13. Decoherence

- Classical probabilities are based on *classical* (Boolean) logic.
- The probabilities defined by the Born Rule are based on *quantum* (non-Boolean) logic.
- *One consequence*: QM probabilities do not satisfy the classical *Or-Addition* Rule.

1. Classical Probabilities and the Classical Or-Addition Rule

A **classical probability theory** is a triple $(\Omega, \mathcal{F}, \mathrm{Pr}_C)$:
- Ω is a set of *simple events* (the *sample space*).
- $\mathcal{F}$ is a set of *compound events* obtained by taking all combinations of simple events using set complement and union.
- Pr_C is a *probability function* that maps elements of $\mathcal{F}$ to [0, 1] and satisfies the following axioms:

(C1) $\mathrm{Pr}_C(\emptyset) = 0$

(C2) $\mathrm{Pr}_C(\neg A) = 1 - \mathrm{Pr}_C(A)$

(C3) $\mathrm{Pr}_C(A \cup A') = \mathrm{Pr}_C(A) + \mathrm{Pr}_C(A') - \mathrm{Pr}_C(A \cap A')$

The Classical Or-Addition Rule

Example:

- Let $\Omega = \{1, 2, 3, 4, 5, 6\}$ represent all possible results of a single roll of a die.
- $\mathcal{F} = \{\{1\}, \{2\}, ..., \{1\} \cup \{2\}, \{1\} \cup \{3\}, ..., \neg\{1\}, \neg\{2\}, ...\ \}$
- Let $\mathrm{Pr}_C(\{i\}) = 1/6$, for $i = 1...6$. (*Principle of Indifference*)
- *Then*: The probability of getting either 1 or 3 on a single roll is given by:

$$\begin{aligned}\mathrm{Pr}_C(\{1\} \cup \{3\}) &= \mathrm{Pr}_C(\{1\}) + \mathrm{Pr}_C(\{3\}) - \mathrm{Pr}_C(\{1\} \cap \{3\}) && \text{(C3)}\\ &= 1/6 + 1/6 - 0 = 1/3\end{aligned}$$

- *And*: The probability of getting either a value in the range $\{1, 2, 3\}$ or a value in the range $\{3, 4, 5\}$ on a single roll is:

$$\begin{aligned}&\mathrm{Pr}_C(\{1, 2, 3\} \cup \{3, 4, 5\})\\ &\quad = \mathrm{Pr}_C(\{1, 2, 3\}) + \mathrm{Pr}_C(\{3, 4, 5\}) - \mathrm{Pr}_C(\{1, 2, 3\} \cap \{3, 4, 5\}) && \text{(C3)}\\ &\quad = [\mathrm{Pr}_C(\{1\}) + \mathrm{Pr}_C(\{2\}) + \mathrm{Pr}_C(\{3\})] + [\mathrm{Pr}_C(\{3\}) + \mathrm{Pr}_C(\{4\}) + \mathrm{Pr}_C(\{5\})] - \mathrm{Pr}_C(\{3\})\\ &\quad = [1/6 + 1/6 + 1/6] + [1/6 + 1/6 + 1/6] - 1/6 = 5/6\end{aligned}$$

2. Quantum Probabilities and Interference

- Replace the classical sample space Ω with a Hilbert space $\mathcal{H}$.

A **quantum probability theory** is a triple $(\mathcal{H}, \mathcal{L}, \mathrm{Pr}_Q)$:
- $\mathcal{H}$ is a Hilbert space of states (*simple events*).
- $\mathcal{L}$ is the collection of subspaces of $\mathcal{H}$ (*compound events*) obtained by taking all combinations of simple events using orthocomplement and linear span.
- Pr_Q is defined by $\mathrm{Pr}_Q(|a\rangle, |\psi\rangle) = |\langle a|\psi\rangle|^2$, for any $|a\rangle, |\psi\rangle \in \mathcal{H}$.

- *Main Result*: Quantum probabilities, so-defined, do not in general satisfy C3!
- They do satisfy the following (where V, W are subspaces of $\mathcal{H}$ and $\mathbf{0}$ is the "zero" subspace):

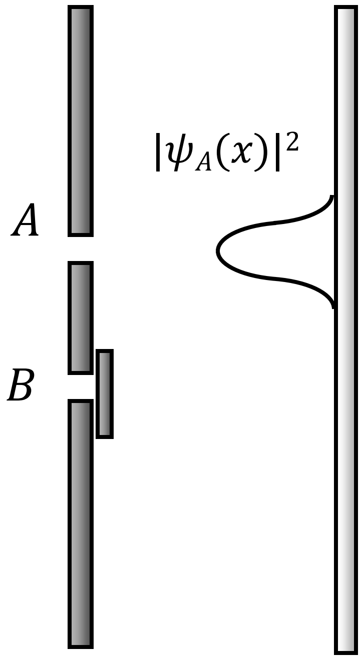
(Q1) $\mathrm{Pr}_Q(\mathbf{0}) = 0$

(Q2) $\mathrm{Pr}_Q(V^\perp) = 1 - \mathrm{Pr}_Q(V)$

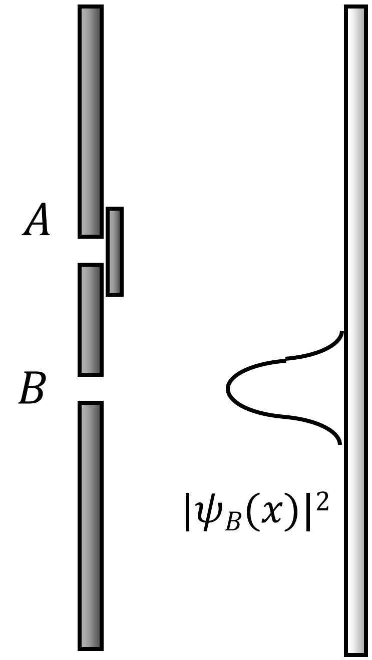
(Q3) $\mathrm{Pr}_Q(V \oplus W) = \mathrm{Pr}_Q(V) + \mathrm{Pr}_Q(W)$, when $V \perp W$

- *Recall*: Linear span $\oplus$ does *not* correspond to classical "or".

<u>*Example*</u>: 2-slit probabilities and interference



A-distribution



B-distribution

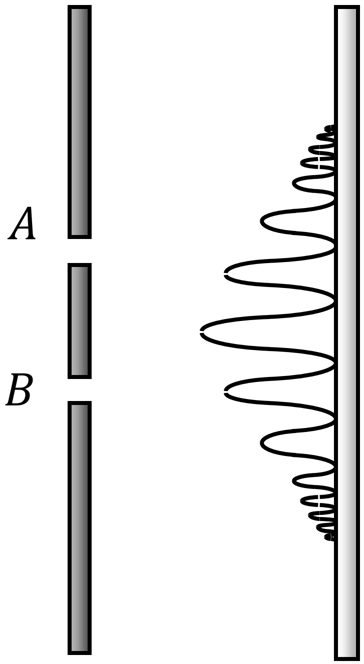
With Slit A open,

$\Pr_Q(e \textit{ is at } x \textit{ in state } \psi_A(x)) = |\psi_A(x)|^2$

With Slit B open,

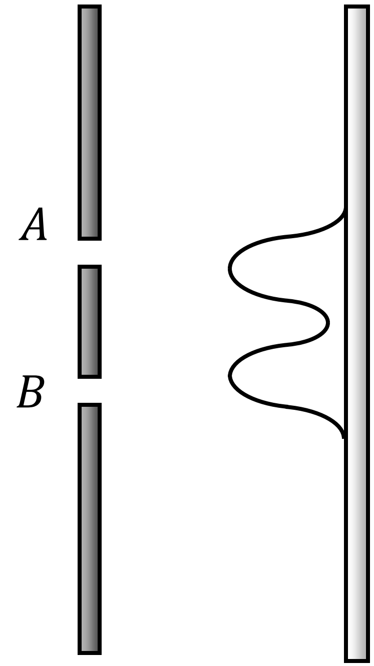
$\Pr_Q(e \textit{ is at } x \textit{ in state } \psi_B(x)) = |\psi_B(x)|^2$

Example: 2-slit probabilities and interference



$|\psi_A(x) + \psi_B(x)|^2$

Interference distribution (what happens)



$|\psi_A(x)|^2 + |\psi_B(x)|^2$

A or B distribution (what doesn't happen)

- With both slits open, the probability that e is located at x is $|\psi_A(x) + \psi_B(x)|^2$.

> - *The state corresponding to the prob distribution* $|\psi_A(x) + \psi_B(x)|^2$ *is* $|\psi_A(x)\rangle + |\psi_B(x)\rangle$.
> - *This is in the subspace* $V \oplus W$ *which is the linear span of the subspace* V *containing the state* $\psi_A(x)$ *and the subspace* W *containing the state* $\psi_B(x)$.

- This is *not* equal to $|\psi_A(x)|^2 + |\psi_B(x)|^2$, which, according to (C3), represents the probability that the electron *either* went through slit *A or* slit *B*.

Let's see how this works using projection operators...

- ***Recall***: The projection operator $P_{|a_i\rangle} = |a_i\rangle\langle a_i|$ corresponds to the 1-dim subspace defined by $|a_i\rangle$ (*i.e.*, the ray in which $|a_i\rangle$ is pointing).

- ***And***: $\sum_i P_{|a_i\rangle} = 1$

Def. Suppose Q is a linear operator on an N-dim vector space $\mathcal{H}$ with orthonormal basis $|b_1\rangle, \ldots |b_N\rangle$. Then the *trace* $\mathrm{Tr}(Q)$ of Q is given by:

$$\mathrm{Tr}(Q) \equiv \sum_{i=1}^{N} \langle b_i|Q|b_i\rangle = \langle b_1|Q|b_1\rangle + \langle b_2|Q|b_2\rangle + \cdots + \langle b_N|Q|b_N\rangle$$

- $\mathrm{Tr}(Q) =$ *the sum of the diagonal elements of any matrix representation of* Q.
- ***Fact***: All such representations have this sum in common: The trace is independent of the basis it's calculated in.
- Properties of the trace:

$\mathrm{Tr}(\lambda A) = \lambda \mathrm{Tr}(A)$, where λ is any number

$\mathrm{Tr}(A + B) = \mathrm{Tr}(A) + \mathrm{Tr}(B)$

$\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$

The Born Rule in terms of projection operators:

$$\mathrm{Pr}_Q(\textit{value of } A \textit{ is } a_i \textit{ in state } |\psi\rangle) = |\langle a_i|\psi\rangle|^2$$

$$= \langle\psi|a_i\rangle\langle a_i|\psi\rangle$$

$$= \sum_j \langle\psi|P_{|a_j\rangle}|a_i\rangle\langle a_i|\psi\rangle \qquad \text{where } \sum_j P_{|a_j\rangle} = 1$$

$$= \sum_j \langle\psi|a_j\rangle\langle a_j|a_i\rangle\langle a_i|\psi\rangle$$

$$= \sum_j \langle a_j|a_i\rangle\langle a_i|\psi\rangle\langle\psi|a_j\rangle$$

$$= \mathrm{Tr}(|a_i\rangle\langle a_i|\psi\rangle\langle\psi|)$$

$$= \mathrm{Tr}(P_{|a_i\rangle}P_{|\psi\rangle})$$

- $P_{|a_i\rangle}$ is the projection operator corresponding to the state $|a_i\rangle$ (more precisely, the 1-dim subspace defined by $|a_i\rangle$).
- $P_{|\psi\rangle}$ is the projection operator corresponding to the state $|\psi\rangle$ (more precisely, the 1-dim subspace defined by $|\psi\rangle$).
- The projection operator corresponding to a state is called the *statistical operator* (or *density matrix*) for the state.

Consider composite state of *Hardness* measuring device and *black* electron:

$$|\psi\rangle = \sqrt{1/2}\,\{|'h'\rangle|h\rangle + |'s'\rangle|s\rangle\}$$

- Its statistical operator $P_{|\psi\rangle} = |\psi\rangle\langle\psi|$ is given by:

$$P_{|\psi\rangle} = 1/2\{|'h'\rangle|h\rangle + |'s'\rangle|s\rangle\}\{\langle'h'|\langle h| + \langle's'|\langle s|\}$$

$$= 1/2\{|'h'\rangle|h\rangle\langle'h'|\langle h| + |'s'\rangle|s\rangle\langle's'|\langle s| + |'h'\rangle|h\rangle\langle's'|\langle s| + |'s'\rangle|s\rangle\langle'h'|\langle h|\}$$

$$= 1/2\{|'h'\rangle\langle'h'| \otimes |h\rangle\langle h| + |'s'\rangle\langle's'| \otimes |s\rangle\langle s|$$

$$+ |'h'\rangle\langle's'| \otimes |h\rangle\langle s| + |'s'\rangle\langle'h'| \otimes |s\rangle\langle h|\}$$

$$= 1/2\{\underbrace{P_{|'h'\rangle} \otimes P_{|h\rangle}}_{\text{stat operator for } |'h'\rangle|h\rangle} + \underbrace{P_{|'s'\rangle} \otimes P_{|s\rangle}}_{\text{stat operator for } |'s'\rangle|s\rangle} + \underbrace{|'h'\rangle\langle's'| \otimes |h\rangle\langle s| + |'s'\rangle\langle'h'| \otimes |s\rangle\langle h|}_{\text{interference terms!}}\}$$

- <u>*So*</u>: $\Pr_Q(\textit{value of } A \textit{ is } a_i \textit{ in state } |\psi\rangle) = \mathrm{Tr}(P_{|a_i\rangle}P_{|\psi\rangle})$

$$= \mathrm{Tr}(1/2 P_{|a_i\rangle}P_{|'h'\rangle} \otimes P_{|h\rangle}) + \mathrm{Tr}(1/2 P_{|a_i\rangle}P_{|'s'\rangle} \otimes P_{|s\rangle})$$

$$+ \mathrm{Tr}(1/2 P_{|a_i\rangle}|'h'\rangle\langle's'| \otimes |h\rangle\langle s|) + \mathrm{Tr}(1/2 P_{|a_i\rangle}|'s'\rangle\langle'h'| \otimes |s\rangle\langle h|)$$

$$= \Pr_Q(\textit{value of } A \textit{ is } a_i \textit{ in state } |'h'\rangle|h\rangle) + \Pr_Q(\textit{value of } A \textit{ is } a_i \textit{ in state } |'s'\rangle|s\rangle)$$

$$+ \textit{ interference terms}$$

3. Decoherence Zeh (1970)

H. Dieter Zeh
(1932-2018)

- *Claim*: When an observer ends up in an entangled state with a measuring device, environmental interactions destroy interference effects and *decohere* the entangled state into one associated with a definite measurement outcome.
- Let $|hard\rangle_E$, $|soft\rangle_E$ be states of the environment E in which it's correlated with a hard electron and a soft electron, respectively.
- *Then*: It is experimentally impossible to distinguish between:

(1) The state $\sqrt{½}\,\{\,|'hard'\rangle_m|hard\rangle_e|hard\rangle_E + |'soft'\rangle_m|soft\rangle_e|soft\rangle_E\}$
(2) Either of the states: $|'hard'\rangle_m|hard\rangle_e|hard\rangle_E$ or $|'soft'\rangle_m|soft\rangle_e|soft\rangle_E$.

- *Recall*: To distinguish between (1) and (2), we would need a very complex multi-particle property that (1) possesses and that neither state in (2) possesses.
- Given that E realistically has a huge number of degrees of freedom, it is experimentally impossible to measure such a property.
- So (1) and (2) are *indistinguishable* for all practical purposes!

3. Decoherence   Zeh (1970)

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(1) The state $\sqrt{½}\,\{\,|'hard'\rangle_m|hard\rangle_e|hard\rangle_E + |'soft'\rangle_m|soft\rangle_e|soft\rangle_E\}$

(2) Either of the states: $|'hard'\rangle_m|hard\rangle_e|hard\rangle_E$ or $|'soft'\rangle_m|soft\rangle_e|soft\rangle_E$.

What this is supposed to mean:

- Whenever the post-measurement state of a composite system is of the form of (1), it *does*, for all practical purposes, describe a situation in which a definite measurement outcome occurred.
- The environment, *for all practical purposes*, collapses the entangled superposition.

3. Decoherence Zeh (1970)

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- Claim: When an observer ends up in an entangled state with a measuring device, environmental interactions destroy interference effects and *decohere* the entangled state into one associated with a definite measurement outcome.
- Let $|hard\rangle_E$, $|soft\rangle_E$ be states of the environment E in which it's correlated with a hard electron and a soft electron, respectively.
- Then: It is experimentally impossible to distinguish between:

(1) The state $\sqrt{½}\,\{\,|'hard'\rangle_m|hard\rangle_e|hard\rangle_E + |'soft'\rangle_m|soft\rangle_e|soft\rangle_E\}$
(2) Either of the states: $|'hard'\rangle_m|hard\rangle_e|hard\rangle_E$ or $|'soft'\rangle_m|soft\rangle_e|soft\rangle_E$.

"During decoherence, entanglement does not really disappear, but goes further and further into the environment; in practice, it becomes rapidly completely impossible to detect." (Laloe 2012, pg. 136.)

<u>Let's see how this is supposed to work using statistical operators...</u>

- The statistical operator $P_{|\psi\rangle} = |\psi\rangle\langle\psi|$ for the state in (1) is:

$$P_{|\psi\rangle} = ½\{|'h'\rangle|h\rangle|E_h\rangle + |'s'\rangle|s\rangle|E_s\rangle\}\{\langle 'h'|\langle h|\langle E_h| + \langle 's'|\langle s|\langle E_s|\}$$

$$= ½|'h'\rangle|h\rangle|E_h\rangle\langle 'h'|\langle h|\langle E_h| + ½|'s'\rangle|s\rangle|E_s\rangle\langle 's'|\langle s|\langle E_s|$$

$$+ ½|'h'\rangle|h\rangle|E_h\rangle\langle 's'|\langle s|\langle E_s| + ½|'s'\rangle|s\rangle|E_s\rangle\langle 'h'|\langle h|\langle E_h|$$

$$= ½|'h'\rangle\langle 'h'| \otimes |h\rangle\langle h| \otimes |E_h\rangle\langle E_h| + ½|'s'\rangle\langle 's'| \otimes |s\rangle\langle s| \otimes |E_s\rangle\langle E_s|$$

$$+ ½|'h'\rangle\langle 's'| \otimes |h\rangle\langle s| \otimes |E_h\rangle\langle E_s| + ½|'s'\rangle\langle 'h'| \otimes |s\rangle\langle h| \otimes |E_s\rangle\langle E_h|$$

$$= ½P_{|'h'\rangle}\otimes P_{|h\rangle}\otimes P_{|E_h\rangle} + ½P_{|'s'\rangle}\otimes P_{|s\rangle}\otimes P_{|E_s\rangle} + (\textit{interference terms})$$

- *<u>Now</u>*: Take the "partial trace" of $P_{|\psi\rangle}$ with respect to the Environment basis:

$$\mathrm{Tr}_E(P_{|\psi\rangle}) = \langle E_h|P_{|\psi\rangle}|E_h\rangle + \langle E_s|P_{|\psi\rangle}|E_s\rangle$$

$$= ½P_{|'h'\rangle}\otimes P_{|h\rangle} + ½P_{|'s'\rangle}\otimes P_{|s\rangle}$$

"Tracing over the environment" kills the interference terms!

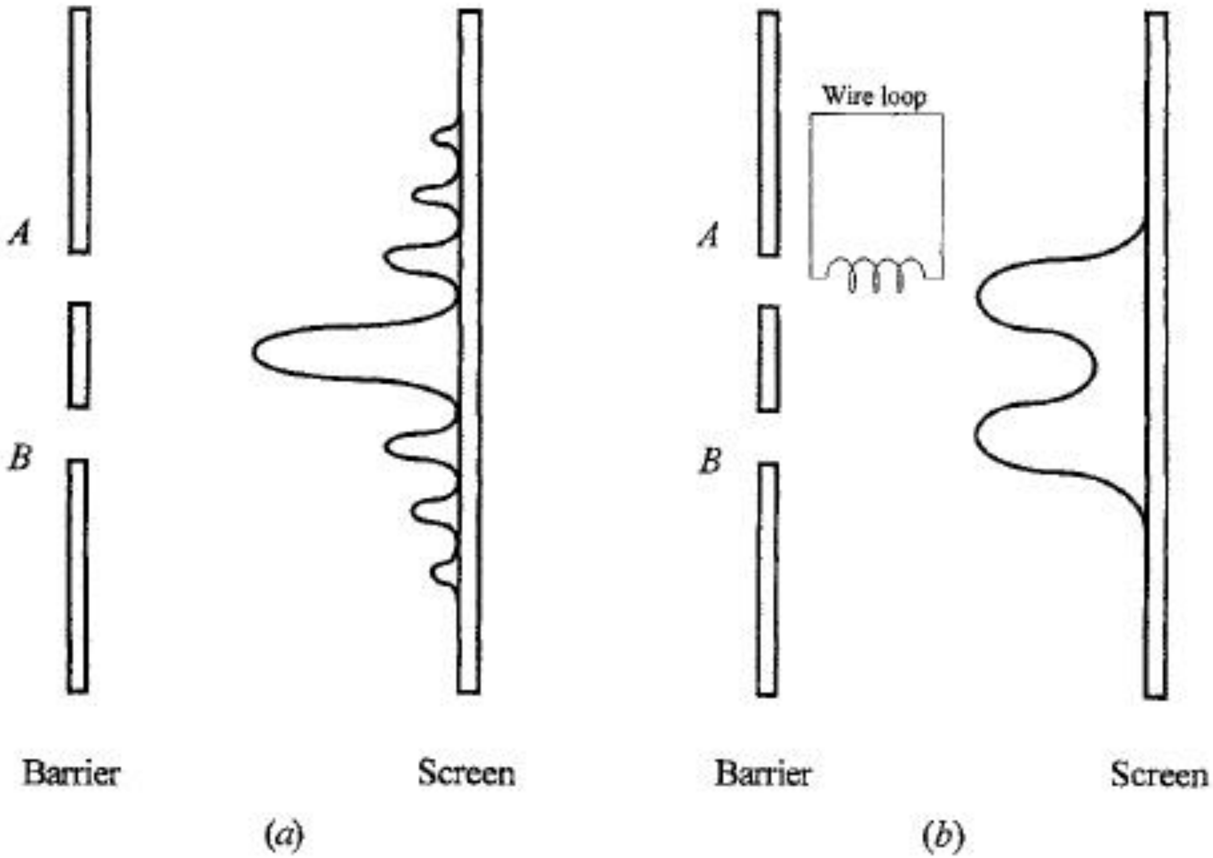


FIG. 8.1 How environmental correlations destroy simple interference effects. (*a*) The interference distribution. (*b*) With a wire loop at *A*.

"...just as the environmentally correlated superposition in the second experiment [b] is empirically indistinguishable from a state where the electron passes through either one slit or the other, an environmentally correlated superposition of different measurement records is empirically indistinguishable from a particular record."   (Barrett, pg. 223.)

Does decoherence solve the measurement problem?

- *No!*
- When we "trace over the environment", we're left with the statistical operator

$$\tfrac{1}{2}P_{|'hard'\rangle} \otimes P_{|hard\rangle} + \tfrac{1}{2}P_{|'soft'\rangle} \otimes P_{|soft\rangle}$$

- This is the statistical operator for a "mixed state": This is how the state of a system is represented when its exact form is known only to lie within a set of possible states.
- In this case, the state of the system is *either* of the pair

$$\{|'hard'\rangle_m |hard\rangle_e, |'soft'\rangle_m |soft\rangle_e\}$$

each with equal weight ½.

- *But*: The result of a measurement (as given by the *Projection Postulate* and by our experience) is a *definite* outcome.
  - *In this case, the result is either* $|'hard'\rangle_m |hard\rangle_e$ *or* $|'soft'\rangle_m |soft\rangle_e$.
  - *It's definitely one of these two alternatives*.
  - *It's not a weighted sum of them both!*

4. Consistent Histories
Griffiths (1984)

Robert Griffiths

- Recall: A *state* of a system is a description of the system in terms of the values its properties take at an *instant of time*.
- A *history* of a system is a description of the system in terms of the values its properties take over an *interval of time*.

Def. 1. A *history h* is a time-indexed sequence of facts, represented by time-indexed projection operators:

$$h = \big(P_1(t_1), P_2(t_2), \ldots, P_n(t_n)\big).$$

- $P_1(t_1)$ might be $P_{|hard\rangle}(t_1)$ which represents the property, at time t_1, "The value of *Hardness* is *hard*".
- Or: It might be $P_{|a\rangle}(t_1)$ which represents the property, at time t_1, "The value of the property A is a".
- Projection operators evolve *via* the Schrödinger dynamics:

$$P(t) = e^{iHt/\hbar}P(0)e^{-iHt/\hbar}$$

Def. 2. The *probability* associated with a history *h* is given by:

$$\mathrm{Pr}_Q(h) = \mathrm{Tr}\big(P_n(t_n)...P_2(t_2)P_1(t_1)P_{|\psi\rangle}P_1(t_1)P_2(t_2)...P_n(t_n)\big)$$

where $P_{|\psi\rangle}$ is the statistical operator associated with an initial state $|\psi\rangle$.

- All the terms inside a trace commute with each other, so:

$$\mathrm{Pr}_Q(h) = \mathrm{Tr}\big(P_n(t_n)P_n(t_n)...P_2(t_2)P_2(t_2)P_1(t_1)P_1(t_1)P_{|\psi\rangle}\big)$$

- Since projection operators are *idempotent*, this is equal to:

$$\mathrm{Pr}_Q(h) = \mathrm{Tr}\big(P_n(t_n)...P_2(t_2)P_1(t_1)P_{|\psi\rangle}\big)$$

- <u>*And*</u>: This can be thought of as the trace version of the *Born Rule* for the probability that the system in the state $|\psi\rangle$, has the "historical property" represented by the operator $P_n(t_n)...P_2(t_2)P_1(t_1)$.

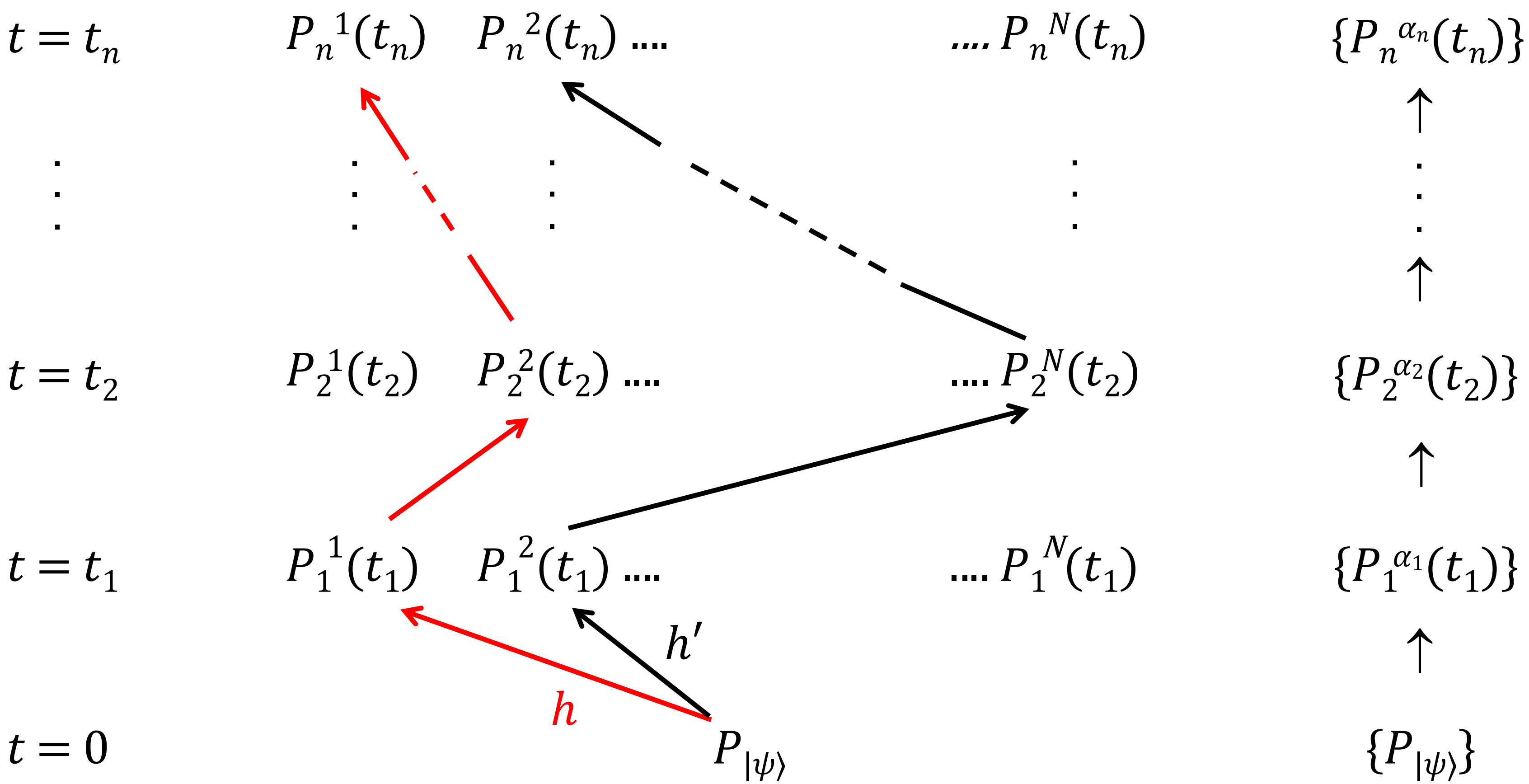
Def. 3. A *family of histories* is a time-indexed sequence of sets of "exhaustive" facts:

$$\left(\{P_1^{\alpha_1}(t_1)\}, \{P_2^{\alpha_2}(t_2)\}, \ldots, \{P_n^{\alpha_n}(t_n)\}\right)$$

where each index $\alpha_i = 1, \ldots, N$ and $\{P_i^{\alpha_i}(t_i)\} = \{P_i^1(t_i), P_i^2(t_i), \ldots, P_i^N(t_i)\}$, such that $P_i^1(t_i) + P_i^2(t_i) + \cdots + P_i^N(t_i) = I_N$.

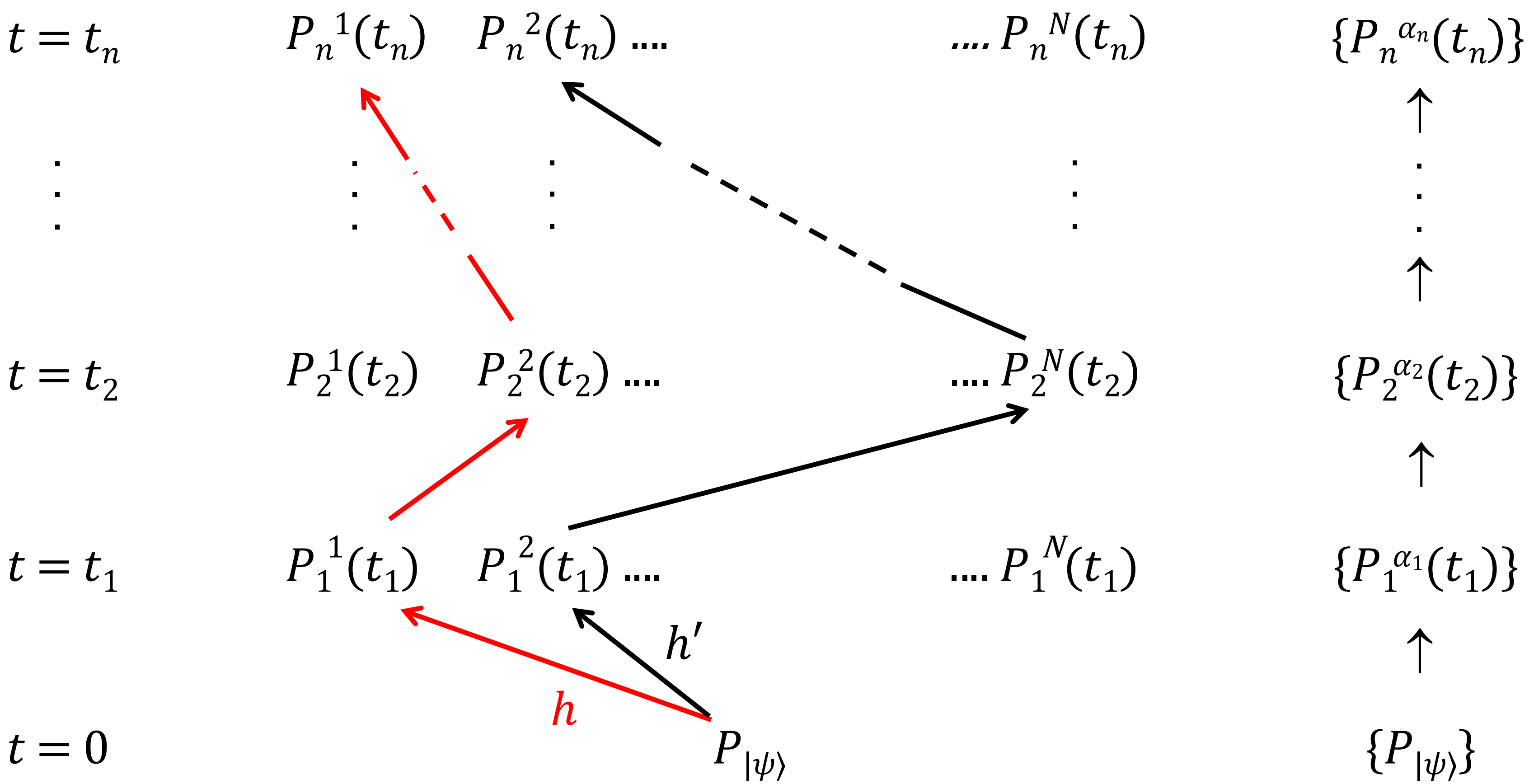
- The projection operators in any set $\{P_i^{\alpha_i}(t_i)\}$ represent all the possible values of the property associated with $P_i(t_i)$.

Histories can be embedded in families of histories:



- $h = (P_{|\psi\rangle}, P_1^{\,1}(t_1), P_2^{\,2}(t_2), \ldots, P_n^{\,1}(t_n))$
- $h' = (P_{|\psi\rangle}, P_1^{\,2}(t_1), P_2^{\,N}(t_2), \ldots, P_n^{\,2}(t_n))$
- h and h' are two histories within the family $(\{P_{|\psi\rangle}\}, \{P_1^{\,\alpha_1}(t_1)\}, \ldots, \{P_n^{\,\alpha_n}(t_n)\})$.

Histories can be embedded in families of histories:



- We can assign probabilities to histories within a family by means of *Def*. 2.
- These are *quantum* probabilities that exhibit interference effects.

Are there histories within a given family that can be assigned classical probabilties?

Are there histories within a given family that do not interfere with each other?

- These would be histories h, h' whose probabilities obey the classical Or-Addition Rule:

$$\Pr_Q(h\ or\ h') = \Pr_Q(h) + \Pr_Q(h')$$

- <u>*First*</u>: Need an expression for the disjunction, $h\ or\ h'$, of two histories h, h'.

<u>*Simple case*</u>: Suppose h_A and h_B are histories that differ *only* in the property at $t = t_i$:

$$h_A = \big(P_1(t_1), \ldots, P_i^A(t_i), \ldots, P_n(t_n)\big)$$

$$h_B = \big(P_1(t_1), \ldots, P_i^B(t_i), \ldots, P_n(t_n)\big)$$

- Now let the history, $h_A\ or\ h_B$, be given by:

$$h_A\ or\ h_B = \big(P_1(t_1), \ldots, P_i^A(t_i) + P_i^B(t_i), \ldots, P_n(t_n)\big)$$

Simple case, continued:

- h_A *or* $h_B = \big(P_1(t_1), ..., P_i^A(t_i) + P_i^B(t_i), ..., P_n(t_n)\big)$
- So: For an initial state $|\psi\rangle$:

$$\Pr_Q(h_A \textit{ or } h_B)$$

$$= \mathrm{Tr}\big(P_n(t_n)...[P_i^A(t_i) + P_i^B(t_i)]...P_1(t_1)P_{|\psi\rangle}P_1(t_1)...[P_i^A(t_i) + P_i^B(t_i)]...P_n(t_n)\big)$$

$$= \mathrm{Tr}\big(P_n(t_n)...P_i^A(t_i)...P_1(t_1)P_{|\psi\rangle}P_1(t_1)...P_i^A(t_i)...P_n(t_n)\big)$$

$$+ \mathrm{Tr}\big(P_n(t_n)...P_i^B(t_i)...P_1(t_1)P_{|\psi\rangle}P_1(t_1)...P_i^B(t_i)...P_n(t_n)\big)$$

$$+ \big\{\mathrm{Tr}\big(P_n(t_n)...P_i^A(t_i)...P_1(t_1)P_{|\psi\rangle}P_1(t_1)...P_i^B(t_i)...P_n(t_n)\big)$$

$$+ \mathrm{Tr}\big(P_n(t_n)...P_i^B(t_i)...P_1(t_1)P_{|\psi\rangle}P_1(t_1)...P_i^A(t_i)...P_n(t_n)\big)\big\}$$

$$= \Pr_Q(h_A) + \Pr_Q(h_B) + \{\textit{interference terms}\}$$

- Which means: The probabilities assigned to h_A and h_B by *Def.* 2 will be classical (*i.e.*, will obey the classical Or-Addition Rule) just when the interference terms vanish.

Now: Consider the general case of h, h' differing on all properties:

$$h = \big(P_1(t_1), ..., P_n(t_n)\big)$$

$$h' = \big(P_1{}'(t_1), ..., P_n{}'(t_n)\big)$$

$$h \; or \; h' = \big([P_1(t_1) + P_1{}'(t_1)], ..., [P_i(t_i) + P_i{}'(t_i)], ..., [P_n(t_n) + P_n{}'(t_n)]\big)$$

- The probabilities assigned to h and h' by *Def*. 2 will be classical just when the general interference term vanishes:

$$\mathrm{Tr}(P_n(t_n)...P_1(t_1))P_{|\psi\rangle}P_1{}'(t_1)...P_n{}'(t_n)) = 0$$

Def. 4. Two histories $h = \big(P_1(t_1), ..., P_n(t_n)\big)$, $h' = \big(P_1{}'(t_1), ..., P_n{}'(t_n)\big)$ are *consistent* just when $\mathrm{Tr}(P_n(t_n)...P_1(t_1))P_{|\psi\rangle}P_1{}'(t_1)...P_n{}'(t_n)) = 0$.

Def. 5. A *consistent family of histories* is a family of histories such that any two histories embeddable in it are consistent.

- A *consistent family of histories* is a collection of histories that defines a classical sample space! You can assign classical probabilities to its members.

Def. 6.

(1) h is a *fine-grained history* just when all projection operators in h are 1-dim.

(2) h' is a *coarse-graining of h* just when some projection operators in h' are sums of projection operators in h.

- Fine-grained histories cannot in general be assigned classical probabilities.
- Course-grained histories can be assigned *approximate* classical probabilities, and these get more classical as $\mathrm{Tr}(P_n(t_n)...P_1(t_1))P_{|\psi\rangle}P_1{}'(t_1)...P_n{}'(t_n)) \to 0$.
- As $\mathrm{Tr}(P_n(t_n)...P_1(t_1))P_{|\psi\rangle}P_1{}'(t_1)...P_n{}'(t_n)) \to 0$, such course-grained histories "decohere".

- *Coarse-graining* a family of histories corresponds to tracing out the environment.
- The environment interacts with the coarse-grained histories to damp out the interference effects, rendering the family approximately consistent.

Def. 7. Two histories $h = (P_1(t_1), ..., P_n(t_n))$, $h' = (P_1'(t_1), ..., P_n'(t_n))$ are *decoherent* just when $\mathrm{Tr}(P_n(t_n)...P_1(t_1))P_{|\psi\rangle}P_1'(t_1)...P_n'(t_n)) \to 0$.

Def. 8. A *decoherent family of histories* is a family of histories such that any two histories embeddable in it are decoherent.

Characteristics of the Consistent/Decoherent Histories (CH) Approach

- Replaces *states* of a physical system with *histories* a physical system.
- The properties (projection operators) that make up a history evolve *only via* the Schrödinger dynamics (no Projection Postulate).
- Identifies a way to associate a probability with a history (*Def*. 2).
- Identifies a condition that picks out those families of histories that are classical (or approximately classical) (*Defs*. 4, 5).

Problems

1. How are alternative histories within a decoherent family to be interpreted?

- Is one history actual and the others just possible?
- Or do all histories within a decoherent family occur? If so, then how are probabilities explained?
  - *This is the Problem of Probabilities that Many Worlds faces.*

2. How are alternative decoherent families to be interpreted?

- Any history h can be embedded in many different mutually incompatible decoherent families (any one of which defines an approximately classical probability space).
- Which do we choose in order to calculate the probability of h?
  - *This is the Preferred Basis Problem that Many Worlds faces.*

Problems 1 & 2 Combined:

- Seem to indicate that CH isn't fundamentally different from Many Worlds.
  - *All CH does is replace world-talk with history-talk, and adds a criterion for identifying histories that behave "classically".*

3. General Problem with the Notion of Decoherence

- "Tracing over the environment" (or "coarse-graining" histories) does not pick out a unique measurement/interaction outcome.
- It does not effect a "collapse" of superposed states (or "interfering" histories).
- So it cannot be appealed to in order to reconcile superpositions (or "interfering" histories) with our experience of unique outcomes.