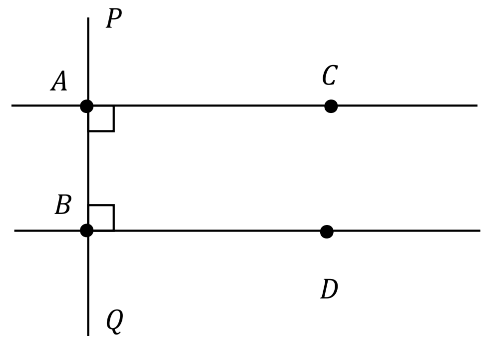
Assignment #8. Non-Euclidean Geometry

1. Consider a geometry in which Euclid's 5th postulate is replaced by:
   Through any point NO line can be drawn parallel to a given line.

   Show that there is at least one triangle in this geometry whose angles sum to more than two right angles.
   Hint: On a line *PQ* select two points *A* and *B*. Construct lines *AC* and *BD* perpendicular to *PQ*. What happens if *AC* and *BD* are extended in both directions?



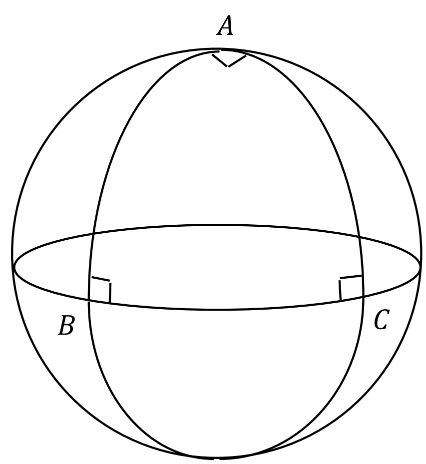
2. In a Euclidean space, what is
   (a) the sum of the angles of any triangle;
   (b) the circumference of a circle with radius 10,000*km*;
   (c) the area of a right triangle if the lengths of the sides enclosing the right angle are both 10,000*km*?

3. The geometry of #1 above is, suitably treated, the geometry of the surface of a sphere. The Earth is, to good approximation, a sphere of circumference 40,000*km*.

   (a) On this sphere, what is the sum of the angles of a triangle all of whose sides are 10,000*km*? (An example of such a triangle is shown as triangle *ABC*. It has one vertex at the North Pole and extends down to the equator.)

   (b) What is the circumference of a circle of radius 10,000*km* in this surface?

   (c) The triangle *ABC* is a right triangle all of whose sides are 10,000*km* long. What is its area? (*Hint*: The area of the Earth is 509,300,000km^2.)



4. If you had before you a two dimensional surface of constant curvature, how could you determine whether the curvature was positive, negative or zero by measuring:
   (a) the sum of angles of a triangle;
   (b) the circumference of a circle of known radius?

5. How could you check whether our three dimensional space has a positive, negative or zero curvature by measuring:

   (a) the sum of angles of a triangle;
   (b) the surface area of a sphere of known radius?