02. Special Relativity: The 2 Postulates and the Michaelson-Morley Experiment

Topics:
1. Motivation
2. Michaelson-Morley Experiment

Albert Einstein
(1879-1955)

"I am more and more convinced that the electrodynamics of moving bodies, as it is presented today, does not agree with the truth, and that it should be possible to present it in a simpler way. The introduction of the name 'ether' into the electric theories has led to the notion of a medium of whose motion one could speak of without being able, I believe, to associate a physical meaning to this statement." (1899.*)

"I believe that electric forces can be directly defined only for empty space... Further, electric currents will have to be regarded not as 'the vanishing of electric polarization in time' but as motion of true electric masses, whose physical reality seems to result from the electrochemical equivalents... Electrodynamics would then be the science of the motions in empty space of moving electricities and magnetisms." (1899.)

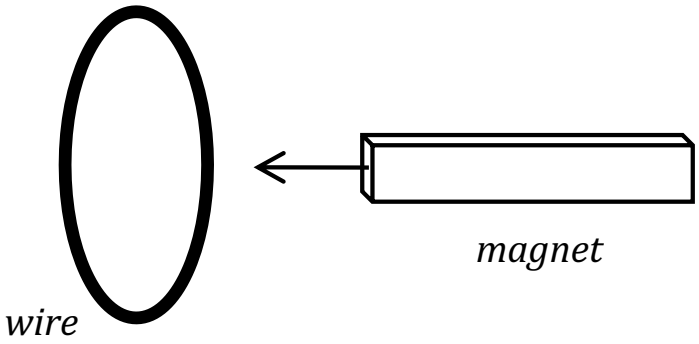
*Letter to Mileva Maric. Quoted in Darrigol, O. (2005) 'The Genesis of the Theory of Relativity', *Poincare Seminar 2005*, pg. 21.

1905. "On the Electrodynamics of Moving Bodies."

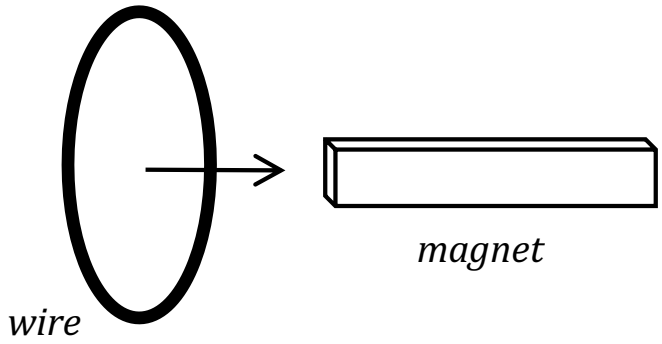
- Central Motivation:

"It is known that Maxwell's electrodynamics--as usually understood at the present time [*i.e.*, Lorentz's theory]--when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena."

- When a magnet moves through a stationary conducting coil, or when the coil moves about the stationary magnet, the induced current is the same.
- *But*: The theoretical explanation is different!
  - *First case*: Moving magnet induces electric field in coil (Faraday's Law). Electric force makes electrons move in coil.
  - *Second case*: Lorentz force due to magnetic field of magnet causes electrons to move in coil.



$$\nabla \times \mathbf{E} = -c^{-1}\partial \mathbf{B}/\partial t$$



$$\mathbf{F} = \rho(c^{-1}\mathbf{v} \times \mathbf{B})$$

"Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the 'light medium' suggest..."

- *1st Postulate ("Principle of Relativity")*:

"The same laws of electrodynamics and optics will be valid in all frames of reference for which the equations of mechanics hold good." "

- *2nd Postulate (Light Postulate)*:
"Only apparently irreconcilable with [the relativity postulate]."

"Light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body."

1. Motivation for Postulates

The Principle of Relativity

"The same laws of electrodynamics and optics will be valid in all frames of reference for which the equations of mechanics hold good."

- Conceptual desire to avoid theoretical "asymmetries" in electrodynamics leads Einstein to reject the aether (the source of such asymmetries).
- "Frames for which the equations of mechanics hold good" = inertial frames = frames moving at constant velocity with respect to each other.
- *Thus*: The P. of R. says: There is no privileged aether frame in which the laws of electrodynamics and optics (*i.e.*, the Maxwell-Lorentz equations) hold; rather, these laws hold in all inertial frames.

- *What this means*: The laws of electrodynamics *and* optics *and* mechanics (*i.e.*, the laws of physics at the time) cannot distinguish one inertial frame from another.
  - *All inertial frames are physically indistinguishable with respect to the laws of physics.*

The Light Postulate

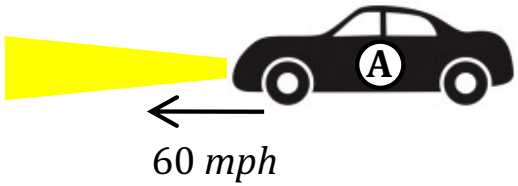
"Light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body."

- According to Maxwell's theory, light always propagates *in the aether* with velocity *c* which is independent of the state of motion of the emitting body.
  - *Light Postulate rephrases this in terms of "empty space".*
- Why state this as a postulate?
  - "The constancy of the velocity of light no longer resulted from the existence of the ether, and had to be postulated separately." (Darrigol, pg. 23.)
  - *It's the essential feature of electrodynamics that Einstein thought would survive in a theory describing light quanta...*
- Why is it "apparently irreconcilable" with the Principle of Relativity?
  - *Principle of Relativity and Light Postulate entail:*

The speed of light is the same in all inertial reference frames.

This violates the way of adding velocities on both emissionist and aether theories of light!

- Consider three cars on a highway:



What is the velocity of the light of car A's headlights with respect to cars B and C?

Emission theory: Velocity of light depends on velocity of its source.

- Car *B* says velocity of light from car *A*'s headlights is $c + 60\ mph + 100\ mph$.
- Car *C* says velocity of light from car *A*'s headlights is $c + 60\ mph + 5\ mph$.

Aether theory: Velocity of light with respect to aether is *c*, indepedent of its source.

- Car *B* says velocity of light from car *A*'s headlights is $c + 100\ mph$.
- Car *C* says velocity of light from car *A*'s headlights is $c + 5\ mph$.

Einstein's theory: Velocity of light is *c* in all inertial frames.

- Car *B* says velocity of light from car *A*'s headlights is *c*.
- Car *C* says velocity of light from car *A*'s headlights is *c*.

- Eintein's two postulates entail that *B* and *C* measure the same velocity for light, even though they are moving with respect to each other!

Is the speed of light independent of inertial reference frame?

2. Michelson/Morley Experiment (1887)

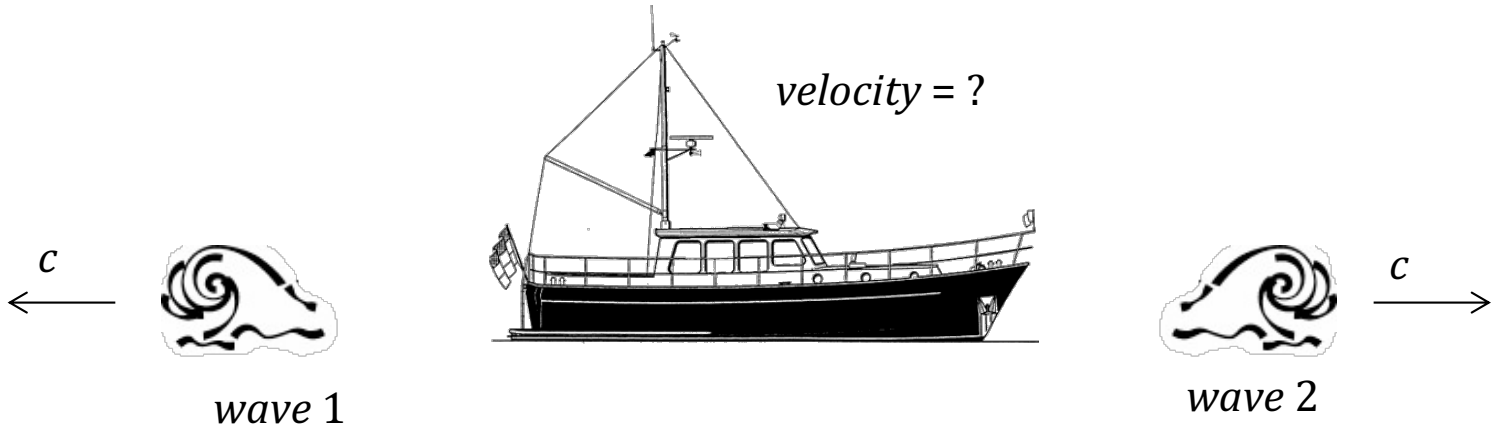
(a) *Purpose*: Measure speed of Earth through aether.

- *Recall*: Aether = in-principle unobservable substance through which observable electromagnetic waves propagate.

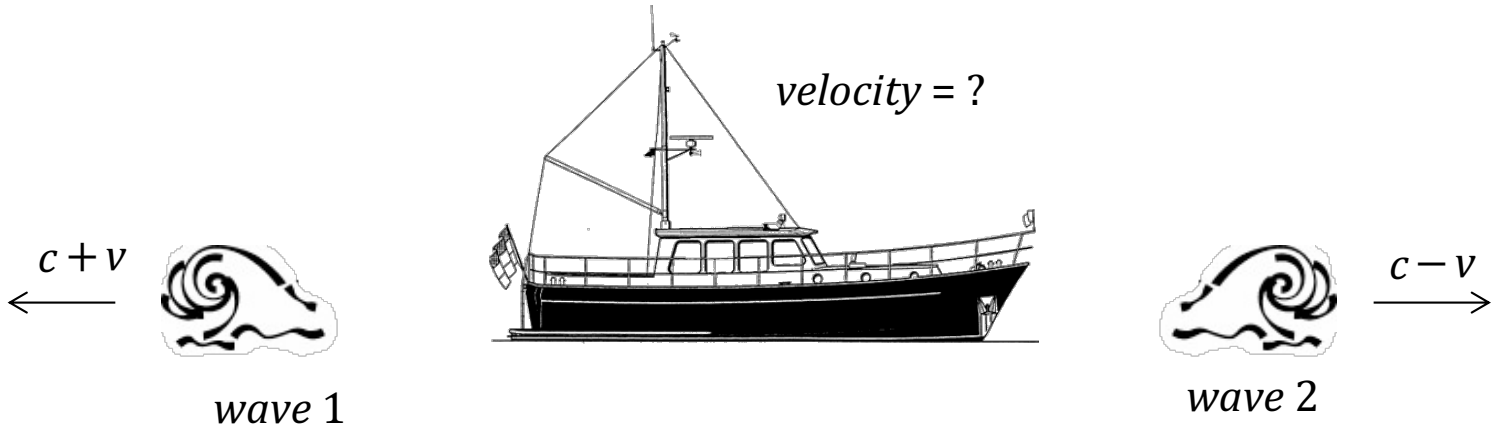
Maxwell's theory: With respect to the aether, all electromagnetic waves travel at constant speed *c* regardless of direction of travel.

How can the speed of the Earth through the aether be measured?

- Suppose water is in-principle unobservable while *water waves* are observable and move at constant speed *c*.
- Consider measuring the speed of a ship with respect to the water through which it's moving.
- Can proceed by measuring the ship's speed with respect to water waves traveling in different directions.



- *Suppose*: Ship measures waves 1 and 2 as traveling at same speed *c*.
  - *Then ship must be at rest with respect to water.*

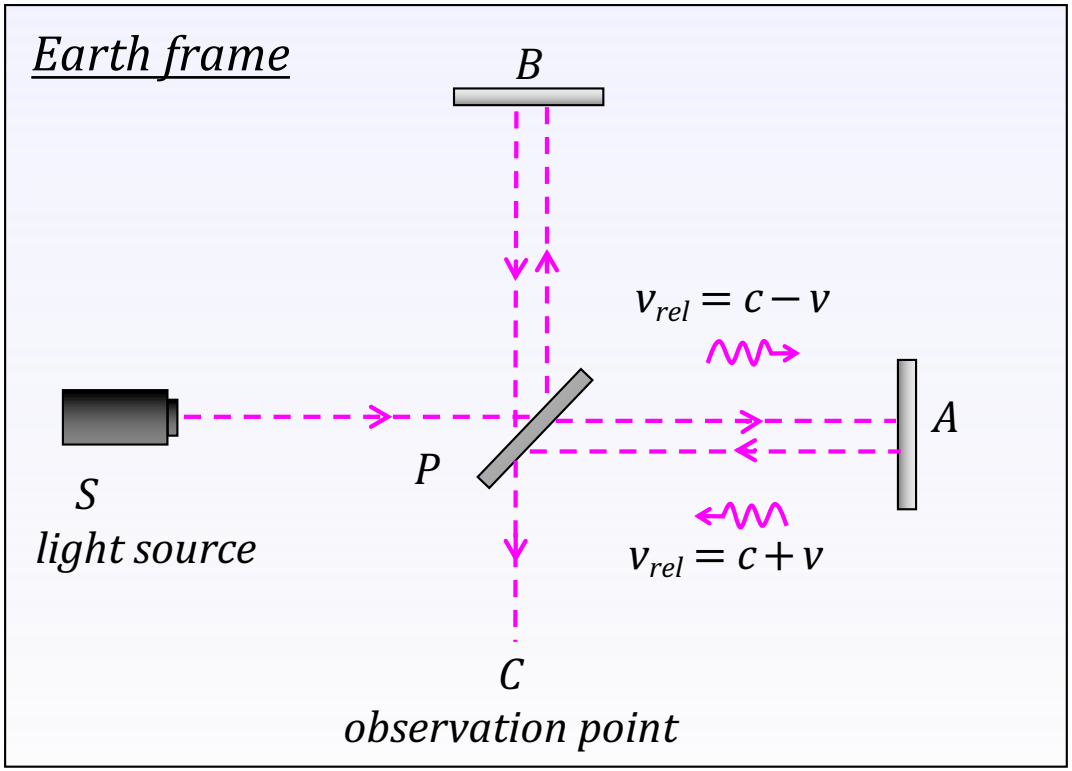


- *Suppose*: Ship measures waves 1 and 2 as traveling at same speed c.
  - *Then ship must be at rest with respect to water.*
- *Suppose*: Ship measures wave 2 as traveling at $c - v$, and wave 1 as traveling at $c + v$.
  - *Then ship must be traveling with speed v in direction of wave 2 with respect to water.*

(b) *Procedure*: Measure the time it takes a light signal to travel a given distance L in the direction of the Earth's motion through the aether, and in a direction perpendicular to this direction.

- *Assumption*: The speed of light as measured on the Earth will be different from c, depending on the Earth's speed and direction of travel.

(c) *Set-Up*: The Michelson Interferometer



Aether frame

$v = ?$

c = speed of light *with respect to* aether

v = speed of interferometer *w.r.t.* aether

v_{rel} = speed of light *w.r.t.* interferometer

L = length of arms *PA* and *PB w.r.t.* interferometer.

Suppose: *PA* is in direction of Earth's motion through aether.

Let: T_1 = time for light signal to travel path *PAP.*

T_2 = time for light signal to travel path *PBP.*

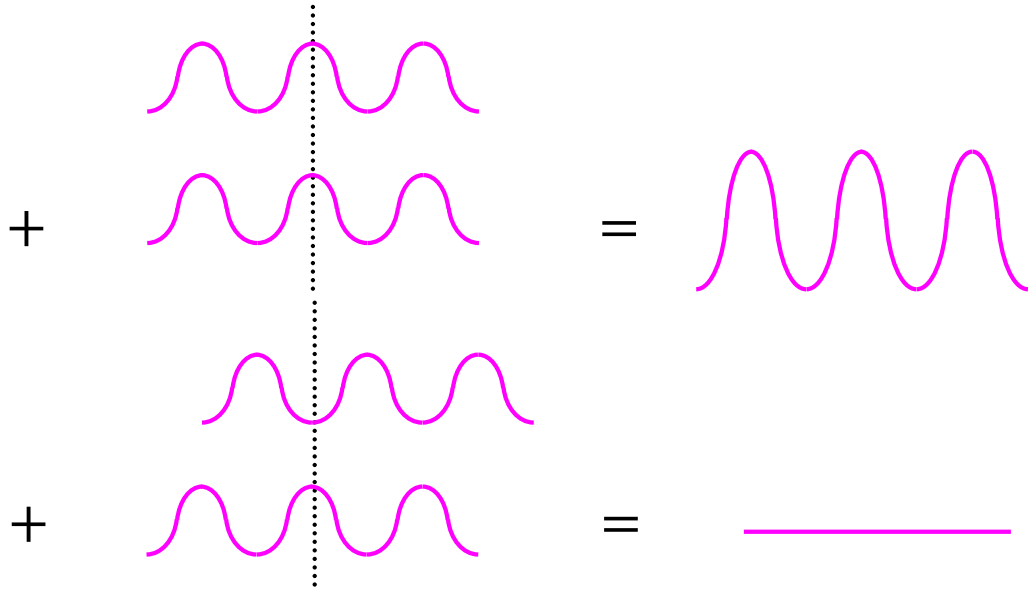
Then: If $T_1 - T_2 = 0$, then no *interference* will be observed at *C*.

If $T_1 - T_2 \neq 0$, then *interference* will be observed at *C*.

Interference of light waves

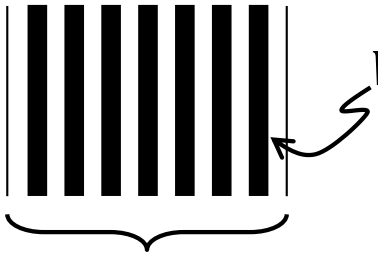
- Two types:

constructive:



destructive:

- Appears as pattern of bright and dark fringes at observation point C:

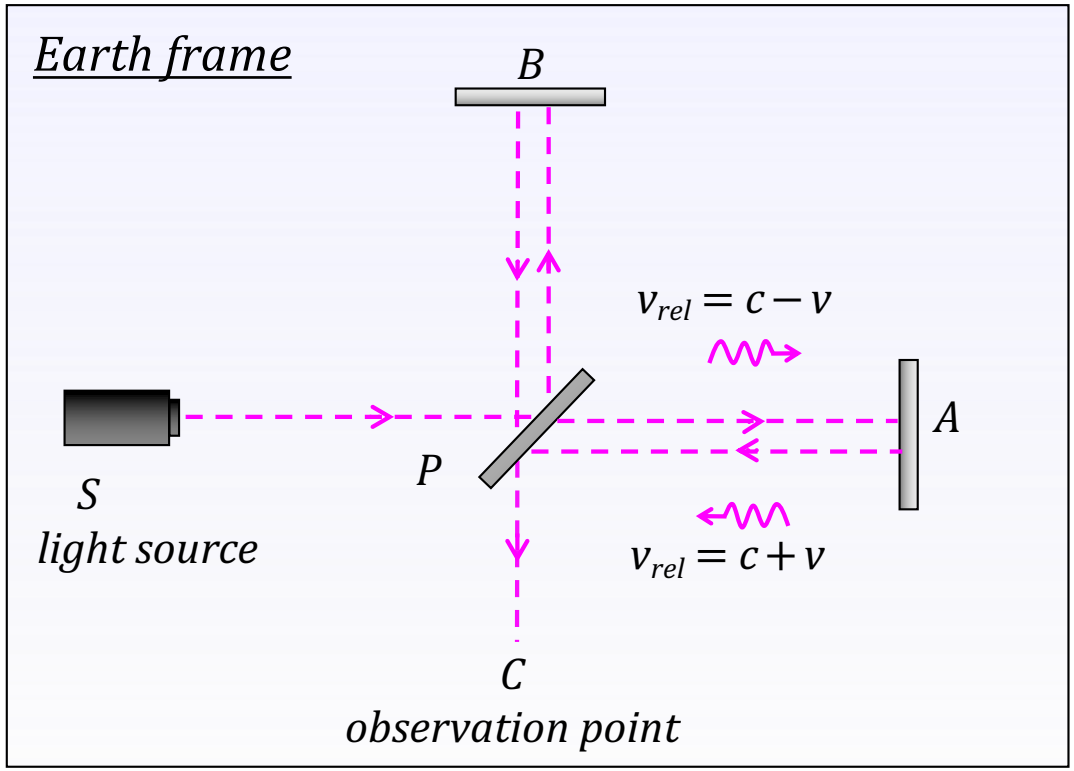


Width of a single fringe = wavelength of light signal.

Width of pattern $= c(T_1 - T_2)$

$=$ *distance traveled by light signal in time* $T_1 - T_2$.

- *Claim*: $T_1 - T_2$ depends explicitly on v*!*
- *So*: v can be calculated if we can determine the width of the observed interference pattern at C.



Aether frame

$v = ?$

T_1 = time for light signal to travel *PAP*

T_2 = time for light signal to travel *PBP*

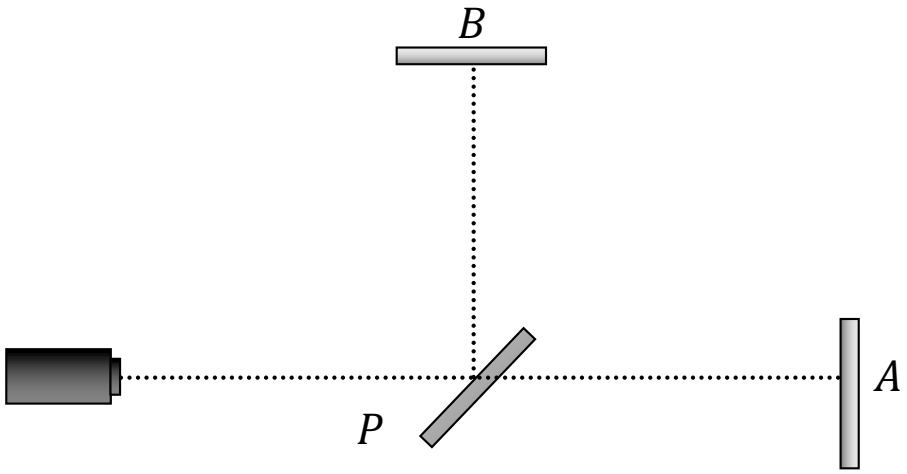
L = length of arms *PA* and *PB* with respect to interferometer

Calculation of T_1

- In Earth frame: $t_{PA} = \dfrac{L}{v_{rel}} = \dfrac{L}{c - v}$ and $t_{AP} = \dfrac{L}{c + v}$

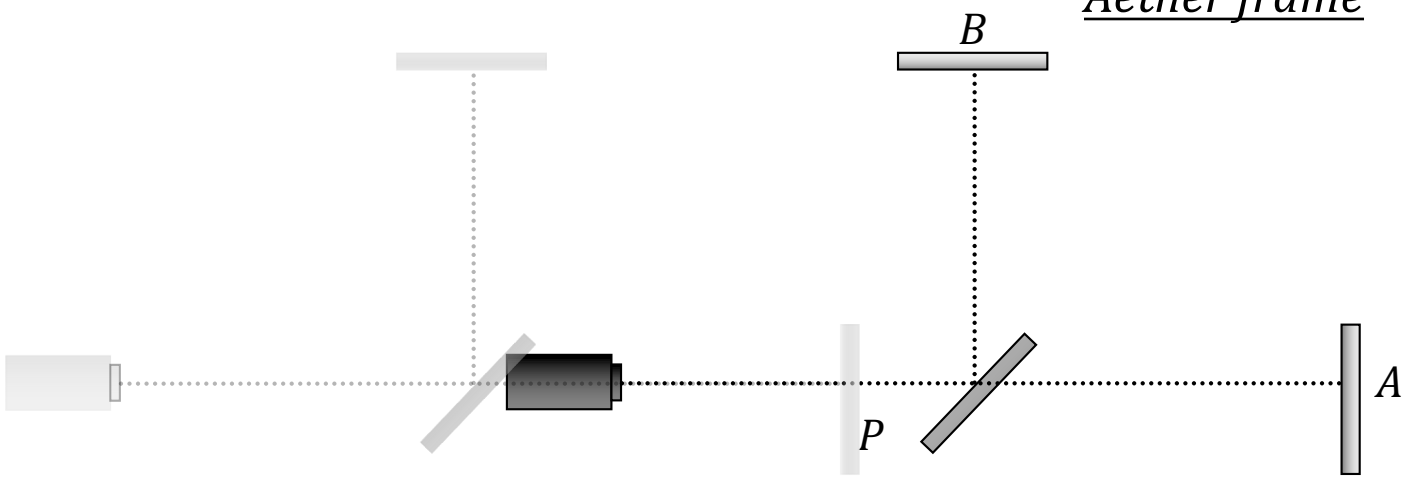
- *So*: $T_1 = \dfrac{L}{c - v} + \dfrac{L}{c + v} = \dfrac{2Lc}{c^2 - v^2} = \dfrac{2L}{c\left(1 - \frac{v^2}{c^2}\right)} = \dfrac{2L}{c}\gamma^2$ where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Aether frame



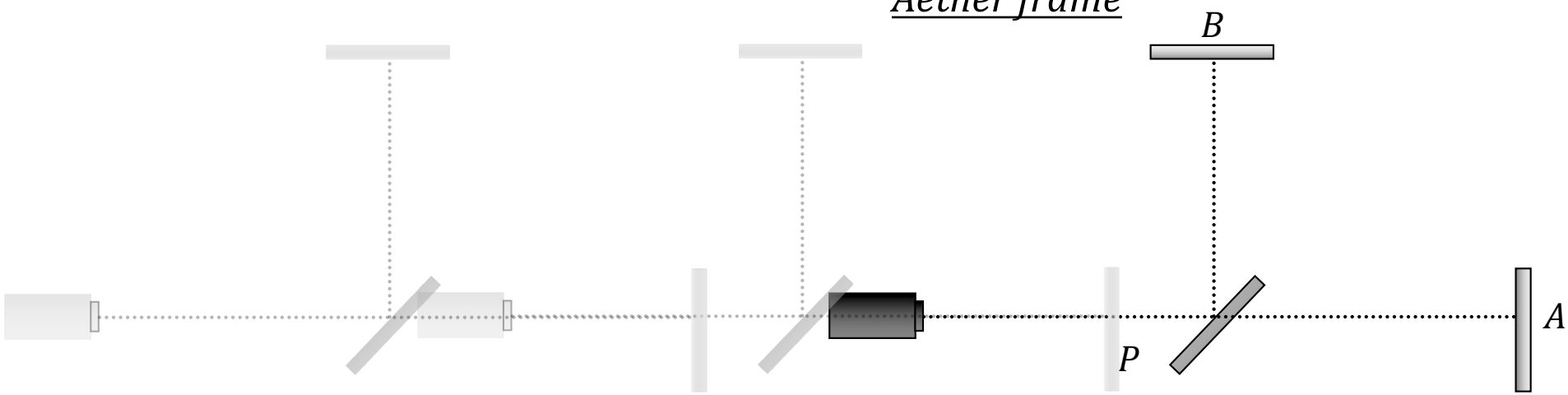
Calculation of T_2

Aether frame

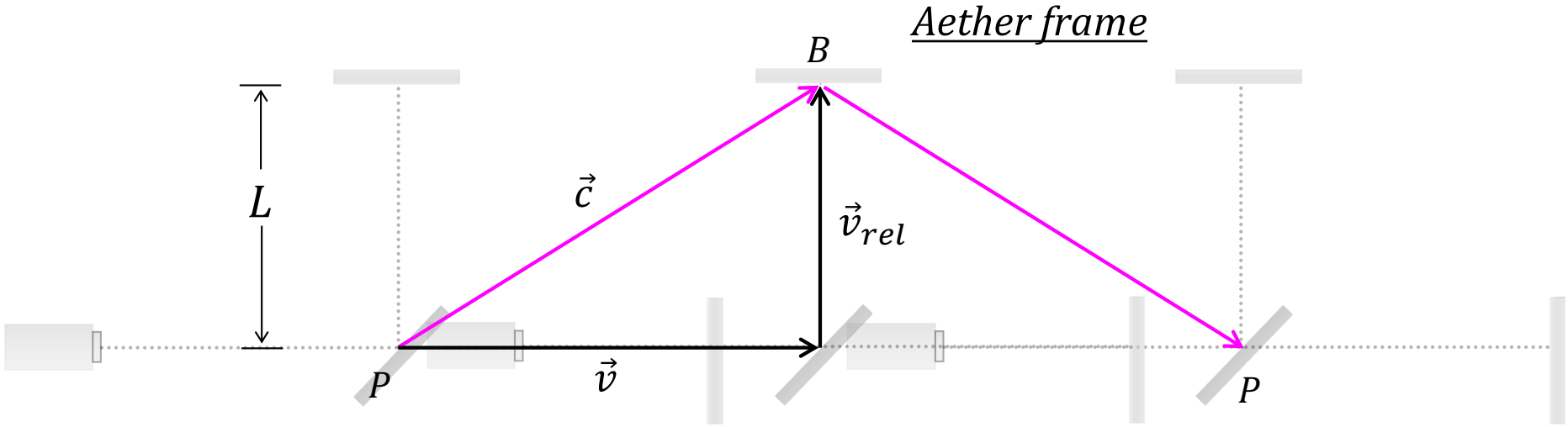


Calculation of T_2

Aether frame



Calculation of T_2



Calculation of T_2

- In aether frame: $c^2 = v_{rel}^2 + v^2$ or $v_{rel} = \sqrt{c^2 - v^2} = c\sqrt{1 - \frac{v^2}{c^2}} = \frac{c}{\gamma}$
- *So*: $t_{PB} = t_{BP} = \frac{L}{v_{rel}} = \frac{L}{c}\gamma$
- *Thus*: $T_2 = \frac{2L}{c}\gamma$

Calculation of $T_1 - T_2$

$$T_1 = \frac{2L}{c}\gamma^2 \quad \text{and} \quad T_2 = \frac{2L}{c}\gamma$$

- Since v is much smaller than c, we can use the *binomial theorem* to approximate γ.

The binomial theorem entails:

For $v \ll c$, $\gamma \approx 1 + \frac{v^2}{2c^2}$ and $\gamma^2 \approx 1 + \frac{v^2}{c^2}$

Check: If v = speed of Earth with respect to Sun, then $v \approx 30\,km/s \ll c$.

- In this case, $v/c \approx 10^{-4}$.
- So $v^2/c^2 \approx 10^{-8}$, which is a millionth of 1% -- *very* small effect!

- *So*:

$$T_1 - T_2 = \frac{2L}{c}\gamma^2 - \frac{2L}{c}\gamma$$

$$= \frac{2L}{c}(\gamma^2 - \gamma)$$

$$\approx \frac{2L}{c}\left(\frac{v^2}{2c^2}\right)$$

$$= \frac{Lv^2}{c^3}$$

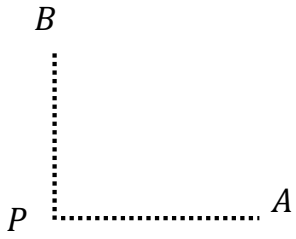
$$T_1 - T_2 = \frac{Lv^2}{c^3}$$

But this assumes PA is in direction of Earth's motion.

- How could we know this?

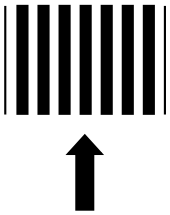
Suppose we rotate interferometer by 90°

From:

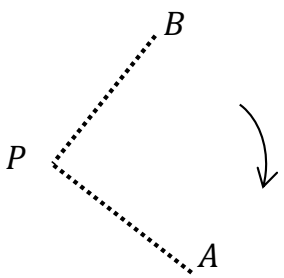


Signal *PAP* is *slower* than signal *PBP* ($T_1 > T_2$) by the *maximum amount*: $T_1 - T_2 = Lv^2/c^3$.

What we see at C:

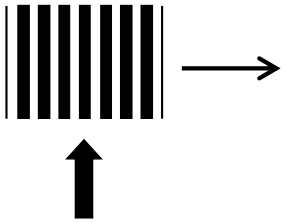


Pattern width given by: $c(T_1 - T_2) = Lv^2/c^2$.



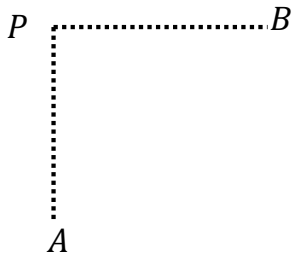
As interferometer rotates, difference in times decreases to zero and then starts to increase, now with signal *PAP faster* than signal *PBP*.

What we see at C:



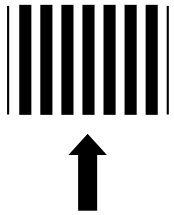
As interferometer is rotated, fringes shift past marker and width of pattern decreases (until $T_1 - T_2 = 0$) and then starts to increase again.

To:



When *PB* arm is aligned with Earth's motion, signal *PAP* is *faster* than signal *PBP* ($T_1 < T_2$) by the *maximum amount*: $T_1 - T_2 = -Lv^2/c^3$.

What we see at C:



Pattern width given by: $c(T_1 - T_2) = -Lv^2/c^2$.

- Any more rotation will cause width to start decreasing again.
- *Some number of fringes have shifted past marker.*

- Maximum difference in *times of travel* is $\frac{Lv^2}{c^3} - \left(-\frac{Lv^2}{c^3}\right) = \frac{2Lv^2}{c^3}$

- <u>So</u>: Maximum difference in *path lengths* is $c \times \frac{2Lv^2}{c^3} = \frac{2Lv^2}{c^2}$

- Measure it by counting the number of fringes that shift past the marker as the pattern's width changes from it's maximum width down to zero and back to its maximum width again, and then multiplying the result by the width of a single fringe (*i.e.*, by the wavelength of the light signals).

$$\left[\begin{array}{c}\#\ of\ shifted \\ fringes\end{array}\right] \times \left[\begin{array}{c}width\ of\ a \\ single\ fringe\end{array}\right] = \frac{2Lv^2}{c^2}$$

what we look for at C

given by wavelength of light signals

- We know L and c, so we can determine v if we can observe a fringe shift!

Michelson & Morely saw no such shift!

Possible Explanations for $T_1 = T_2$

1. *The Earth is at rest in the aether*.

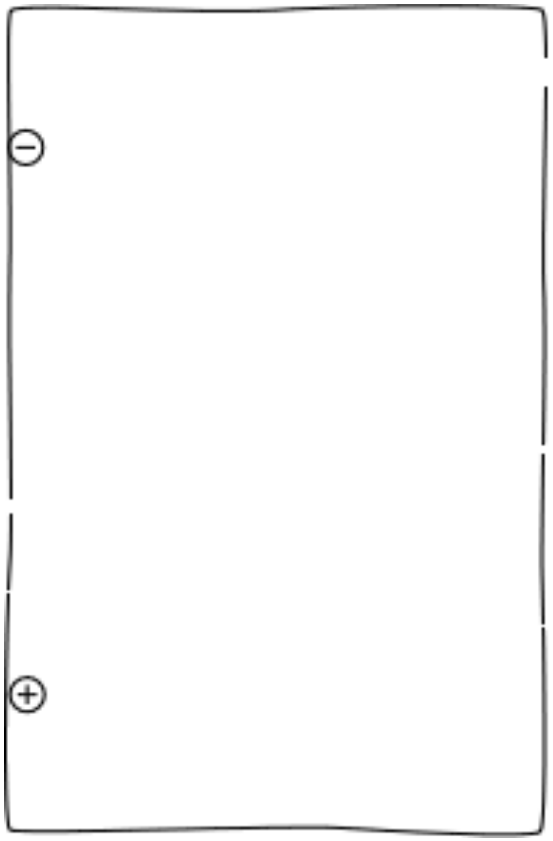
- *But*: We *know* the Earth is moving with respect to the Sun.

2. *Aether drag*. The aether immediately surrounding the Earth is dragged along with it. (MM's conclusion.)

- *How this helps*: The speed of light is a constant c with respect to the aether in all directions. So if the aether is dragged along with the Earth, then $T_1 = T_2 = 2L/c$.
- *But*: Experimental disconfirmation.

3. *Emission Theories of Light.* The speed of light is a constant c with respect to its source, *not* to the aether.

- *How this helps*: The speed of light is thus a constant c with respect to the Earth frame (the interferometer). So $T_1 = T_2 = 2L/c$.
- *But*: Experimental disconfirmation.



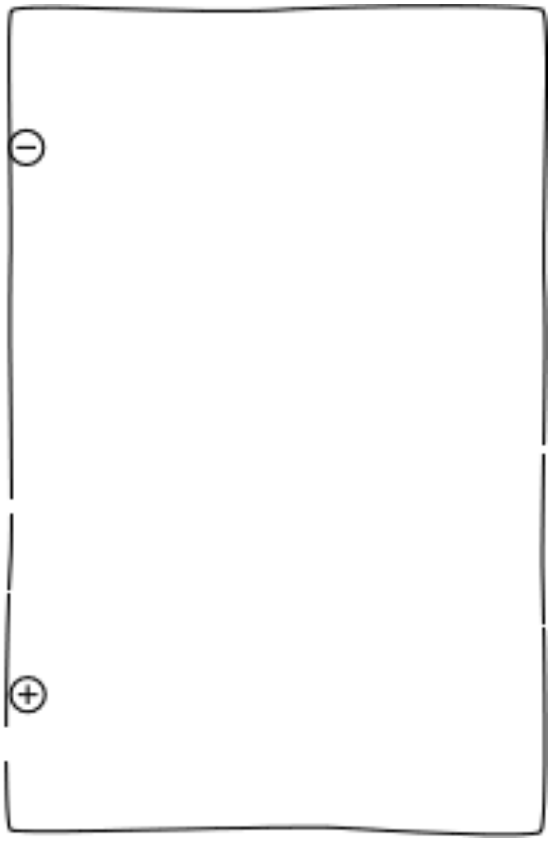


Image from <https://www.pitt.edu/~jdnorton/Goodies/Chasing_the_light/index.html>

4. *Lorentz-Fitzgerald Contraction*. Objects physically contract in the direction of their *motion through the aether*.

How this helps:

- *Recall*: $cT_1 = 2L\gamma^2$ and $cT_2 = 2L\gamma$.
- *So*: $T_1 = T_2 \Rightarrow 2L\gamma^2 = 2L\gamma$.
- *Now*: Suppose L is different for both paths:

  $cT_1 = 2L_{||}\gamma^2$ ($L_{||}$ = length of path parallel to motion)

  $cT_2 = 2L_{\perp}\gamma$ ($L_{\perp}$ = length of path perpendicular to motion)

- *Thus*: $L_{\perp} = L_{||}\gamma$.
- *So*: Arm of interferometer $L_{||}$ parallel to direction of motion is shorter than arm $L_{\perp}$ perpendicular to direction of motion (recall $\gamma > 1$).

5. *Principle of Relativity and Light Postulate*.

- *How this helps*: Recall consequence of light postulate: The speed of light is the same in all inertial reference frames.
- *Thus*: In the Earth frame, $T_1 = T_2 = 2L/c$.