important contribution to the laws of gravity given us since Newton laid down his principles."

Thompson states that the difference between theories of Newton and those of Einstein are infinitesimal in a popular sense, and as they are purely mathematical and can only be expressed in strictly scientific terms it is useless to endeavor to detail them for the man in the street.

"What is easily understandable," he continued, "is that Einstein predicted the deflection of the starlight when it passed the sun, and the recent eclipse has provided a demonstration of the correctness of the prediction.

Example 2

Observations of clocks in uniformly accelerating frame:



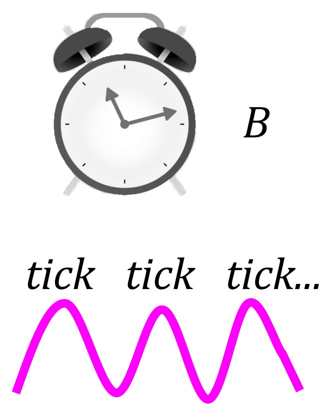
Claim: Clock *B* ticks slower than Clock *A*.

Why? To compare clocks, send light signals:

(i) Correlate *frequency* of a light signal with Clock *B*.

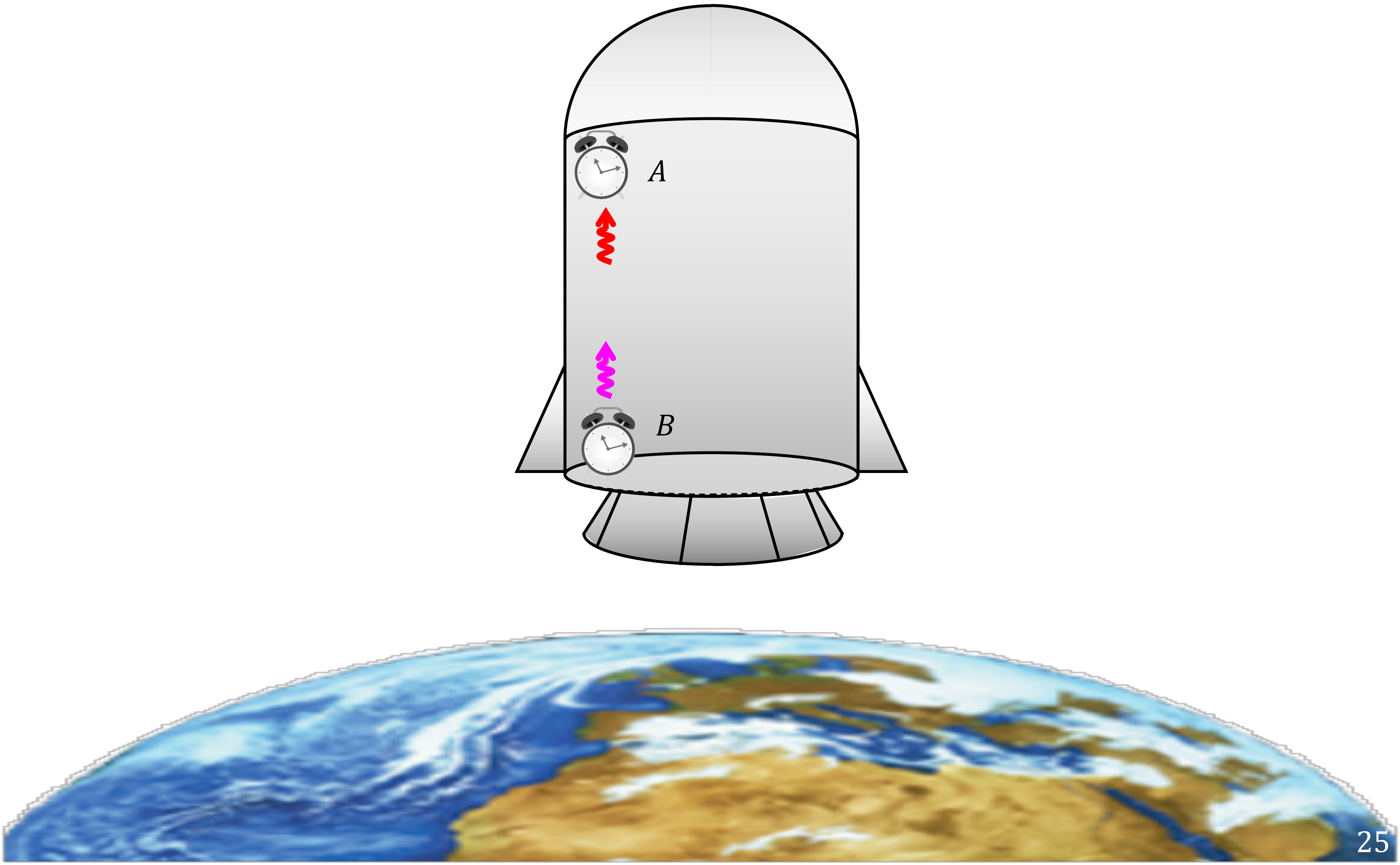
(ii) Send correlated light signal from *B* to *A*.

(iii) Since *A* is accelerating away from light signal, it will receive signal at *lower* frequency (*i.e.*, *shifted to the red*); hence *A* will measure *B* as ticking slower.



Example 2

Indistinguishable frame in presence of homogeneous gravitational field:



Example 2

Prediction: Gravity slows clocks (gravitational "red-shift").

Experimental Evidence: 1956 Pound-Rebka experiment in tower at Jefferson Lab on Harvard campus.

4. The Conventionality of Geometry in General Relativity

Is it a matter of convention whether or not to geometrize the gravitational force?

(A) *Flat geometry & grav. force*

$$\frac{d^2x}{dt^2}m_i = -m_g\nabla\Phi$$

(B) *Curved geometry & no grav. force*

$$\left(\frac{d^2x}{dt^2}m_i + m_g\nabla\Phi\right) = 0$$

- Can the grav force be thought of as an undetectable deformation force?
  - *Present in the "simple" flat geometry, but absent in the complicated curved geometry?*

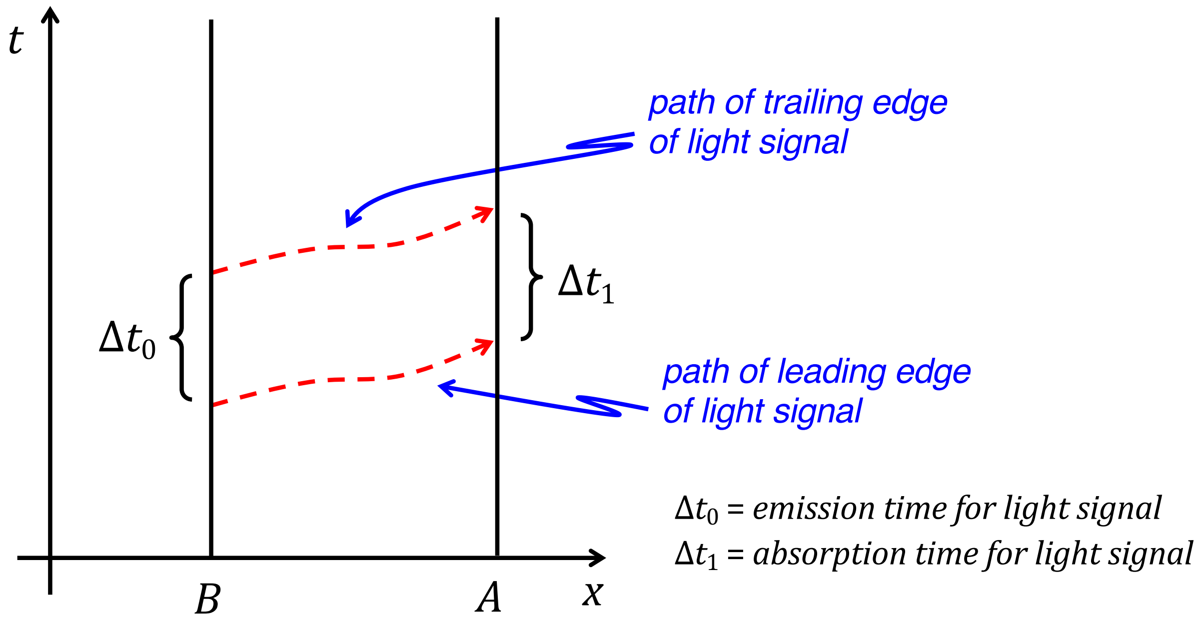
Assumption: There is a *unique* split between *inertial structure* and *gravity* in general relativity.

- *Which means: The contents of the parenthesis in (B) can always be written uniquely as two distinct terms.*

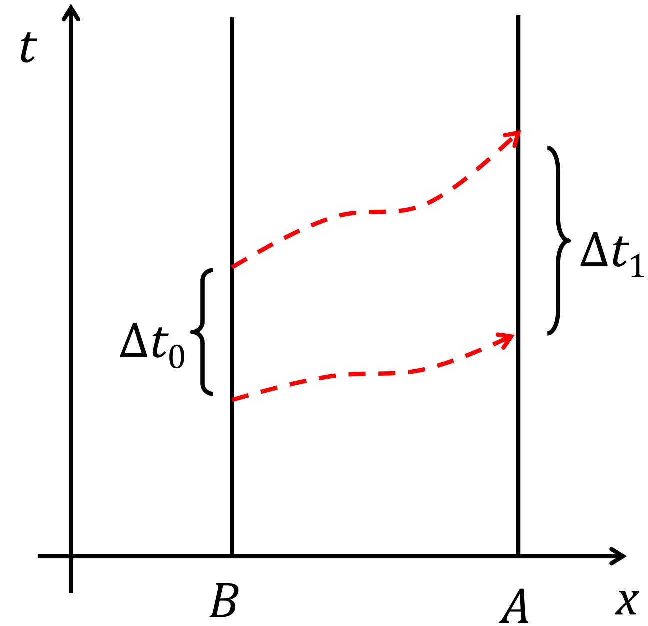
- *Then*: Since all observations indicate $m_i = m_g$, there would be no observational difference between (A) and (B).

- *Realist Response*: The curved geometry description is, arguably, much simplier.

Two ways to explain the gravitational red-shifting of clocks:



Flat Geometry



Curved Geometry

Experimental Result: $\Delta t_1 > \Delta t_0$.

- If spacetime is flat, we should have $\Delta t_1 = \Delta t_0$.

The experimental result must be explained by claiming that gravity affects clocks in a way different from its effects on other objects.

- If spacetime is curved, then the experimental result is explained by the fact that the paths taken by leading and trailing edges of light signal are not "parallel".

- The experimental result can be explained without reference to a force acting on clocks in a way different from how it acts on other things.

- We can say that gravity, as the curvature of spacetime, affects all objects in the same way.

<u>BUT</u>: There isn't a unique split between inertial structure and gravity in GR.

- Under a standard condition (that the connection be symmetric), the term schematically represented by $d^2x/dt^2 + \nabla\Phi$ cannot be split into an inertial part d^2x/dt^2 and a gravitational part $\nabla\Phi$.
- *<u>So</u>*: Under this standard condition, geometry is *not* conventional in GR!

<u>Suppose we relax this standard condition</u>

- We get a theory ("teleparallel gravity") that looks like GR and in which a "split" between (what looks like) inertial structure and (what looks like) gravitation can be achieved.
- *<u>But</u>*: The verdict is still out on whether this is an equivalent way of formulating GR, or whether it counts as an entirely different theory!