03. Simultaneity

Topics:

0. Initial Concepts
1. Spacetime Diagrams
2. Composition of Velocities
3. Relativity of Simultaneity
4. Tachyons & Causality
5. Conventionality of Simultaneity

0. Initial Concepts

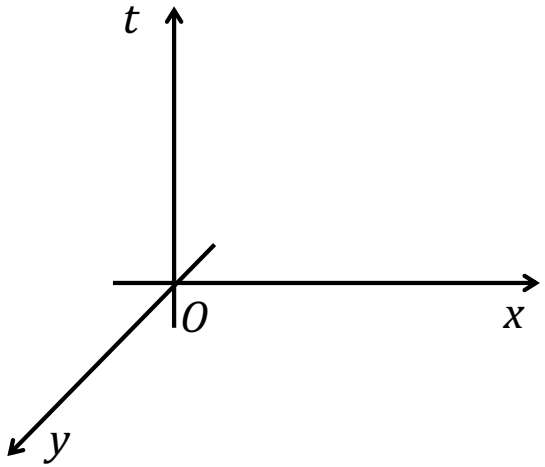
(1) A *spacetime* is a 4-dim collection of points with additional structure.

(2) A *coordinate system* is a way of assigning 3 spatial quantities and 1 temporal quantity to every event in spacetime.

- So every point in spacetime is assigned 4 quantities: (x, y, z, t)

3 spatial *1 temporal*

(3) A *reference frame* is an object *O* that defines the origin of a coordinate system.



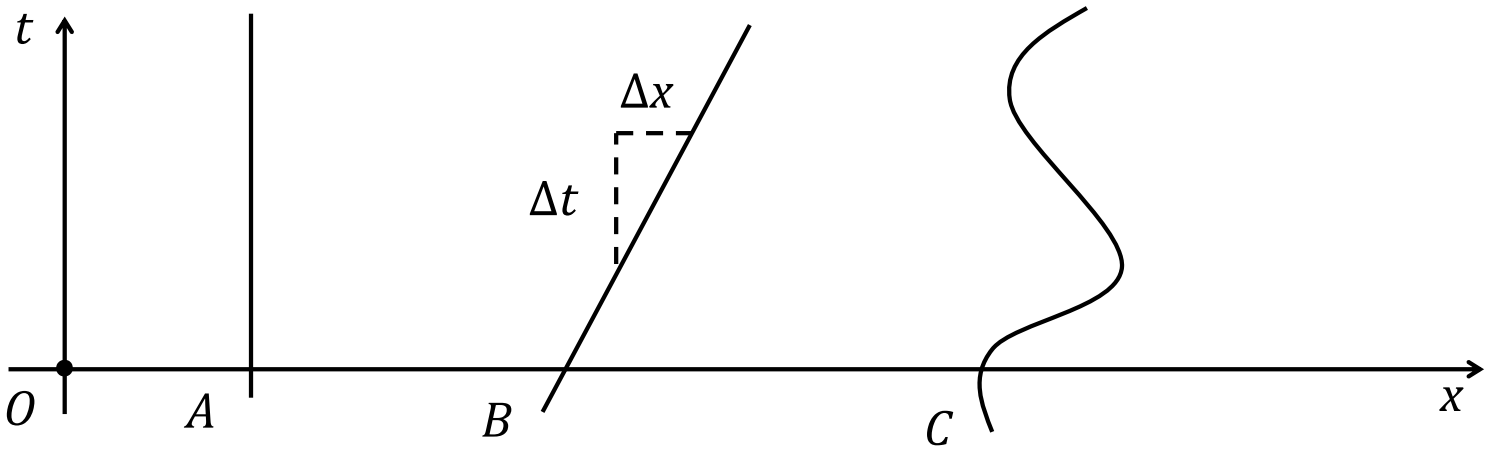
1. Spacetime Diagrams

(a) Pick an origin O in spacetime.

(b) Draw t-coordinate axis and x-axis (supress y- and z-axes for convenience).

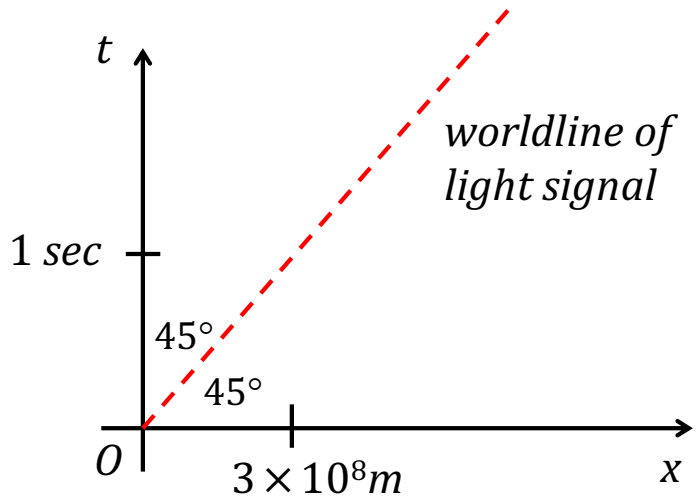
(c) Associate paths with trajectories ("worldlines") of objects in space and time.

(d) Speed v of a world-line with respect to $O = \dfrac{1}{slope} = \dfrac{\text{change in } x}{\text{change in } t}$



- Object A has speed $v_A = 0/t = 0$. So object A is at *rest* with respect to O.
- Object B has *constant speed* $v_B = \Delta x/\Delta t$ with respect to O.
- Object C has non-constant slope, so it is in *accelerated* motion with respect to O.

- *Convention*: Pick units of O-coordinate system so that the worldline of a light signal is inclined 45° with respect to O-coordinate axes.
- Speed of light $c = 3\times10^8 m/s = 186{,}000\, mi/s = 1\, light\text{-}year/year = etc...$



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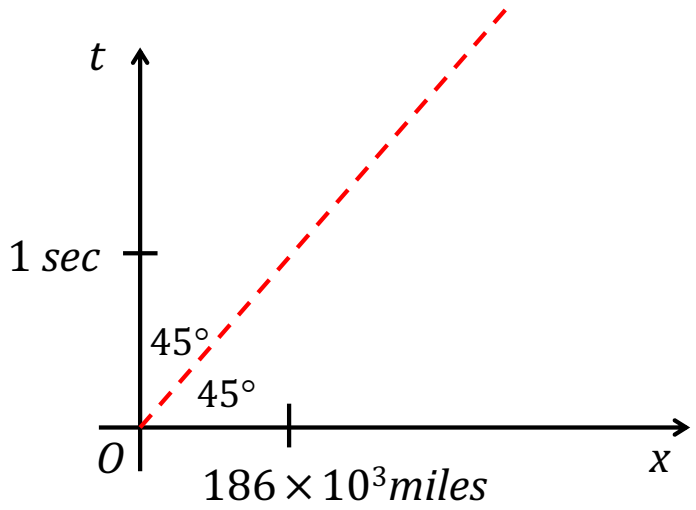

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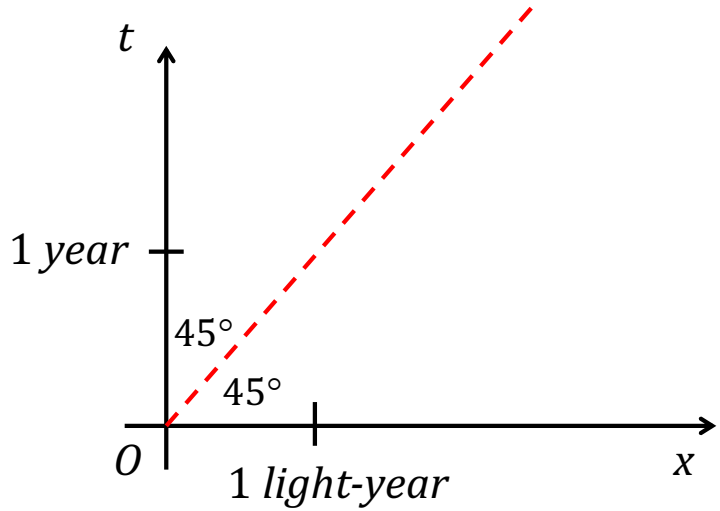
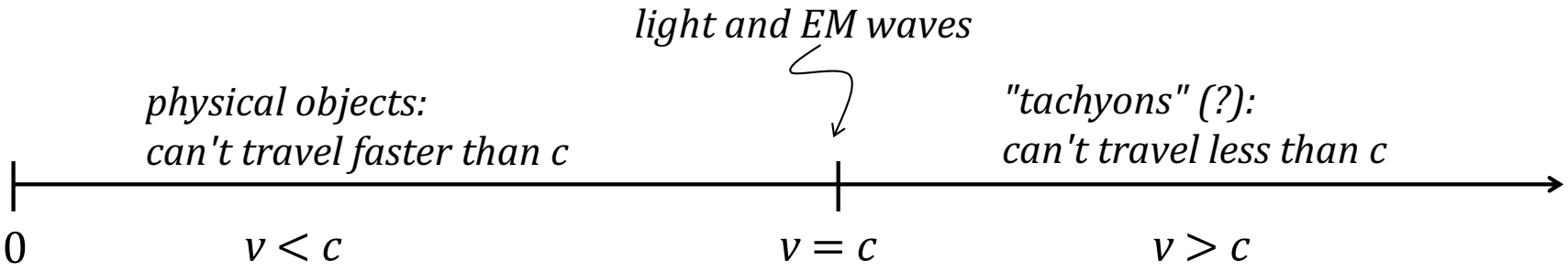


2. Composition of Velocities and Lightcone Structure

- *Principle of Relativity*: The laws of physics are the same in all non-accelerating (inertial) reference frames.
- *Consequence of Principle of Relativity and Light Postulate*: The speed of light is the same in all inertial frames.

General claim: This entails that c acts as a *"speed limit"*.

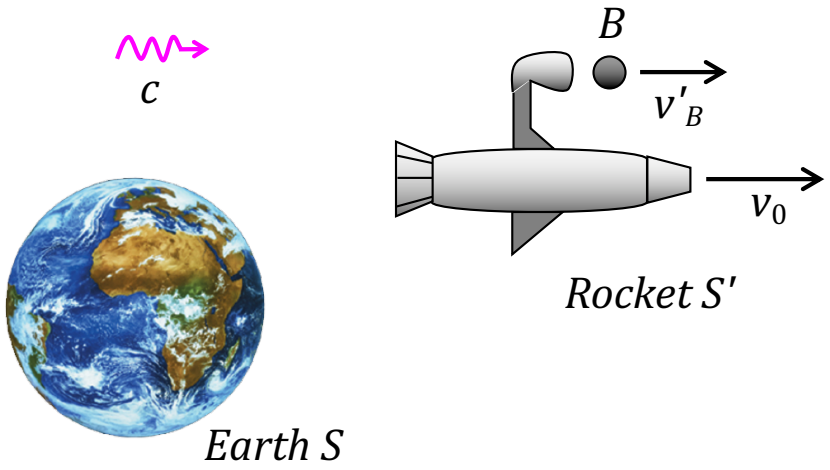


Claim A: Objects traveling $< c$ with respect to a given inertial frame cannot travel $\geq c$ with respect to any other inertial frame.

Claim B: Objects traveling $> c$ with respect to a given inertial frame cannot travel $\leq c$ with respect to any other inertial frame.

Claim C: Objects traveling at c with respect to a given inertial frame cannot travel less than or greater than c with respect to any other inertial frame.

Claim A: Objects traveling $< c$ with respect to a given inertial frame cannot travel $\geq c$ with respect to any other inertial frame.



v'_B = speed of B with respect to S'
v_0 = speed of S' with respect to S
c = speed of light (same in *both* S and S')

- *Suppose*: $v_0 = v'_B = 200{,}000\,km/s$. Recall $c = 300{,}000\,km/s$.
- What is speed v_B of B with respect to S? $\quad v_B \stackrel{?}{=} v_0 + v'_B = 400{,}000\,km/s$
- This would mean that B is traveling faster than c with respect to S; so,
  (a) According to S', light signal overtakes B.
  (b) According to S, B overtakes light signal.

  } *Violation of Principle of Relativity!*

- *So*: An object traveling $< c$ with respect to one inertial frame cannot travel $> c$ with respect to another.
  - *Similar arguments for Claims B and C.*

How should velocities be "composed" in Special Relativity?

- Consider the *inverse* Lorentz transformations:

$$x = \gamma_0(x' + v_0 t')$$

$$t = \gamma_0\left(t' + \frac{v_0 x'}{c^2}\right)$$

- Bullet's S'-coordinates (x', t') satisfy $x' = v't'$.

  Substitute this into the inverse Lorentz transforms:

$$x = \gamma_0(v' + v_0)t'$$

$$t = \gamma_0\left(1 + \frac{v_0 v'}{c^2}\right)t'$$

- Bullet's speed in S-coordinates is $v = x/t$, which is thus:

$$v = \frac{v_0 + v'}{1 + \frac{v_0 v'}{c^2}}$$

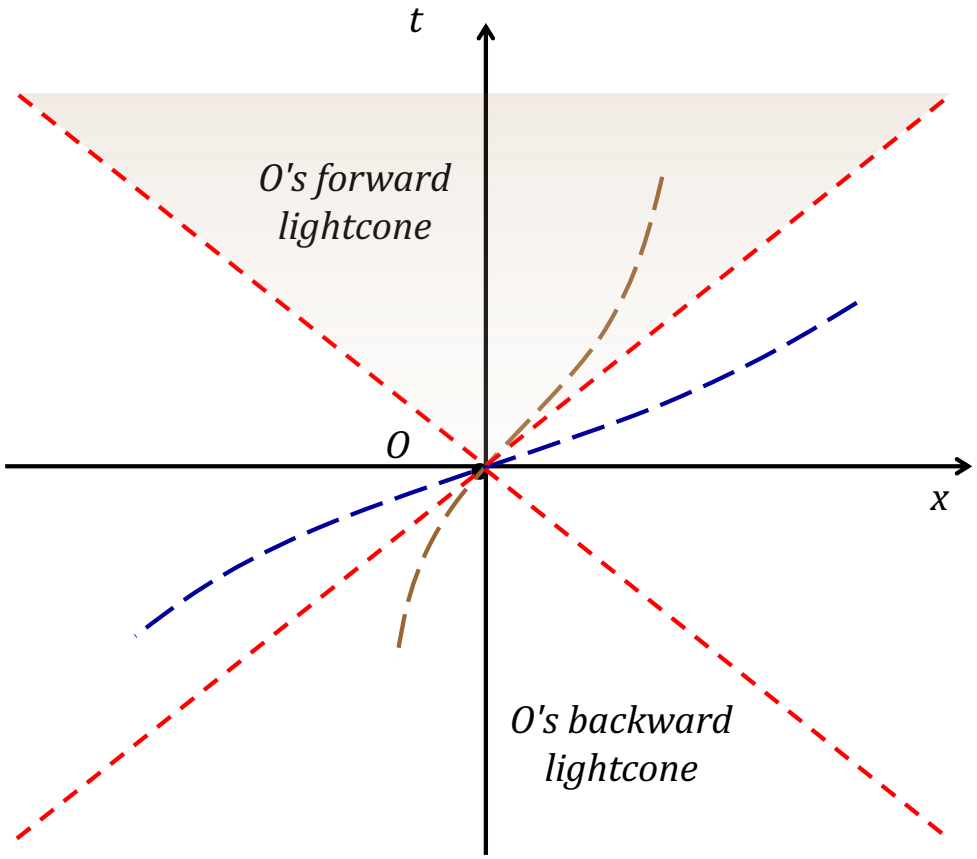
v = speed of object *w.r.t.* stationary frame S

v' = speed of object *w.r.t.* moving frame S'

v_0 = speed of moving frame S' *w.r.t.* stationary frame S

Lightcone Structure

- Let's represent the fact that c is a speed limit in a spacetime diagram:



Timelike worldline
- possible path of an object traveling $< c$

Lightlike worldline
- possible path of an object traveling at c

Spacelike worldline
- possible path of an object traveling $> c$

- Lightcone structure at O splits spacetime into 4 regions:

  (1) Events in O's forward lightcone. (Events in O's future.)

Lightcone Structure

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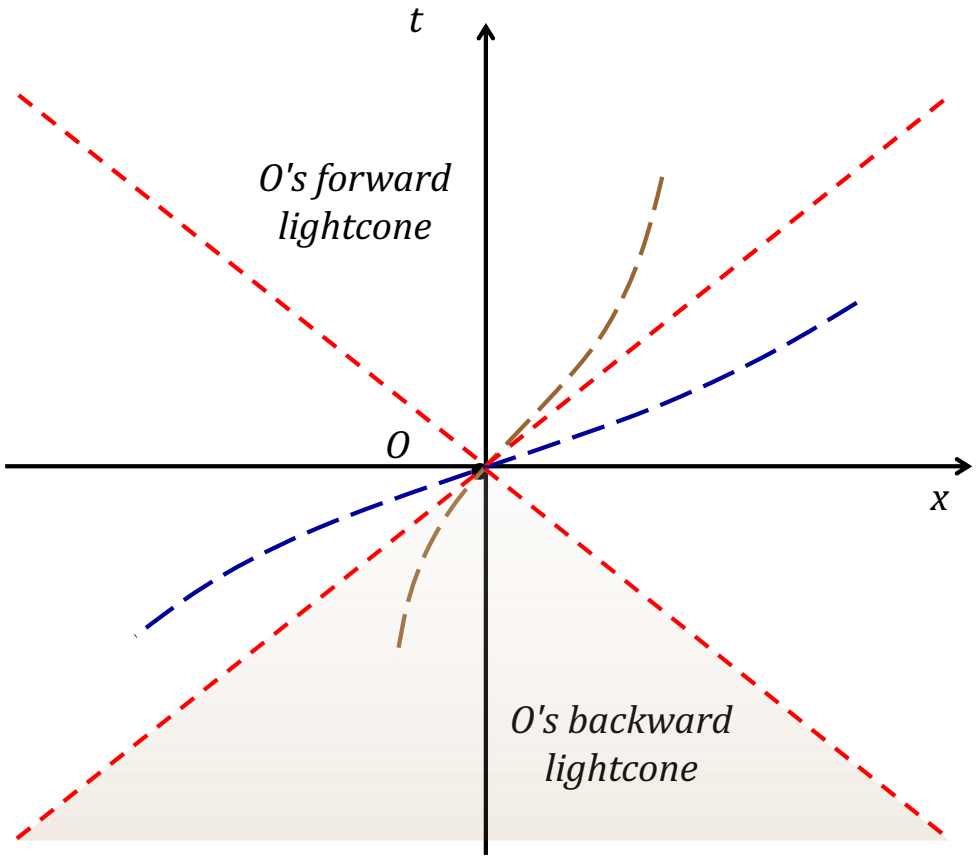

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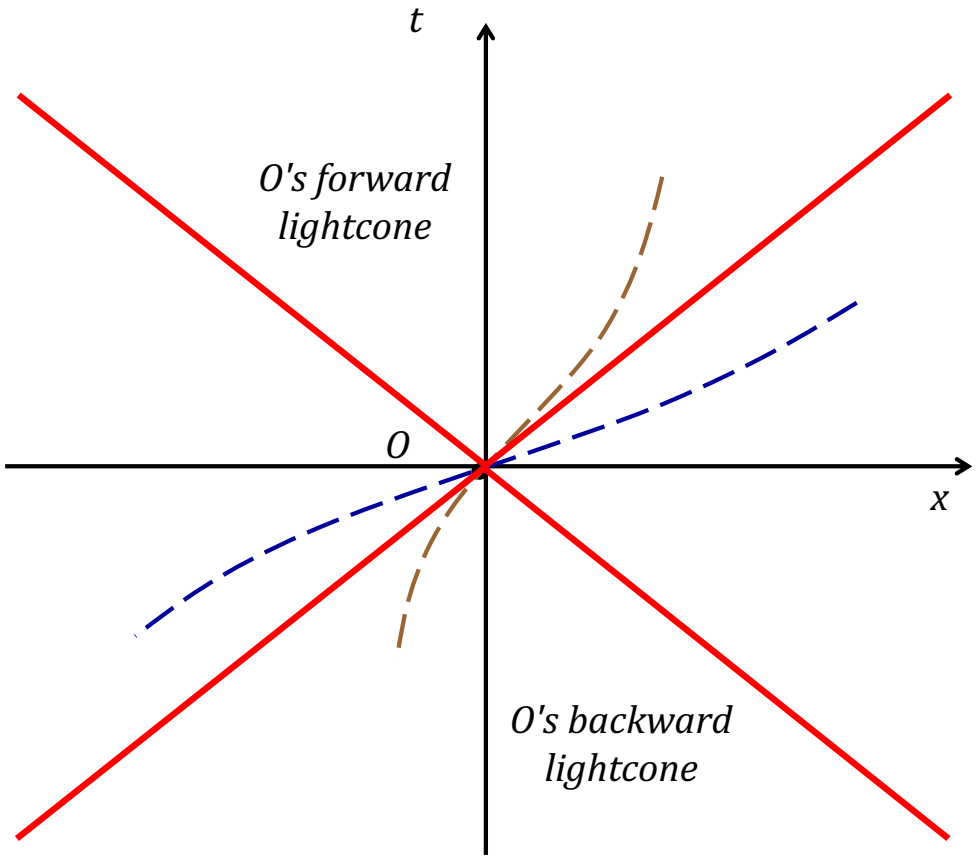

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(2) Events in O's backward lightcone. (Events in O's past.)

(3) Events on O's lightcone. (Events connectible to O by lightlike worldlines.)

Lightcone Structure

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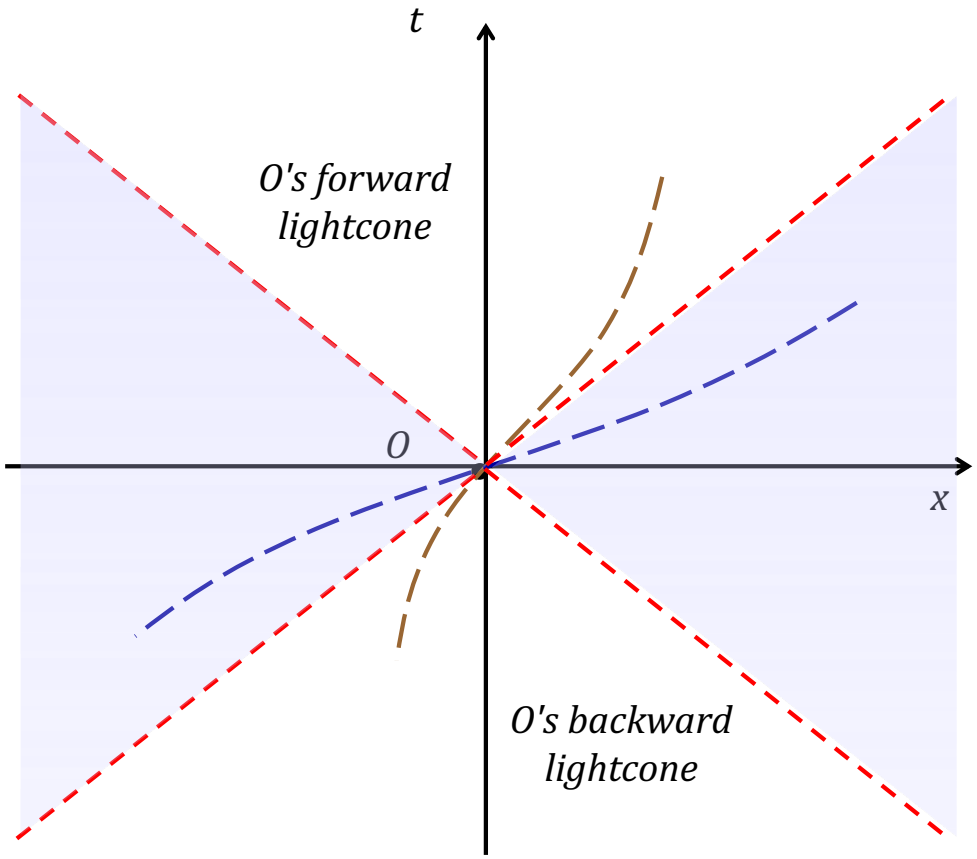

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(1) Events in O's forward lightcone. (Events in O's future.)

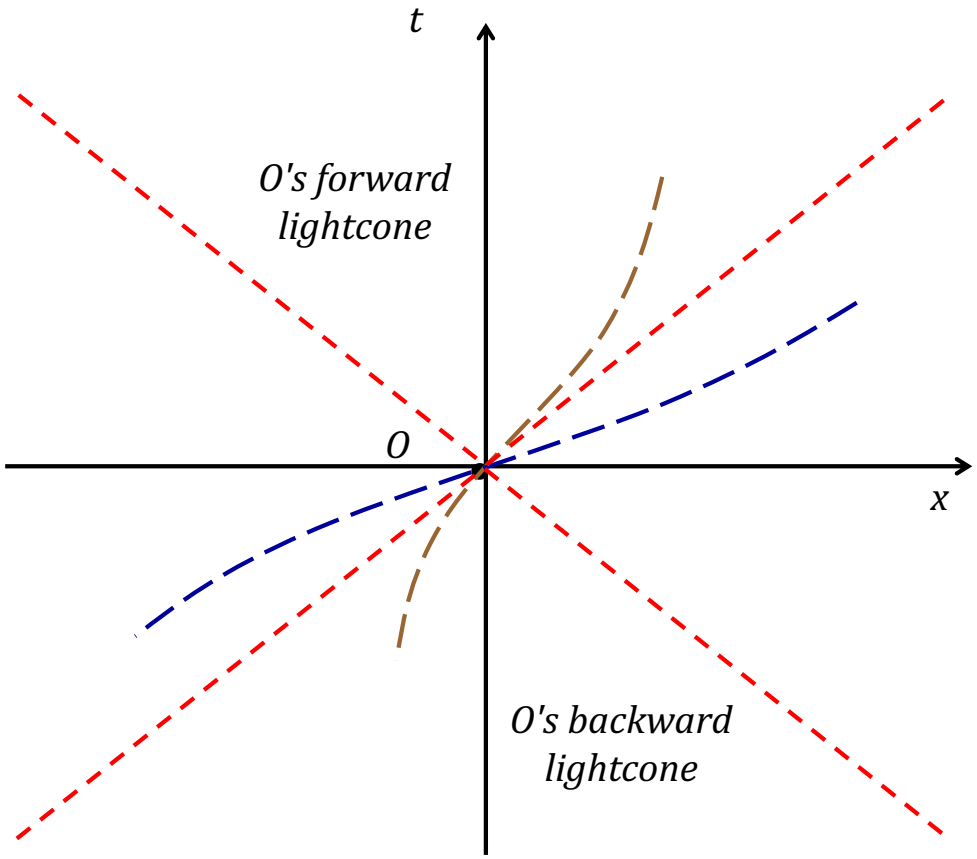
(2) Events in O's backward lightcone. (Events in O's past.)

(3) Events on O's lightcone. (Events connectible to O by lightlike worldlines.)

(4) Events outside O's lightcone. (Events connectible to O by spacelike worldlines.)

Lightcone Structure

- Let's represent the fact that c is a speed limit in a spacetime diagram:



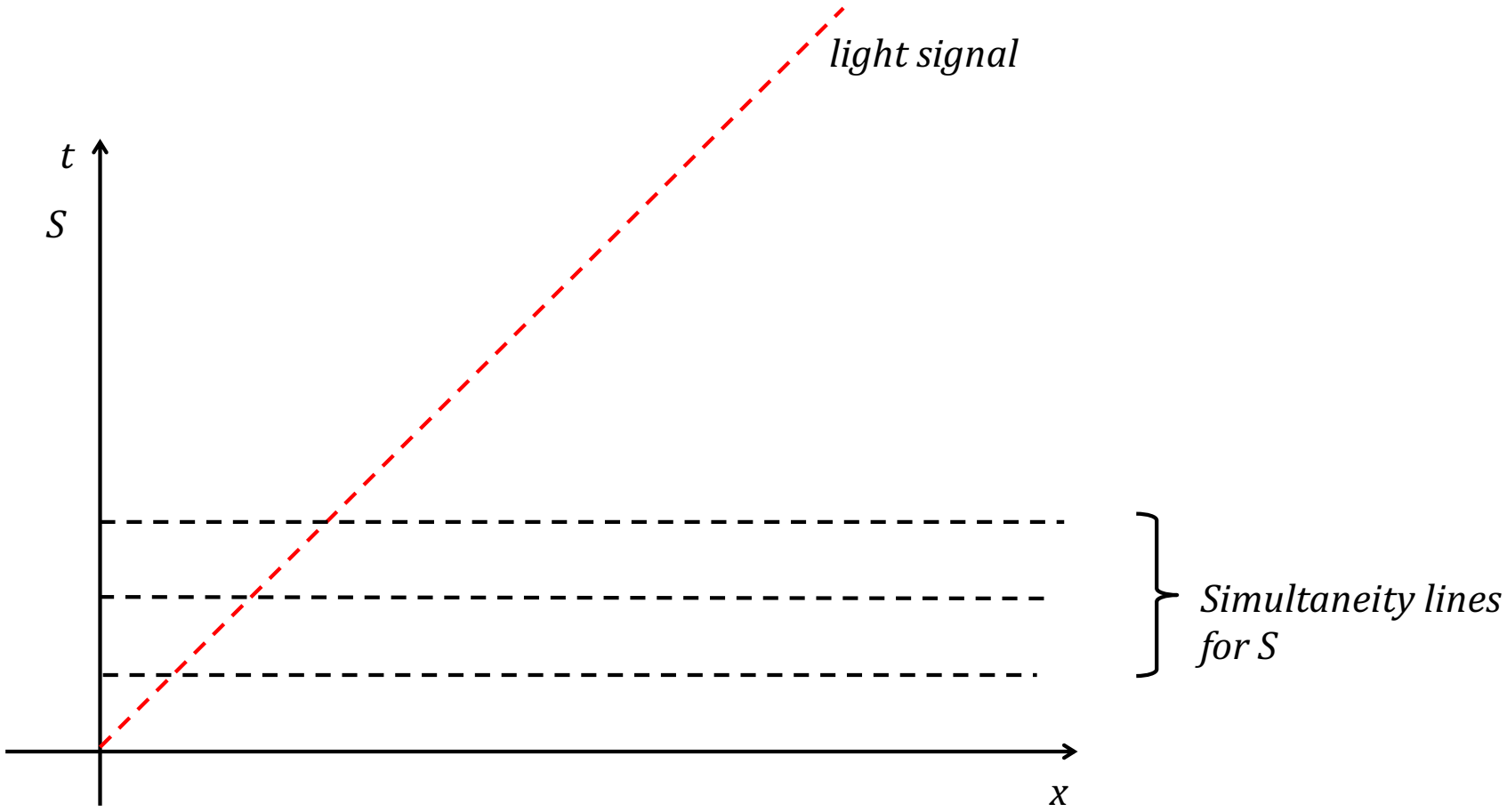
Timelike worldline
- possible path of an object traveling $< c$

Lightlike worldline
- possible path of an object traveling at c

Spacelike worldline
- possible path of an object traveling $> c$

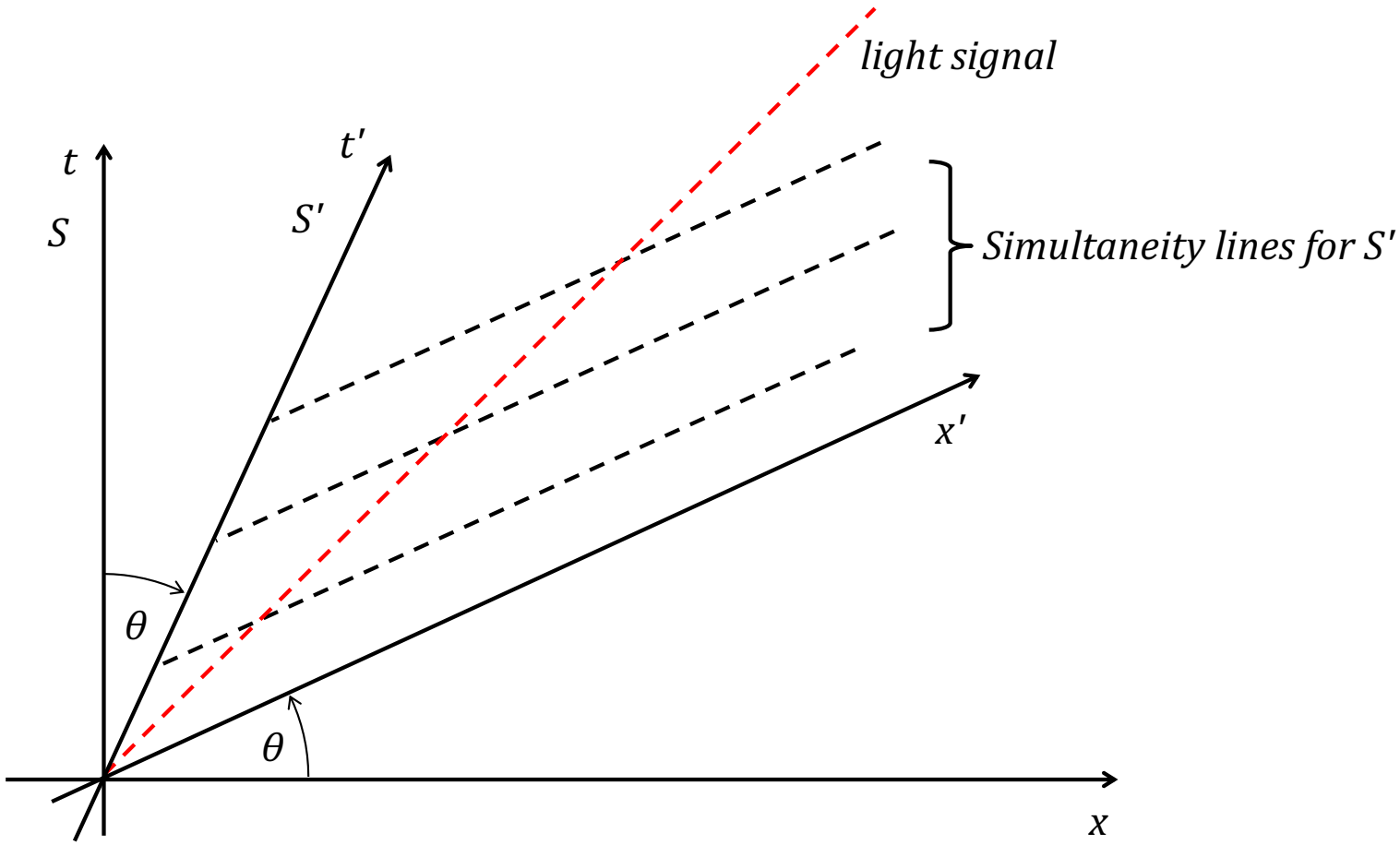
- Spacetime of special relativity is called *Minkowski spacetime*.
- Minkowski spacetime = 4-dim collection of points with lightcone structure at every point.

3. The Relativity of Simultaneity



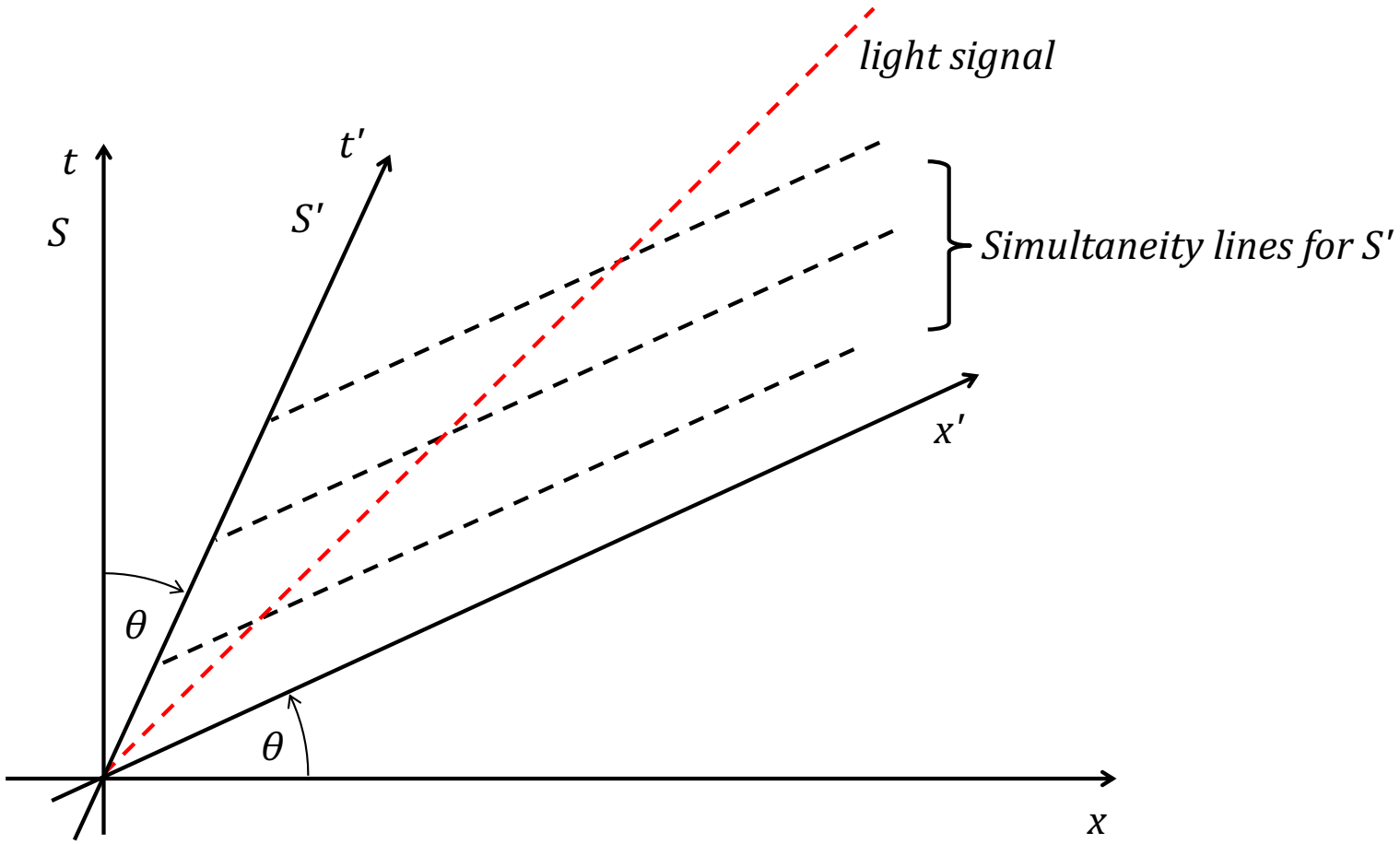
- *Simultaneity line* = line consisting of all events that occur at the same time *w.r.t. S*.
- Simultaneity lines for *S* are parallel to the *x*-axis of *S*.

3. The Relativity of Simultaneity



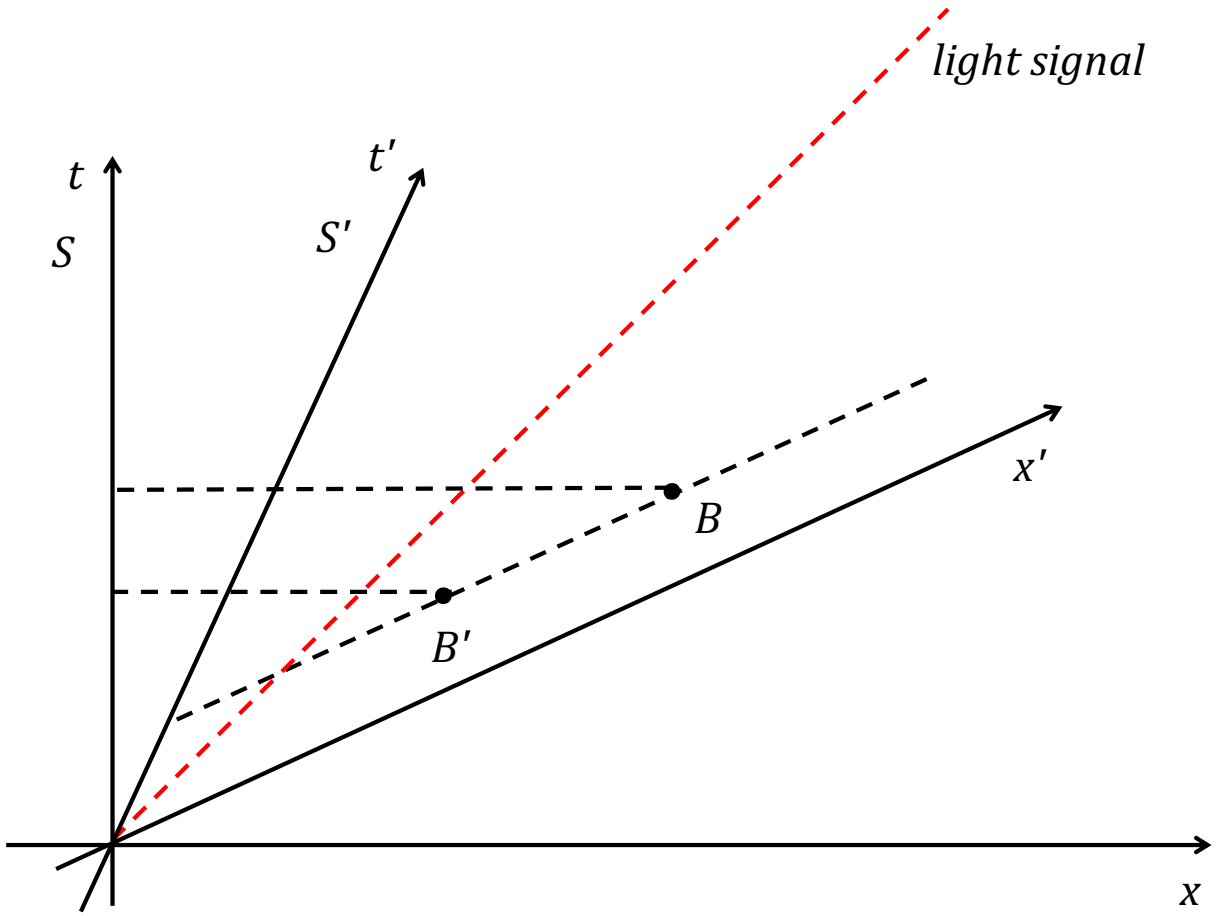
- To uphold Light Postulate, worldline of light signal must have *same slope w.r.t. S and S′*.
- Thus *x′*-axis must tilt away from *x*-axis by *same amount* as *t′*-axis tilts from *t*-axis.

3. The Relativity of Simultaneity



- Simultaneity lines for S and S' are different!
- So S and S' will make different judgements of simultaneity!

3. The Relativity of Simultaneity



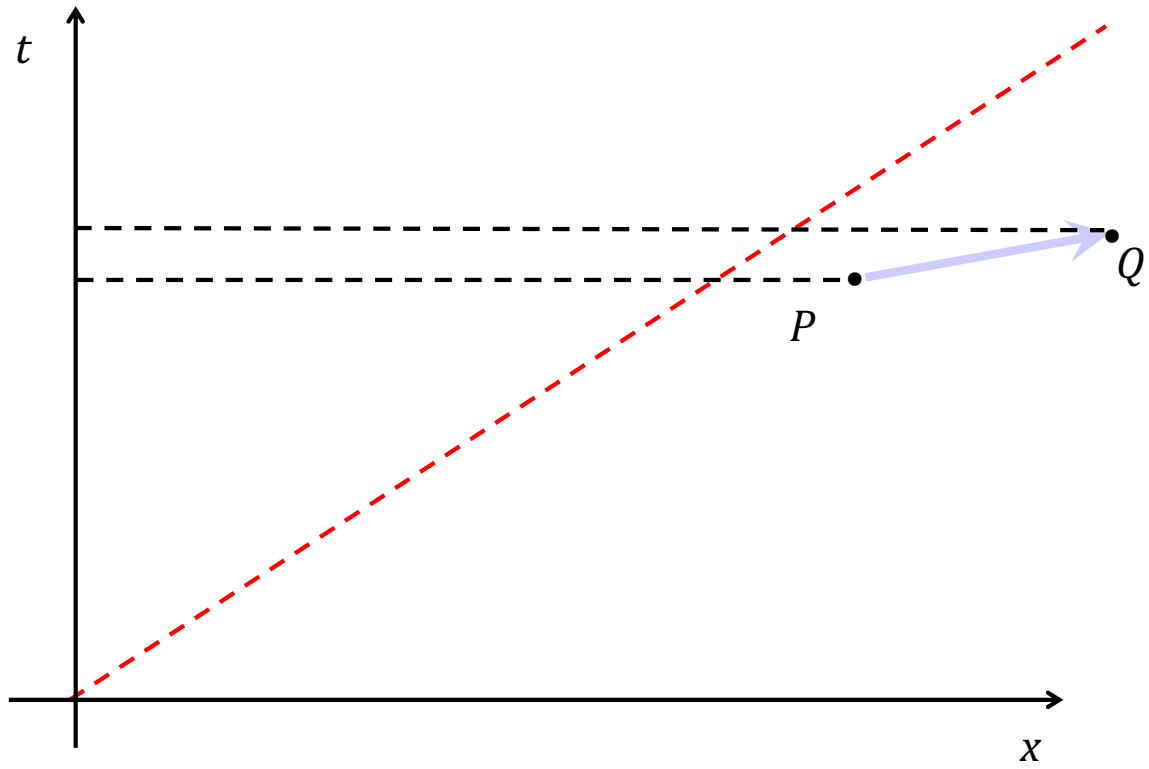
- B' and B are simultaneous with respect to S'.
- B' happens before B with respect to S.
- *Relativity of simultaneity = Events are simultaneous only with respect to an inertial reference frame.*

4. Tachyons and Causality

- *Recall*: Tachyons are objects that cannot travel less than or equal to the speed of light.
- If they exist, tachyons have the following properties:

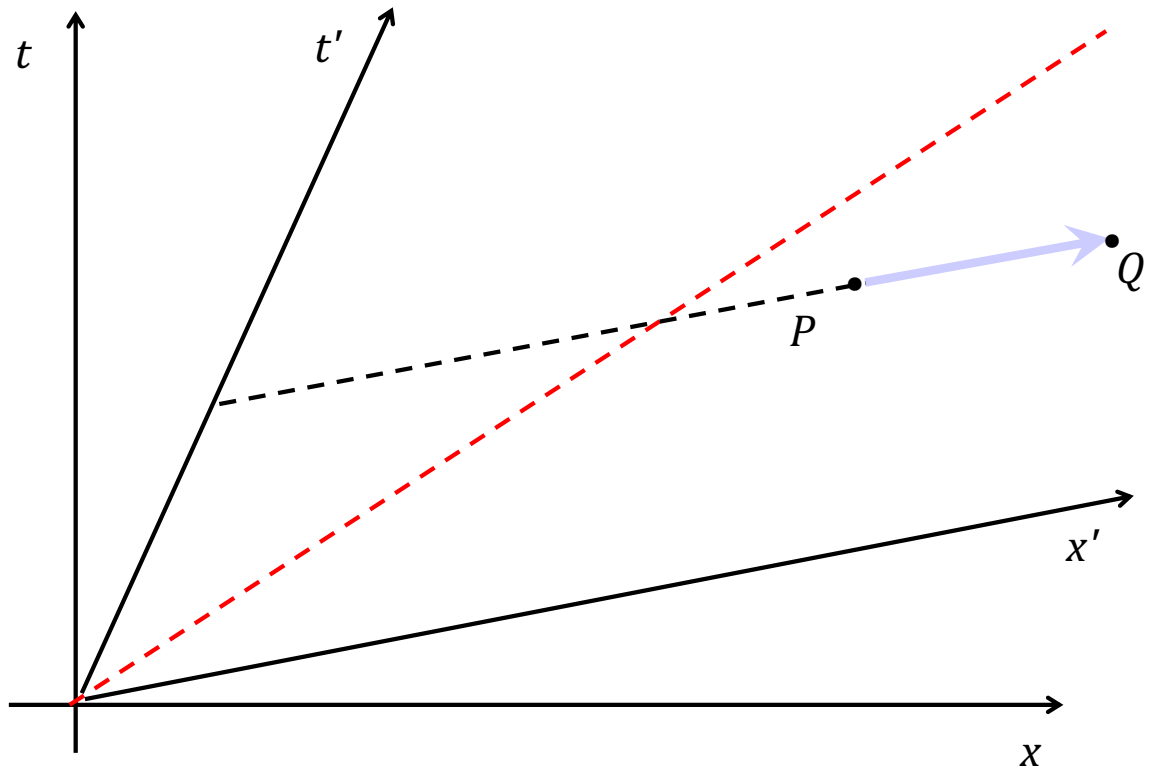
  (1) They are always traveling at speeds $> c$.

  (2) A tachyon travels forward in time, backward in time, or instantaneously, depending on the frame of reference.



A tachyon travels from P to Q in Minkowski spacetime.

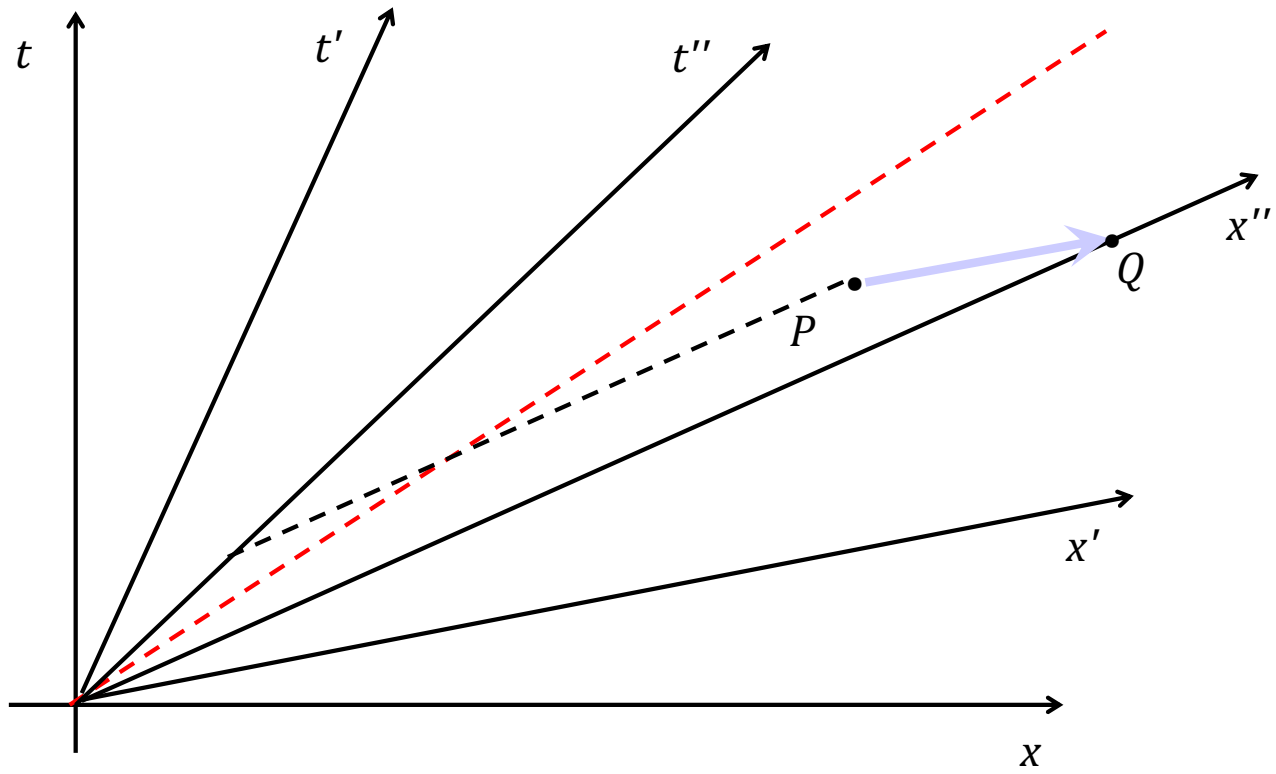
(a) According to S, it travels forward in time.



A tachyon travels from P to Q in Minkowski spacetime.

(a) According to S, it travels forward in time.

(b) According to S', it travels instantaneously.



A tachyon travels from P to Q in Minkowski spacetime.

(a) According to S, it travels forward in time.

(b) According to S', it travels instantaneously.

(c) According to S'', it travels backwards in time.

Assumption 1: Tachyons exist.

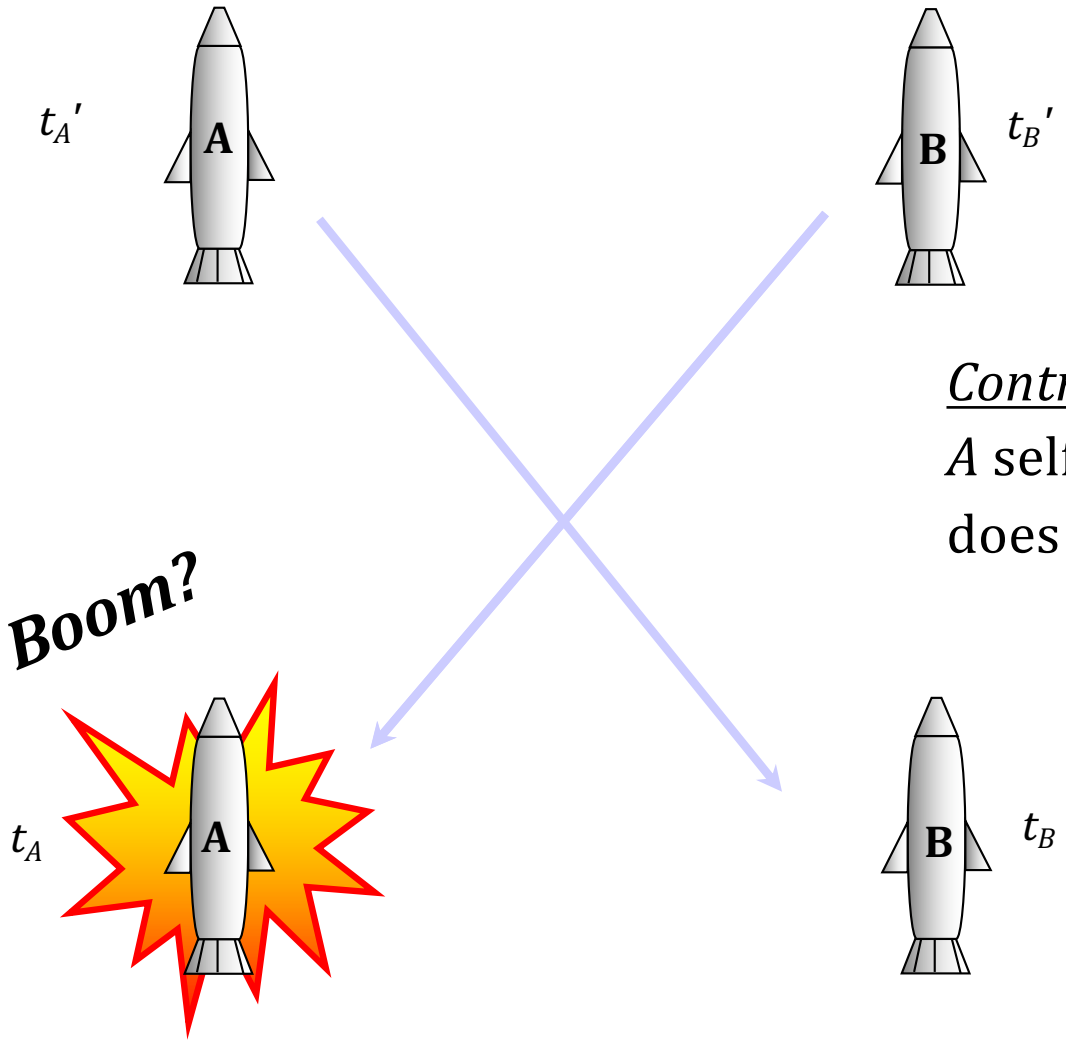
Assumption 2: Tachyons can be used to transmit information.

Claim: Assumptions 1 & 2 are contradictory.

Set-up: Two rockets A, B at constant speed *w.r.t.* each other and programmed as follows:

(1) If A recieves a tachyon at time t_A then, *and only then*, it self-destructs.
If not, then at $t_A' > t_A$, it transmits a tachyon into the past to B that arrives at t_B.

(2) If B recieves a tacyon at t_B then, *and only then*, at time $t_B' > t_B$, it transmits a tachyon into the past to A that arrives at t_A.



Contradiction:

A self-destructs *if and only if* it does not self-destruct.

Possible Responses

(a) Tachyons do not exist.

(b) Tachyons exist, but cannot be used to transmit information.

(c) Tachyons exist and can be used to transmit information, but only under very restricted conditions.

- Both (a) and (b) entail that *causal signals* (= signals that can transmit information) cannot travel faster than the speed of light. So:

Possible Resolutions

(a') Causal signals cannot travel faster than the speed of light.

(b') Causal signals can travel faster than the speed of light, but only under very restricted conditions.

One diagnosis of rocket paradox

Violates intuitive notion of cause-effect relationship:

Causes always precede their effects (No Backward Causation).

- Signal from *B* (*cause*) is sent *after A* explodes (*effect*).

Two Options:

A. [Special Relativity] & [No Backward Causation] ⇒ [Causal signals cannot travel faster than *c*.]

⇒ [No rocket-type paradoxes]

B. [Special Relativity] & [Causal signals can travel faster than the speed of light, but only under very restricted conditions.] ⇒ [No rocket-type paradoxes]

- Does Special Relativity *necessarily* entail that causal signals cannot travel faster than the speed of light? *No!*
- Does Special Relativity entail that nothing can travel faster than the speed of light? *No!*
- Does Special Relativity allow for the possibility of communicating with the past? *Yes!*
- Do causes always precede their effects? *Maybe!*

5. The Conventionality of Simultaneity

Hans Reichenbach
(1891-1953)

Claim: Given an event *A*, there is no objective fact of the matter as to what *distant* events at rest with respect to *A* are simultaneous with *A*. The choice is a matter of convention.

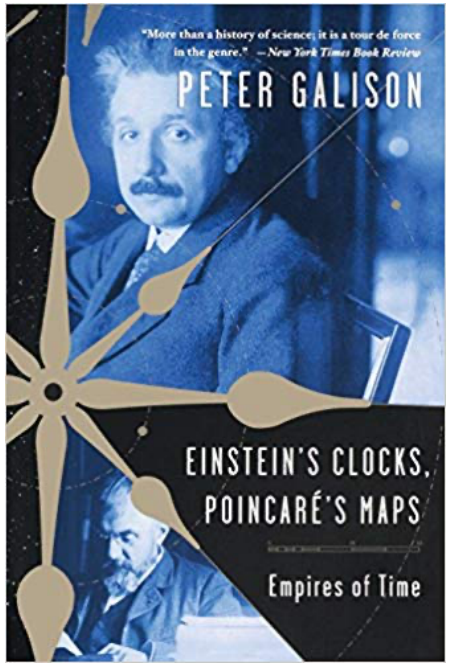
Relativity of simultaneity: Different inertial frames judge the simultaneity of events in different ways. (Entailed by the 2 Postulates.)

Conventionality of simultaneity: *Within a single inertial frame*, the simultaneity of *distant* events is not fixed and can be judged in different ways. (*Not* entailed by the 2 Postulates.)

- How can the simultaneity of distant events in the same inertial frame be established?
  - *Einstein (1905): By setting up synchronized clocks at these events.*

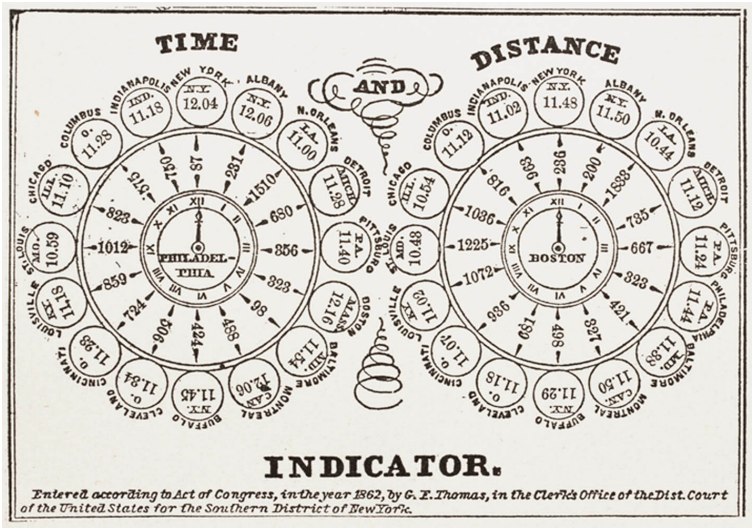
- How can distant clocks in the same inertial frame be synchronized?
  - *Einstein (1905): Use light signals.*

Aside: Why did Einstein focus on clock synchronization?

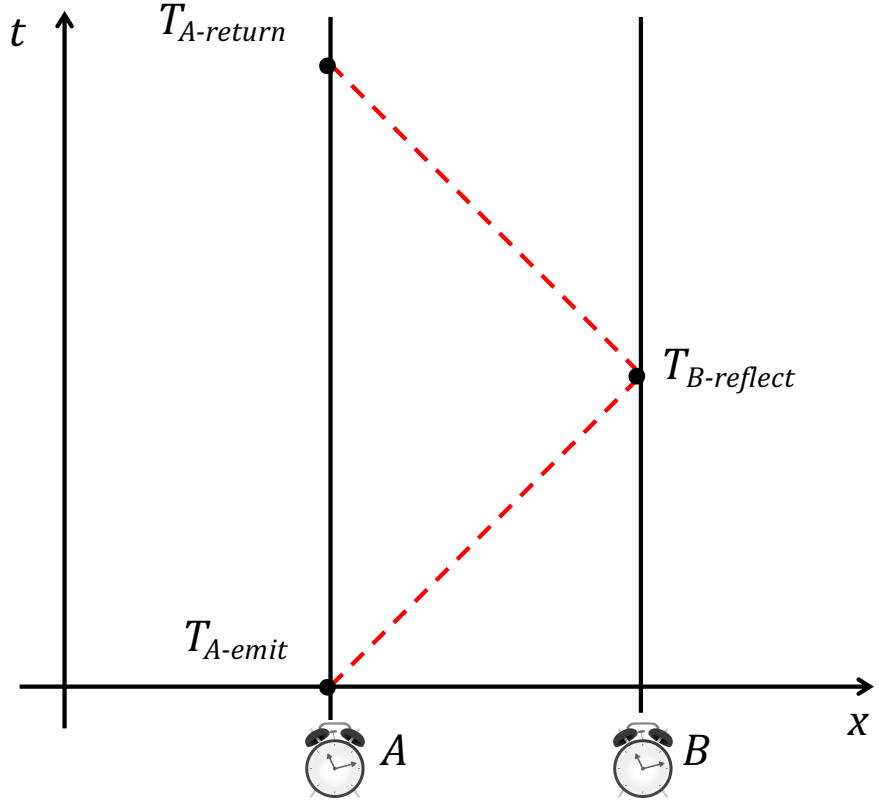


Answer: Clock synchronization was on the cutting edge of technology at the end of the 19th century:

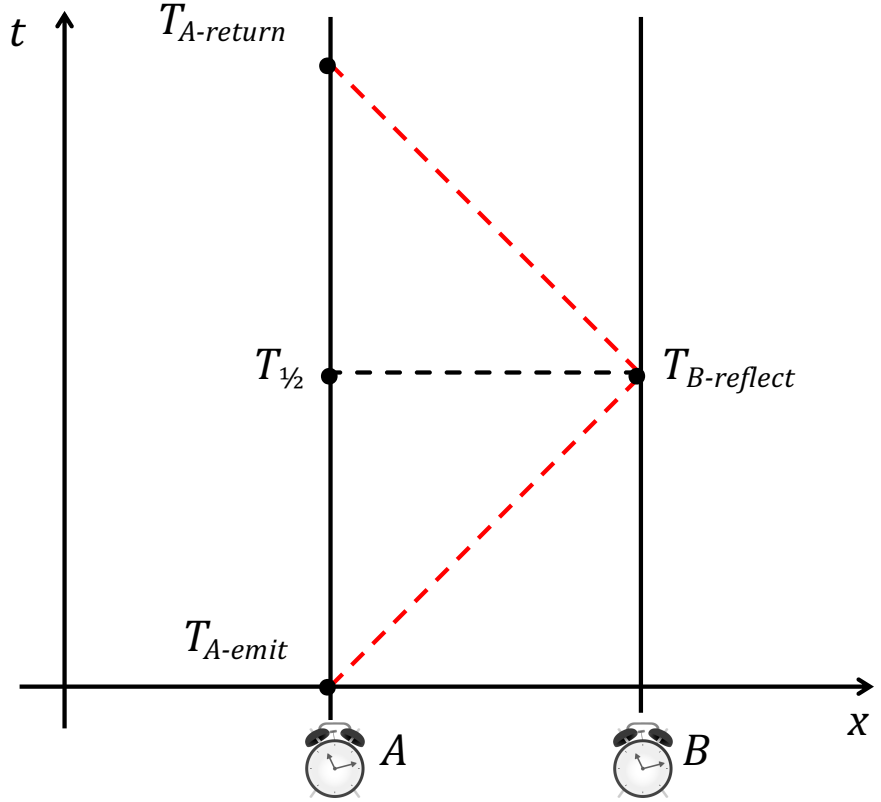
- *Railway technology*: Needed highly accurate (synchronized) clocks for dependable, efficient service.
- *Electrification of clocks*: To synchronize clocks to "railway time", send electric signals from central clock.



- Galison (2003): Example of how technology drives theoretical advances.



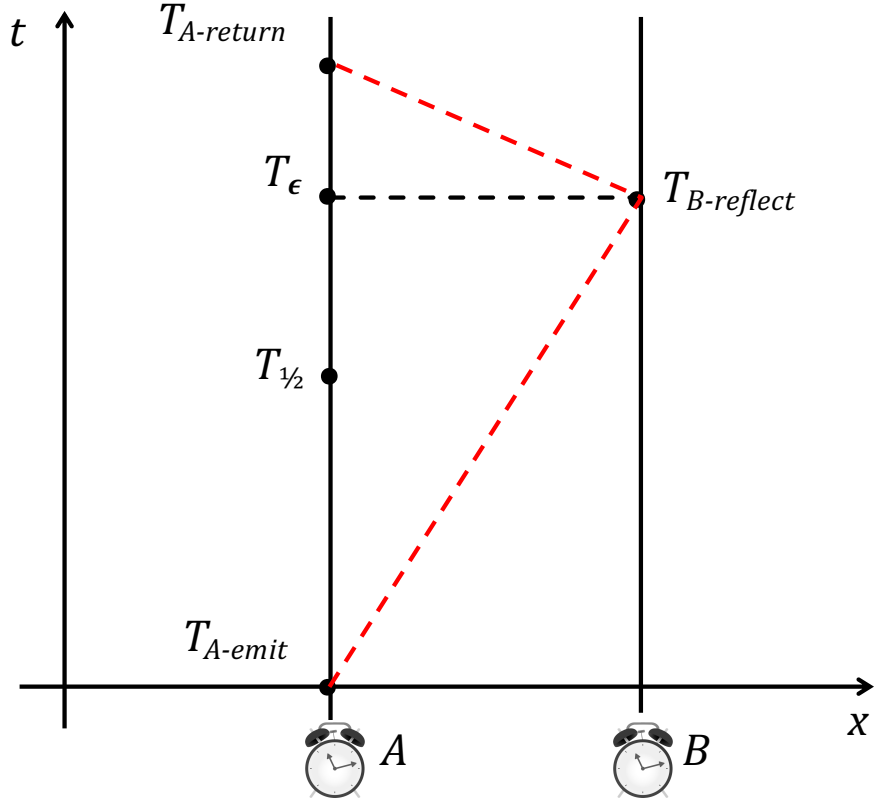
- To synchronize Clock B a given distant from Clock A,
  (1) Emit a light signal from A to B and record the time $T_{A\text{-emit}}$ on A.
  (2) Have B reflect the signal back to A. Record the time on B, $T_{B\text{-reflect}}$.
  (3) Record the A time $T_{A\text{-return}}$ when the light signal returns.



Standard Simultaneity

The event at $T_{B\text{-reflect}}$ is simultaneous with the event at $T_{½}$.

- *Einstein's Stipulation*: A and B may be said to be in synchrony just when $T_{B\text{-reflect}} = T_{½} \equiv T_{A\text{-emit}} + ½(T_{A\text{-return}} - T_{A\text{-emit}})$.
  - *Assumption*: Light travels at the same speed c in all directions.



Standard Simultaneity

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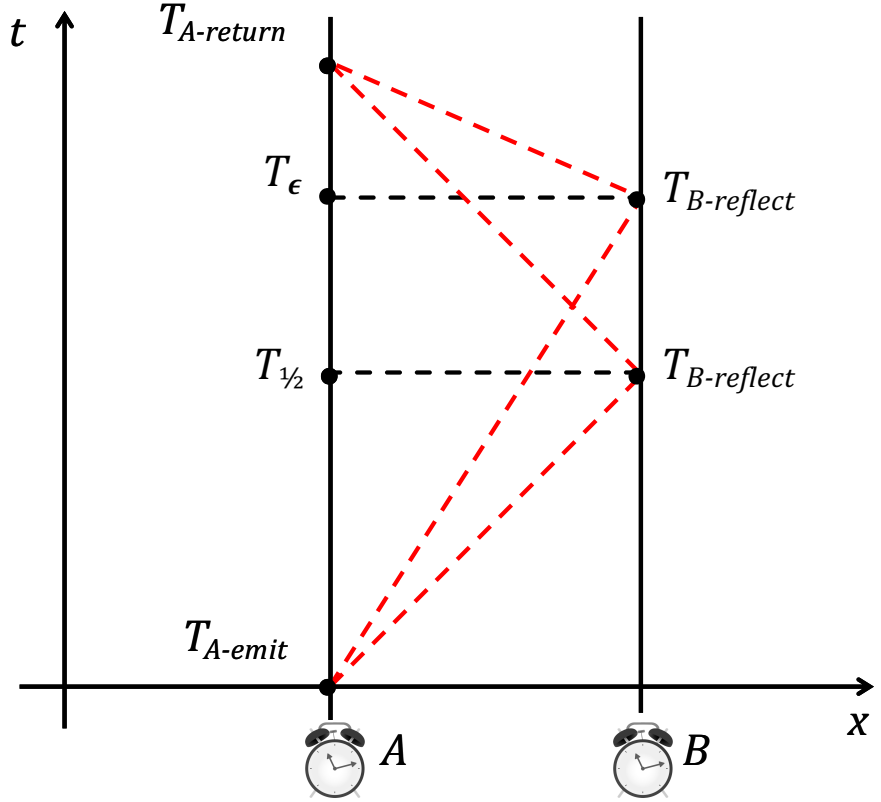
Non-Standard Simultaneity

The event at $T_{B\text{-reflect}}$ is simultaneous with the event at T_{ϵ}.

- *Einstein's Stipulation*: A and B may be said to be in synchrony just when $T_{B\text{-reflect}} = T_{½} \equiv T_{A\text{-emit}} + ½(T_{A\text{-return}} - T_{A\text{-emit}})$.
  - *Assumption*: Light travels at the same speed c in all directions.

- *Reichenbach's Conventionalism*: A and B are in synchrony just when $T_{B\text{-reflect}} = T_{\epsilon} \equiv T_{A\text{-emit}} + \epsilon(T_{A\text{-return}} - T_{A\text{-emit}})$, for *any* value of ϵ, where $0 < \epsilon < 1$.
  - *Assumption*: Light does *not* necessarily travel at the same speed c in all directions.



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The event at $T_{B\text{-reflect}}$ is simultaneous with the event at $T_{½}$.

Non-Standard Simultaneity

The event at $T_{B\text{-reflect}}$ is simultaneous with the event at T_{ϵ}.

- Who's right: Einstein or Reichenbach?
  - *Does light travel at the same speed in all directions or not?*

How can the "one-way" speed of light be measured?

Reichenbach's Claim

(a) To measure the one-way speed of light, we need synchronized clocks.

(b) But we can only synchronize our clocks if we have prior knowledge of distant simultaneity, which requires prior knowledge of the one-way speed of light.

Realist Response:

- Agree that there is no observational difference between the standard simultaneity relation and any non-standard simultaneity relation.
- *So*: If empirical adquacy (i.e., agreement with observation) is the criterion for how one chooses between competing theories, then there's no reason to prefer the standard relation to any non-standard relation.
- *But*: Why think empirical adquacy is the only criterion of theory choice?

- Suppose simplicity is a criterion of theory choice.
- *Then*: We should prefer the standard simultaneity relation, since it assumes light travels at the same speed in all directions.
- *However*: Simplicity is a highly subjective concept...

Einstein

General relativity is much more simple than Newton's theory of gravity!

???

Average Joe

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- *But*: Why think empirical adquacy is the only criterion of theory choice?

- Suppose unifying power is a criterion of theory choice (i.e., we should choose that theory that fits better with other theories).
- *Then*: We should prefer the standard simultaneity relation, since Friedman-Robertson-Walker spacetimes in general relativity (*i.e.*, "Big Bang" spacetimes) are isotropic in a way that singles out the standard definition.
- *But*: Adopting such spacetimes as descriptions of our universe requires many assumptions, one of which just is isotropy.