14. Quantum Particles: Identity and Individuality

1. Two Views on Individuals

1. Haecceitism ("hak-SEE-uh-tism")

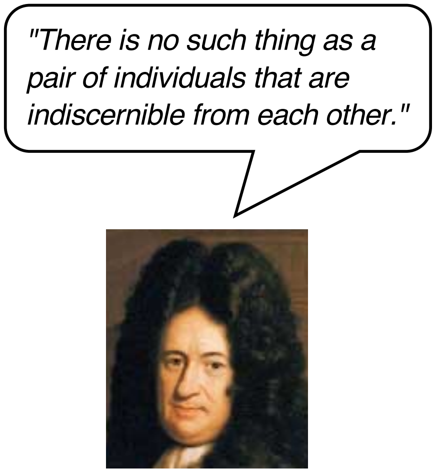
- Every individual possesses a "primitive thisness" (haecceity) that makes it unique.
  - Self-identity formulation: *Every individual is identical to itself*.

2. Bundle View

- An individual = a bundle of properties.
  - *So*: Properties individuate objects. No two individuals can have all the same properties.
- *Motivation*:

> *Principle of the Identity of Indiscernibles*
> If two objects are *indiscernible*, then they are identical.

- *What about quantum objects?*
- *Can they be considered individuals?*



Gottfried Wilhelm Leibniz
(1646-1716)

2. Fermions and Bosons

- *Consider first*: electrons.

Electron properties:	*possible values*:
- energy	$n = 1, 2, \ldots$
- orbital angular momentum	$\ell = 0, 1, 2, \ldots (n-1)$
- z-component of orbital angular momentum	$m_\ell = -\ell, \ldots 0, \ldots, \ell$
- spin	$m_s = -½, +½$

- *So*: The state of an electron is characterized by four values (n, ℓ, m_ℓ, m_s).

Pauli Exclusion Principle (1925)

No two electrons can be in the same state; *i.e.*, no two electrons can have all the same values of (n, ℓ, m_ℓ, m_s).

Wolfgang Pauli
(1900-1958)

		n:	1	2		3			4			
Z	*Element*	ℓ:	0	0	1	0	1	2	0	1	2	3
1	H hydrogen		1									
2	He helium		2									
3	Li lithium		2	1								
4	Be beryllium		2	2								
5	B boron		2	2	1							
6	C carbon		2	2	2							
7	N nitrogen		2	2	3							
8	O oxygen		2	2	4							
9	F fluorine		2	2	5							
10	Ne neon		2	2	6							

Energy shells

K shell ($n = 1$)

L shell ($n = 2$)

M shell ($n = 3$)

N shell ($n = 4$)

etc.

Orbitals

s orbital ($\ell = 0$)

p orbital ($\ell = 1$)

d orbital ($\ell = 2$)

f orbital ($\ell = 3$)

etc.

Ex: The 3 electrons in a lithium atom are characterized by: $(1, 0, 0, +½)$, $(1, 0, 0, -½)$, $(2, 0, 0, +½)$.

- *Recall*: Electrons possess spin-½ properties (*Hardness*, *Color*, *etc*.).
  - Each is defined with respect to a spatial axis.
  - Each is two-valued, with values represented by $m_s = +½, -½$.

There are other more complex multi-valued spin properties.

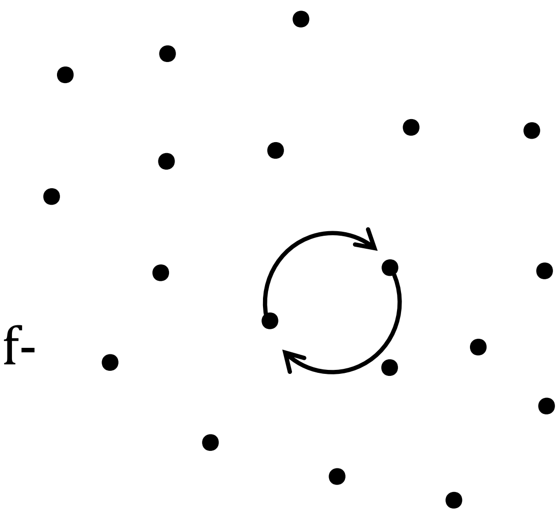
Spin Properties

- *Two basic types*: "half-integer-spin" and "integer-spin".

Property	*number of m_s values*	*m_s values*
Spin-$\frac{1}{2}$	two	$-\frac{1}{2}, +\frac{1}{2}$
Spin-$\frac{3}{2}$	four	$-\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$
Spin-$\frac{5}{2}$	six	$-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}, +\frac{5}{2}$
⋮		
Spin-0	one	0
Spin-1	three	$-1, 0, +1$
Spin-2	five	$-2, -1, 0, +1, +2$
⋮		

- *Theoretically (Standard Model)*:
  - Matter consists of spin-½ particles (leptons, quarks).
  - Forces (EM, strong, weak) consist of spin-1 particles (γ, g, $W^{\pm}$, Z).
  - Mass is mediated by a spin-0 particle (Higgs).
- *Not as successful*: Gravitational force is mediated by spin-2 particle (graviton).

Spin and Statistics



- *Statistics*: describes how a multi-particle system behaves under single-particle exchanges.

- *Experimentally*:
  - A multi-particle system cannot be made up of both half-integer-spin and integer-spin particles.
  - Half-integer-spin multi-particle states and integer-spin multi-particle states are *Permutation Invariant*.
  - Half-integer-spin multi-particle states obey the *Exclusion Principle*, integer-spin multi-particle states do not.

Permutation Invariance: Exchanging single-particle states in a multi-particle state results in a state that is physically indistinguishable from the original state.

Exclusion Principle: Two or more identical single-particle states cannot appear in the same multi-particle state.

<u>Fermi-Dirac (FD) Statistics: Group Rules for Half-Integer-Spin Particles</u>

- Multi-particle states are *Permutation Invariant*.
- Two or more particles with same (non-spatiotemporal) properties *cannot* be in the same state. (*Exclusion Principle*)

<u>Bose-Einstein (BE) Statistics: Group Rules for Integer-Spin Particles</u>

- Multi-particle states are *Permutation Invariant*.
- Two or more particles with same (non-spatiotemporal) properties *can* be in the same state. (*No Exclusion Principle*)

Def. A *fermion* is a particle that obeys FD Statistics.

A *boson* is a particle that obeys BE Statistics.

- *<u>So (Standard Model)</u>*:
  - Matter consists of spin-½ particles = fermions (leptons, quarks).
  - Forces (EM, strong, weak) consist of spin-1 particles = bosons (γ, g, $W^{\pm}$, Z).
  - Mass is mediated by a spin-0 particle = boson (Higgs).

How to encode Permutation Invariance

- Let $|\Phi\rangle$ represent a multi-particle state.
- Let $|\Phi'\rangle$ be obtained from $|\Phi\rangle$ by exchanging any two of its single-particle substates.

> $|\Phi\rangle$ is *permutation invariant* just when $\langle\Phi|A|\Phi\rangle = \langle\Phi'|A|\Phi'\rangle$ for any operator A representing an observable quantity.

Two ways to guarantee this:

$|\Phi'\rangle = |\Phi\rangle$ (*symmetric under permutations*)

$|\Phi'\rangle = -|\Phi\rangle$ (*anti-symmetric under permutations*)

Examples:

(a) $|\Phi_s\rangle = \sqrt{1/2}\,\{|\phi\rangle_1|\psi\rangle_2 + |\psi\rangle_1|\phi\rangle_2\}$ *symmetric*

(b) $|\Phi_{as}\rangle = \sqrt{1/2}\,\{|\phi\rangle_1|\psi\rangle_2 - |\psi\rangle_1|\phi\rangle_2\}$ *anti-symmetric*

(c) $|\Phi_{ns}\rangle = \sqrt{1/2}\,|\phi\rangle_1|\phi\rangle_2 + \sqrt{1/4}\{|\phi\rangle_1|\psi\rangle_2 - |\psi\rangle_1|\phi\rangle_2\}$ *non-symmetric*

(a) and (b) are permutation invariant

How to encode the Exclusion Principle

(a) $|\Phi_s\rangle = \sqrt{1/2}\,\{|\phi\rangle_1|\psi\rangle_2 + |\psi\rangle_1|\phi\rangle_2\}$ *symmetric*

(b) $|\Phi_{as}\rangle = \sqrt{1/2}\,\{|\phi\rangle_1|\psi\rangle_2 - |\psi\rangle_1|\phi\rangle_2\}$ *anti-symmetric*

(c) $|\Phi_{ns}\rangle = \sqrt{1/2}\,|\phi\rangle_1|\phi\rangle_2 + \sqrt{1/4}\{|\phi\rangle_1|\psi\rangle_2 - |\psi\rangle_1|\phi\rangle_2\}$ *non-symmetric*

- Suppose we allow particles 1 and 2 to be in identical states.
  - *Let* $\psi = \phi$ *in* (a)-(c).
- *Then*: The anti-symmetric multi-particle vector vanishes! The others don't.
- *Suggests*: Use anti-symmetric vectors to represent the states of a multi-particle system that is both Permutation Invariant and obeys the Exclusion Principle.

Bosonic 2-particle states:

$\sqrt{1/2}\{|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle\},\ \ |\phi\rangle|\phi\rangle,\ \ |\psi\rangle|\psi\rangle$ Symmetric under permutations

Fermionic 2-particle state:

$\sqrt{1/2}\{|\phi\rangle|\psi\rangle - |\psi\rangle|\phi\rangle\}$ Anti-symmetric under permutations

3. Classical *vs*. Quantum Statistics

Fermi-Dirac (FD) Statistics: Group Rules for Half-Integer-Spin Particles

- Multi-particle states are *Permutation Invariant*.
- Two or more particles with same (non-spatiotemporal) properties *cannot* be in the same state. (*Generalized Exclusion Principle*).

Bose-Einstein (BE) Statistics: Group Rules for Integer-Spin Particles

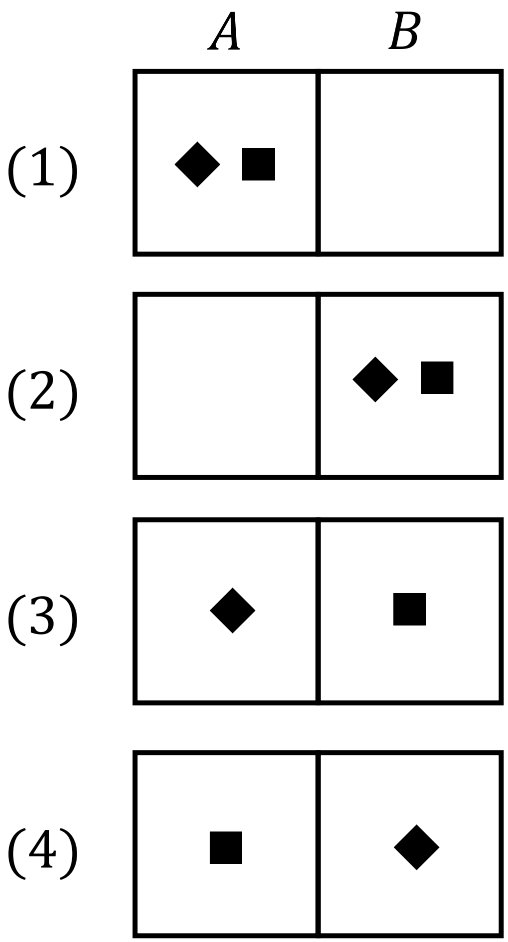
- Multi-particle states are *Permutation Invariant*.
- Two or more particles with same (non-spatiotemporal) properties *can* be in the same state. (*No Exclusion Principle*.)

Maxwell-Boltzman (MB) Statistics: Group Rule for Classical Particles

- Multi-particle states are not *Permutation Invariant*.
- Two or more particles with same (non-spatiotemporal) properties *can* be in the same state. (*No Exclusion Principle*.)

Suppose: We have two particles in a 2-particle state composed of two single-particle states A, B. How can we calculate the probability that one of the particles is in state A and the other is in B?

Case 1. Classical particles

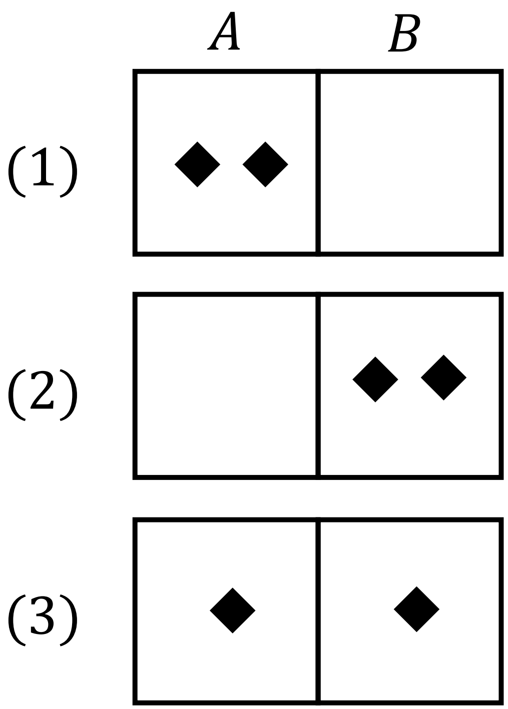


- Use MB statistics: There are 4 possible 2-particle states. (4 possible ways to distribute two classical particles over two states.)
- Assign each of these possible 2-particle states equal probability of 1/4 (Principle of Indifference).

$$\Pr(\textit{one particle in } A \textit{ and one particle in } B)$$
$$= \Pr(\textit{state } 3) + \Pr(\textit{state } 4) = 1/4 + 1/4 = 1/2$$

Suppose: We have two particles in a 2-particle state composed of two single-particle states A, B. How can we calculate the probability that one of the particles is in state A and the other is in B?

Case 2. Bosons

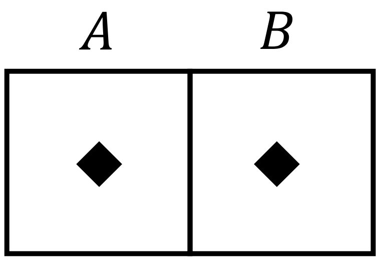


- Use BE statistics: There are 3 possible 2-particle states. (3 possible ways to distribute two bosons over two states.)
- Assign each of these possible 2-particle states equal probability of 1/3 (Principle of Indifference).

$$\Pr(\textit{one particle in } A \textit{ and one particle in } B)$$
$$= \Pr(\textit{state } 3) = 1/3$$

Suppose: We have two particles in a 2-particle state composed of two single-particle states A, B. How can we calculate the probability that one of the particles is in state A and the other is in B?

Case 3. Fermions

(1) 

- Use FD statistics: There is only one possible 2-particle states (due to the Exclusion Principle).

$$\Pr(\textit{one particle in A and one particle in B})$$
$$= \Pr(\textit{state}\ 1) = 1$$

4. Quantum Individuals

Question: What does Permutation Invariance of quantum states say about the status of quantum particles as individuals?

Initial Response

- Classical particles are individuals: switching two of them makes a difference.
- Quantum particles are not individuals: switching two of them does not make a difference.

But: Permutation Invariance applies to states.

- *It is a constraint on the possible states that a given multi-particle system can be in.*
- *The individuality of an object need not depend on constraints placed on the possible states it can be in.*

Let's consider two approaches to viewing quantum particles as individuals...

1. Haecceitism

Claim: Classical and quantum particles possess "primitive thisness" (haecceity).

The Main Difference:

- Classical haecceities are physically distinguishable: you can tell them apart based on the states they occupy.
- Quantum haecceities are physically indistinguishable: no experiment can distinguish between two quantum haecceities.

So: Quantum particles might be considered individuals...

...but at the cost of accepting the strange metaphysical notion of haecceity.

Does the bundle view offer a better option?

2. Bundle View

Claim: Classical and quantum particles consist of bundles of properties.

Question: Under the Bundle View, are classical and quantum particles individuals?

Do they satisfy the Principle of the Identity of Indiscernibles?

PII: If two objects are *indiscernible*, then they are identical.

- *Bundle view says*: Two objects are *indiscernible* when they have all the same properties.

Two types of properties

1. A **monadic property** = a "single-place" property that an object can possess without reference to other objects. (*Ex*. mass.)
2. A **relational property** = a "multi-place" property that an object can only possess with respect to one or more other obejcts. (*Ex.* Being taller than.)

This suggests that there are (at least) three versions of the PII...

Three versions of PII

PII.v1: If two objects agree on *all* monadic and relational properties, then they are identical.

PII.v2: If two objects agree on all monadic and relational properties, *excluding spatiotemporal properties*, then they are identical.

PII.v3: If two objects agree on all monadic properties, then they are identical.

Claims:

(a) Classical particles can violate *PII*.v2 and *PII*.v3, but must satisfy *PII*.v1.

They can always be discerned by their positions in space (and time).

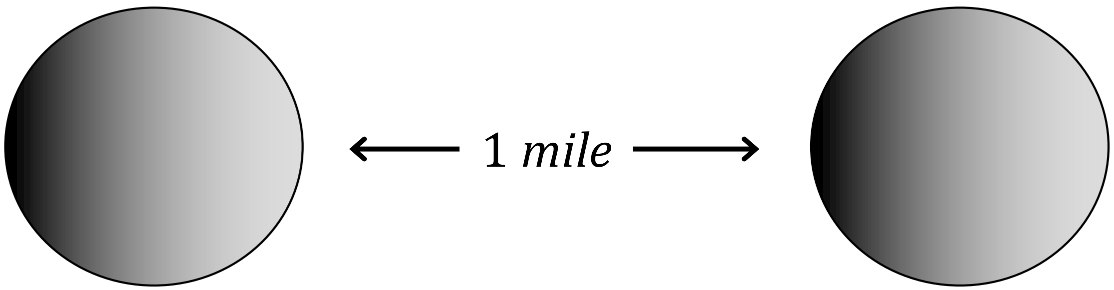
(b) Quantum particles can violate *PII*.v2, *PII*.v3, and *PII*.v1.

Claim: Two bosons/fermions in a symmetric/anti-symmetric state possess all the same monadic and relational properties.

So: Either quantum particles are not individuals, or they must possess haecceities.

But: Do these versions of the *PII* exhaust all the possible ways two objects can be discerned from each other?

Potential counterexample:
2 identical iron spheres one mile apart in an otherwise empty universe.



Spheres agree on all monadic and relational properties!

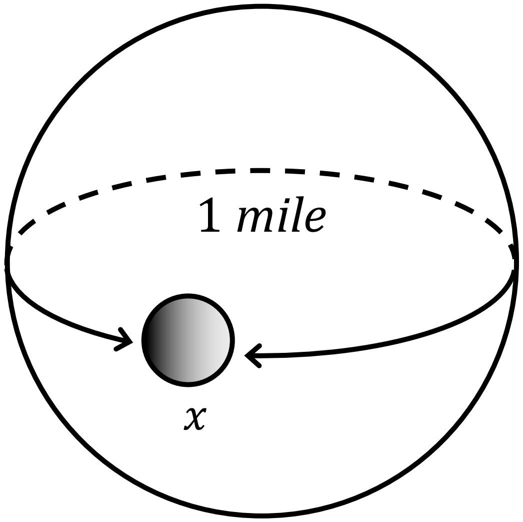
What about spatiotemporal ones?

Relationalism: Spatiotemporal properties are relations between an object and other objects.

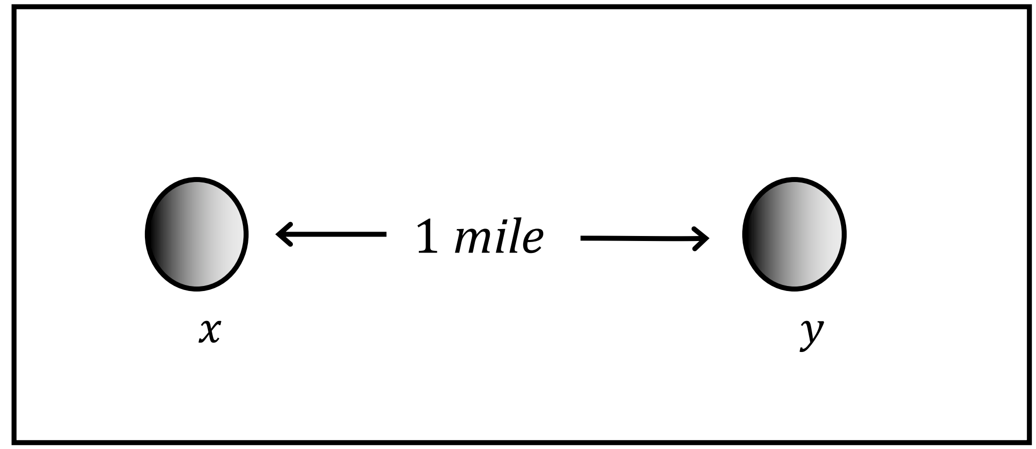
Spheres agree on these: They both stand in the spatial relation of being 1 mile from the other.

Substantivalism: Spatiotemporal properties are relations between an object and absolute spacetime.

Whether or not spheres are identical depends on global topology of spacetime!



<u>Closed global topology</u>: spheres agree on spatial location (so PII entails there's really just one of them).



<u>Open global topology</u>: spheres disagree on spatial locations (so PII entails there really are two of them).

- *<u>Suppose we are relationalists with respect to spacetime</u>*
  Then the spheres violate *PII*.v1, *PII*.v2, and *PII*.v3.
- *<u>But</u>*: They are distinct: there are *two* of them.

The spheres are individuated "solo numero" (by number alone)

How to individuate objects solo numero

See if there is an appropriate irreflexive 2-place relational property...

An **irreflexive 2-place relational property** = a 2-place relation that relates one object with the other, but does not relate either of the objects with itself. (*Ex*: "Being 1 mile apart from.")

Three types of discernibility

(a) *Absolute discernibility*: Differing by a monadic property.

(b) *Relative discernibility*: Differing by a relational property.

(c) *Weak discernibility*: Differing by an irreflexive relational property.

Potential examples of weakly discernible objects

- The two iron spheres.
- The points in a Euclidean space.
- Right and left hands in an empty universe.
- Two anti-correlated fermions!

Claim 1: Fermions are weakly discernible.

- Two spin-½ fermions with the same properties can be in the spin "singlet" state:

$$|\Phi^-\rangle = \sqrt{½}\{|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\}$$

- In $|\Phi^-\rangle$, there is an irreflexive relation that holds between them:

  "Having opposite direction of spin to"

- *Thus*: The fermions are weakly discernible; so they satisfy the PII.

So: Under the Bundle View, fermions can be considered individuals.

Claim 2: Bosons can be in states in which they are not discernible, even weakly.

- *Ex*: $|\phi\rangle_1|\phi\rangle_2$, where both bosons have all the same properties.
- *So*: Bosons do *not* satisfy the PII.

Thus: Under the Bundle View, bosons cannot be considered individuals.

Problems:

1. There are spin-½ fermion states in which the fermions are not discernible, even weakly.

   Ex: The Bell state $|\Psi^-\rangle = \sqrt{½}\{|\uparrow\rangle_1|\uparrow\rangle_2 - |\downarrow\rangle_1|\downarrow\rangle_2\}$

   Anti-symmetric (fermionic) state in which subsystems are correlated.
   - So: No irreflexive relation.

2. There are bipartite boson states in which the bosons are weakly discernible.

   Ex: The Bell state $|\Phi^+\rangle = \sqrt{½}\{|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2\}$

   Symmetric (bosonic) state in which subsystems are anti-correlated.
   - So: Characterized by irreflexive relation "having opposite direction of spin to".

- *Potential response to #1*:
  - For a composite 2-particle spin-½ system, the composite spin properties are represented by the total spin operator $S^2 = (\mathbf{S}_1 + \mathbf{S}_2)\cdot(\mathbf{S}_1 + \mathbf{S}_2)$ and the total spin-along-the-z-axis operator $S_z = S_{z_1} + S_{z_2}$.
  - The *only* anti-symmetric eigenvector of S^2 and S_z is $|\Phi^-\rangle$, and not $|\Psi^-\rangle$.
  - *So*: There is only one 2-particle spin-½ fermion (anti-symmetric) state with definite values of spin, and it is one in which the fermions are weakly discernible (anti-correlated).