06. Dynamics--Energy

Topics:
1. Energy
2. Nuclear Physics
3. Mass/Energy Equivalence

1. Energy

- Energy is the ability to do work:

$$work = (force) \times (distance\ moved)$$

- For a constant force F, a small change in work dW is related to a small change in distance dx by

$$dW = Fdx$$

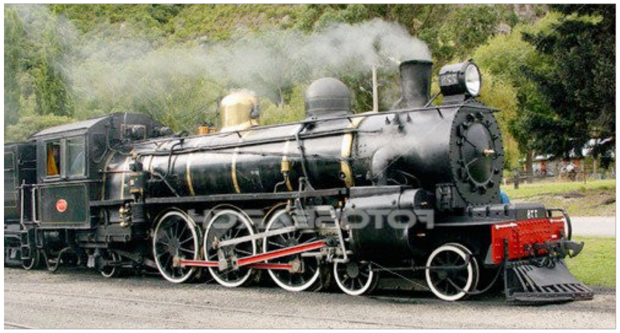
- Many forms of energy, all interconvertible:

(a) *Chemical energy*

(b) *Heat energy*

(c) *Kinetic energy*

(d) *Many others: electrical, potential, pressure, etc...*



Principle of Conservation of Energy
The total energy of a closed system is conserved (doesn't change over time).

Kinetic Energy K

- Energy due to the motion of an object.
- Work done by a force in accelerating the object from rest to some velocity.

Newtonian Kinetic Energy K

$K = \frac{1}{2}mv^2$

- *Recall*: $F = \frac{dp}{dt}$ and $p = mv$.
- *So*: A small change in kinetic energy dK is related to a small change in distance dx by $dK = \frac{dp}{dt}dx = \frac{dx}{dt}dp = vd(mv)$
- *Now integrate*: $\int dK = \int vd(mv)$
- *This yields*: $K = \frac{1}{2}mv^2$

Kinetic Energy K

- Energy due to the motion of an object.
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Relativistic Kinetic Energy K_{SR}

$K_{SR} = mc^2(\gamma - 1)$

- *Recall*: $F = \dfrac{dp_{SR}}{dt}$ and $p_{SR} = m\gamma v$, where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
- *So*: A small change in relativistic kinetic energy dK_{SR} is related to a small change in distance dx by $dK_{SR} = \dfrac{dp_{SR}}{dt}dx = \dfrac{dx}{dt}dp_{SR} = vd(m\gamma v)$
- *Now integrate*: $\int dK_{SR} = \int vd(m\gamma v)$
- *This yields*: $K_{SR} = mc^2(\gamma - 1)$.

Kinetic Energy K

- Energy due to the motion of an object.
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Newtonian Kinetic Energy K

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Relativistic Kinetic Energy K_{SR}

$K_{SR} = mc^2(\gamma - 1)$

- *Recall*: For $v \ll c$, we can approximate γ by $1 + \frac{v^2}{2c^2}$
- *So*: When $v \ll c$, $K_{SR} \approx mc^2\left\{\left(1 + \frac{v^2}{2c^2}\right) - 1\right\} = \frac{1}{2}mv^2$
- *Thus*: Relativistic kinetic energy reduces to Newtonian kinetic energy when $v \ll c$ (the "Newtonian limit").

Relativistic Energy

- *Relativistic kinetic energy*: $K_{SR} = mc^2(\gamma - 1)$, or:

$$K_{SR} = mc^2\gamma - mc^2$$

$$(energy\ of\ motion) = (part\ depending\ on\ v) - (part\ independent\ of\ v)$$

- *Rewrite as*:

$$mc^2\gamma = K_{SR} + mc^2$$

$$(total\ energy) = (energy\ of\ motion) + (\text{"}rest\text{"}\ energy)$$

$$E = K_{SR} + E_0$$

- *Total Energy*: $E = mc^2\gamma$
  - *In units in which $c = 1$, this says "Relativistic mass ($m\gamma$) = total energy".*

- *Rest Energy*: $E_0 = mc^2$.
  - *In units in which $c = 1$, this says "Rest mass (m) = rest energy".*

- *In general*: $E = Mc^2$, where M is either relativistic mass or rest mass!

Relativistic Energy

$$E = Mc^2$$

- An increase ΔE in energy entails an increase in mass by $\Delta M = \Delta E/c^2$.
  - *Very small increase in mass, since c is so large!*

(*Charge battery*) $\Rightarrow$ (*increase in chem energy*) $\Rightarrow$ (*very small increase in mass*)

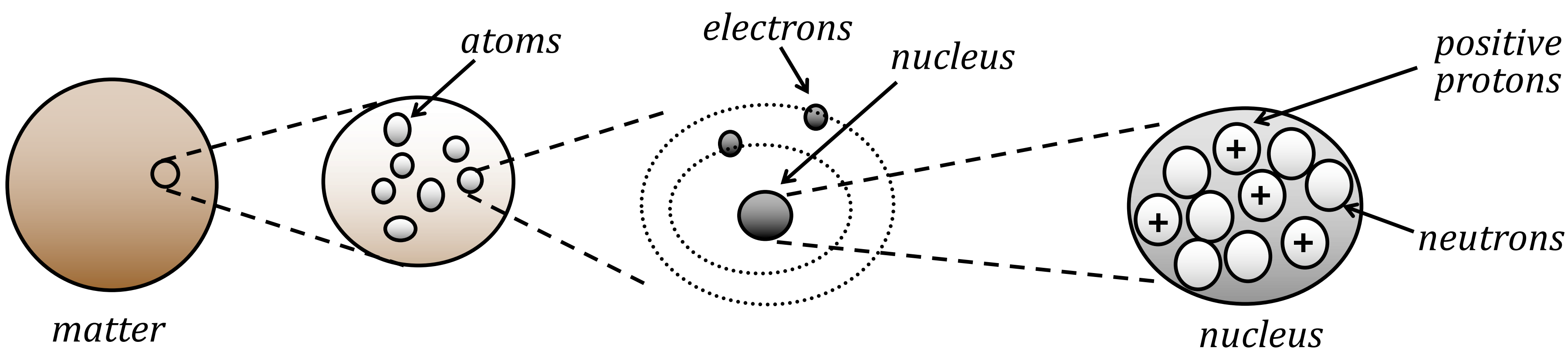
(*Heat water on stove*) $\Rightarrow$ (*increase in heat energy*) $\Rightarrow$ (*very small increase in mass*)

- *What about reverse process*:

  An increase ΔM in mass entails an increase in energy by $\Delta E = \Delta Mc^2$.
  - *Very large increase in energy, since c is so large!*

2. Nuclear Physics

- Theoretical result in 1905: $E = Mc^2$.
  - Experimental confirmation--1930's nuclear chemists.



- The # of protons and neutrons fixes the chemical character of the element.

<u>Ex</u>: Carbon has 6 protons and comes in two versions:

One with 6 neutrons (regular Carbon): $^{12}_{6}C$

One with 7 neutrons (Carbon-13): $^{13}_{6}C$

- Nuclear reactions convert elements into each other (Alchemists' dream):

<u>Ex</u>: $^{14}_{7}N + ^{4}_{2}He \longrightarrow ^{17}_{8}O + ^{1}_{1}H$

Binding energy (BE) = energy needed to bind nucleus together.

- Special relativity entails that binding energy has mass!

$$(\textit{mass of nucleus}) = (\textit{mass of protons}) + (\textit{mass of neutrons}) + (\textit{mass of BE})$$

- *Experimental result*: *BE* of medium-sized elements is greater than *BE* of large or small elements.
  - *Medium-sized elements are more tightly bound.*

"Large" elements

Uranium (^{235}U), Gold (^{197}Au), *etc*...

"Medium" elements

Iron (^{56}Fe), *etc*...

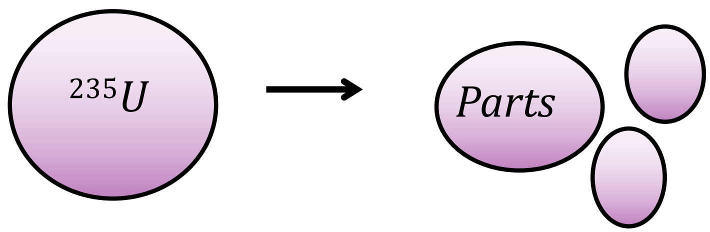
"Small" elements:

Hydrogen (^{1}H), Helium (^{2}He), *etc*...

Two consequences

(1) When a large element breaks apart, the *pieces* are more tightly bound.

- *Thus there's an increase in rest mass!*



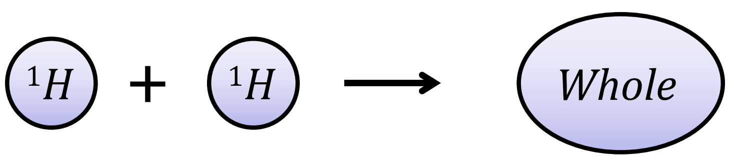
smaller BE *greater BE*

$(\text{mass of } ^{235}U) = (\text{mass of Parts}) + (\text{mass of BE difference})$

(2) If small elements are fused, the *resultant whole* is more tightly bound.

- *Thus there's an increase in rest mass!*

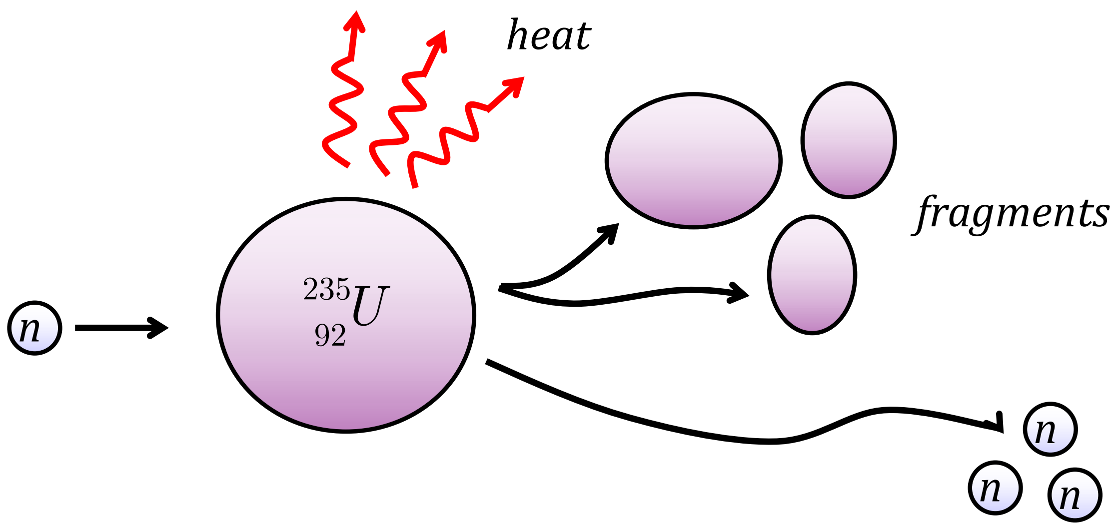
Big energy release!



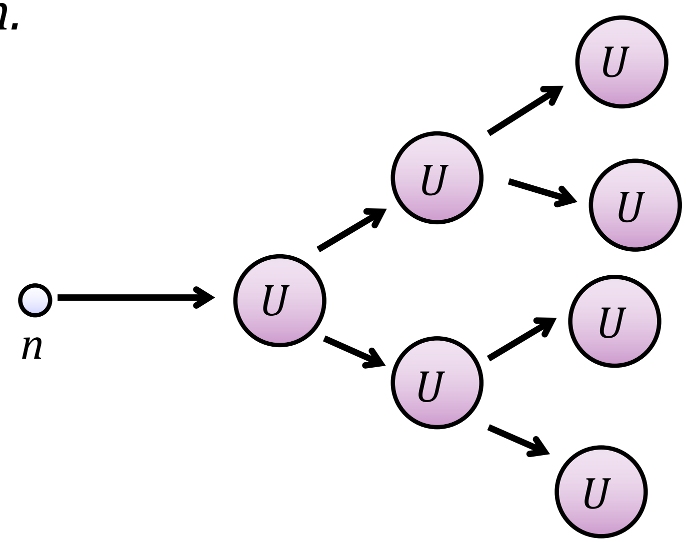
smaller BE *greater BE*

$(\text{mass of } ^{1}H) + (\text{mass of } ^{1}H) = (\text{mass of Whole}) + (\text{mass of BE difference})$

Process (1): Fission



- Natural radioactive decay: Energy released as heat.
- To get larger amounts of energy released, set up a "chain reaction":
  - *Trigger initial reaction with free neutron.*
  - *Additional products: 2-3 more neutrons!*
  - *Use these to trigger more reactions.*

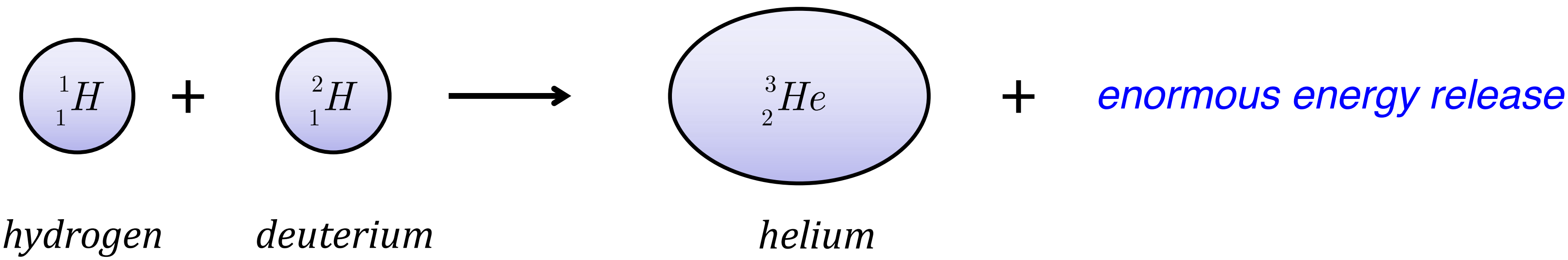


Two methods to prevent neutrons from escaping

(a) *Fission Bomb*: Put "critical mass" in one spot.

(b) *Fission Reactor*: Slow down neutrons with moderator (Cadmium) so they trigger more reactions.

Process (2): Fusion



- Very difficult to initiate: Requires enormous energy input!

<u>*Three methods for producing fusion reactions*</u>

(a) Fusion Bomb

- *Use fission bomb as trigger.*

(b) Fusion Reactor

- *No reliable cost-effective process: Experimental reactors use more energy than they produce.*

(i) Inertial Confinement

- *Use (giant) laser to blast a droplet of water from all sides.*

(ii) Magnetic Bottle

- *Use magnets to squeeze a plasma.*

(c) Stars

- *Intense gravitational pressure initiates and sustains fusion reactions.*

3. Mass/Energy Equivalence

- *Recall*:

$m\gamma$ = relativistic mass (varies with velocity)

m = rest mass (constant)

$E = m\gamma c^2$ = total energy (varies with velocity)

$E_0 = mc^2$ = rest energy (constant)

What does it mean to say mass and energy are "equivalent"?

Option (1): They are the same thing.

Option (2): They are different things but can be converted into each other.

Option (1a). *Same-Property Interpretation*

Claim: In Newtonian physics, mass and energy are *different properties*. In Special Relativity, mass and energy are different names for the *same* property.

- *Objection 1*. We use different units for mass (kg) and energy ($kg \cdot m^2/s^2$).
  - *Doesn't this mean they're different properties?*
  - *No! Can choose units in which $c = 1$.*
  - *In these units, $E = m\gamma$, $E_0 = m$, and mass and energy are measured in the same units (kg, say).*

- *Objection 2*. When $c = 1$, distance and time are also measured in the same units. Should we thus allow that distance and time are also equivalent and are different names for the same property?

Possible Counter-Claim:
- Distance and time intervals are *different* inertial-frame-dependent *aspects* of an invariant property, *spatio-temporal length*.
- Similarly, mass and energy are different inertial-frame-dependent *aspects* of an invariant property, "energy-momentum".

Option (1b). *One-Stuff Interpretation*

Distinctions:

mass = *property* matter = *substance*

energy = *property* field = *substance*

In Newtonian physics:

- Matter has both mass and energy, while fields have energy only.
- Hence matter is distinct from fields: *There are two types of fundamental stuff.*

In Special Relativity:

Weak Claim: Mass/energy equivalence entails we cannot treat matter and fields differently. (*Only one type of stuff*.)

Strong Claim: Energy/fields are fundamental. (*Fields* are the only type of stuff.)

Weak Claim: Mass/energy equivalence entails we cannot treat matter and fields differently. (*Only one type of stuff*.)

Strong Claim: Energy/fields are fundamental. (*Fields* are the only type of stuff.)

- *Objection 1*. *Weak Claim* assumes matter is *essentially* characterized by mass and energy, whereas fields are *essentially* characterized by energy (hence we cannot treat them differently if mass and energy are equivalent).
  - *But why make this assumption?*

- *Objection 2*. If mass and energy are equivalent by being different aspects of the same property (energy-momentum), then the *Strong Claim* owes us an explanation of why the energy aspect of this quantity is more fundamental than the mass aspect.

Possible Counter-Claim:

- There is a type of substance, distinct from matter or fields, that has the property of energy-momentum.
- This substance appears in different aspects (matter or field) in analogy with the way the property of energy-momentum appears in mass and energy aspects.

Option (2). *Conversion Interpretation*

Claim: Mass and energy are different properties, as in Newtonian physics, but Special Relativity indicates that they can be *converted* into each other.

- *Note*: This rejects a "spacetime" interpretation of mass and energy as different aspects of an invariant property *energy-momentum* (in analogy with distance and time as different aspects of an invariant property, *spatiotemporal length*).
- *But*: Perhaps this analogy doesn't work in the case of dynamics. Perhaps the distinction between kinematics and dynamics isn't as clear as we've been assuming...