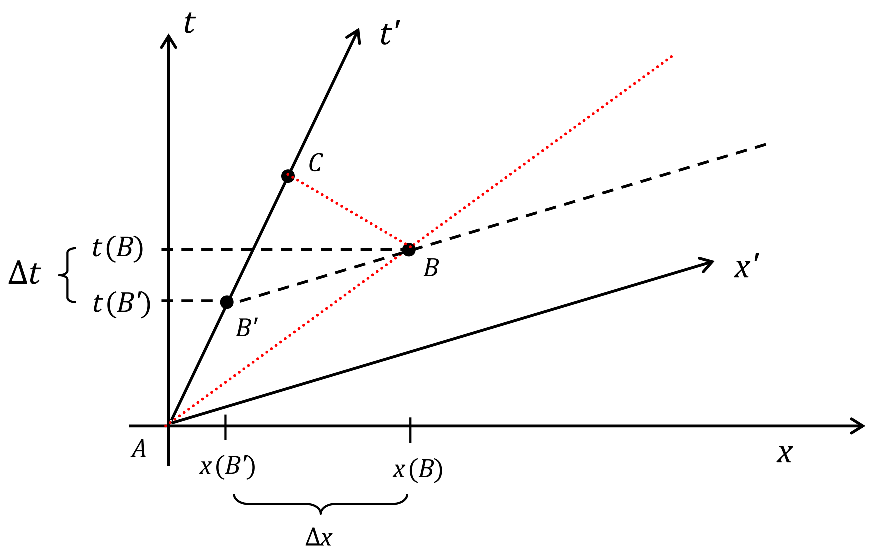
Assignment #3: Simultaneity Shift

Set-Up

If B' and B are simultaneous with respect to S', what is the time difference between them with respect to S? Consider the following spacetime diagram.



Physically: S' moves at speed v with respect to S. S' emits a light signal at A, which is reflected at B and received by S' at C.

Given the standard definition of simultaneity, the time according to S' from emission to reflection equals the time from reflection to final reception:

$$AB' = B'C$$

Note that $B'B$ is a simultaneity line for S'. It's slope $\Delta t/\Delta x$ in S-coordinates will give the change in time Δt with respect to S between the two simultaneous events B' and B in S', provided we know Δx, the distance in S-coordinates between B' and B. The change in time Δt is called the *simultaneity shift* for S. In this assignment, you will derive an expression for this shift and apply it to a particular problem.

Given:

Slope of $B'B$ in S-coordinates $= \dfrac{\Delta t}{\Delta x} = \dfrac{t(B) - t(B')}{x(B) - x(B')}$

From the geometry of the diagram, we have:

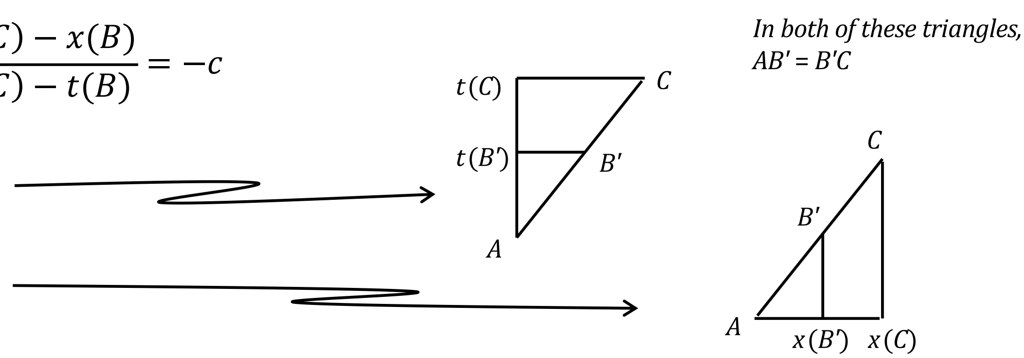
(1) $$\frac{1}{slope\ AB} = \frac{x(B)}{t(B)} = c$$

(2) $$\frac{1}{slope\ AC} = \frac{x(C)}{t(C)} = v$$

(3) $$\frac{1}{slope\ BC} = \frac{x(C) - x(B)}{t(C) - t(B)} = -c$$

(4) $t(B') = \frac{1}{2}t(C)$

(5) $x(B') = \frac{1}{2}x(C)$



1. Now use (1)-(5) to solve for $x(B)$, $x(B')$, $t(B')$ in terms of $t(B)$:

   (a) From (1): $x(B) = ?$

   (b) From (2): $x(C) = ?$

   (c) Substitute your expressions for $x(B)$ and $x(C)$ into (3) and solve for $t(C)$ in terms of $t(B)$.

   (d) Substitute your expression for $t(C)$ into (4) and solve for $t(B')$ in terms of $t(B)$.

   (e) Substitute your expression for $t(C)$ into (2) and solve for $x(C)$ in terms of $t(B)$.

   (f) Substitute this expression for $x(C)$ into (5) and solve for $x(B')$ in terms of $t(B)$.

2. You now have expressions for $x(B)$, $x(B')$, and $t(B')$ in terms of $t(B)$. Substitute these into the expression for the slope of $B'B$ and solve for Δt in terms of Δx. This is the simultaneity shift.

3. Alfred is traveling at $v = 0.9c$ with respect to Batman. Acccording to Alfred, two events B' and B are simultaneous. According to Batman, if these events are separated by a distance of $\Delta x = 10$ *meters*, how much time Δt elapses between them? (*Recall*: $c = 3\times10^8\ m/s$)