12a. Modal Interpretations

1. General Features

- Let's return to using Hilbert spaces to represent QM state spaces, and operators to represent properties.
- *Recall*: The *Kochen-Specker Theorem* says that the properties associated with a Hilbert space $\mathcal{H}$ can't *all* have values at the same time (if $\dim\mathcal{H} \geq 3$).
- *One Way to Avoid KS*: Claim that *some* (not all) properties defined on H always have determinate values (even in superpositions), others do not.

Ex: Bohm's Theory
- One property (position) is always determinate (always has a value).
- All other properties are *contextual* – their values depend on how they are measured.

Modal Interpretations Claim:

(A) For any Hilbert space $\mathcal{H}$, there is a subset of operators that represent properties that are always determinate (always possess values).

(B) The QM probabilities for these properties are *epistemic*: for these properties, probabilities represent our ignorance of their actual values.

Modal Interpretations reject the Eigenvector/Eigenvalue Rule:

$$\left[\begin{array}{l}\text{A physical system}\\ \text{possesses the value}\\ \lambda \text{ of a property.}\end{array}\right] \quad \textit{if and only if} \quad \left[\begin{array}{l}\text{The state of the system is}\\ \text{represented by an eigenvector}\\ \text{of the operator representing the}\\ \text{property with eigenvalue } \lambda.\end{array}\right]$$

- Modal intepretations allow for a state to possess the value of a property, *even when it is not an eigenvector of the associated operator*.
- *They claim*: For any given state $|\psi\rangle$, in addition to those properties for which $|\psi\rangle$ is an eigenvector, there are *other* properties for which $|\psi\rangle$ also possesses values (the always-determinate "modal" propeties).
- *So*: Modal interpretations agree with the *"if"* part ($\Leftarrow$) of EE.
- *But*: They disagree with the *"only if"* part ($\Rightarrow$) of EE.

- *Initial task for Modal Interpretations*:
  Identify the subset of always-determinate "modal" properties.

Different versions pick out different modal properties.

2. KHD Modal Interpretation

Kochen (1985), Healy (1989), Dieks (1991)

Simon Kochen *Richard Healy* *Dennis Dieks*

- *Claim*: At any given time, the subset of always-determinate properties is given by the basis states of the *biorthogonal expansion* of the system's state vector.

Biorthogonal Decomposition Theorem:
Let $|Q\rangle$ be a vector in the product Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$. Then there is a basis $|a_1\rangle, \ldots, |a_N\rangle$ of $\mathcal{H}_1$, and a basis $|b_1\rangle, \ldots, |b_N\rangle$ of $\mathcal{H}_2$ such that $|Q\rangle$ can be expanded as:

$$|Q\rangle = c_{11}|a_1\rangle|b_1\rangle + c_{22}|a_2\rangle|b_2\rangle + \cdots + c_{NN}|a_N\rangle|b_N\rangle$$

And, if $|c_{11}| \neq |c_{22}| \neq \cdots \neq |c_{NN}|$, then these bases are *unique*.

- In general, if $|g_1\rangle, \ldots, |g_N\rangle$ and $|h_1\rangle, \ldots, |h_N\rangle$ are bases of $\mathcal{H}_1$ and $\mathcal{H}_2$, then any vector $|Q\rangle$ can be expanded as

$$|Q\rangle = d_{11}|g_1\rangle|h_1\rangle + d_{12}|g_1\rangle|h_2\rangle + \cdots + d_{21}|g_2\rangle|h_1\rangle + d_{22}|g_2\rangle|h_2\rangle + \cdots .$$

- The *Biorthog Decomp Theorem* says that there are some bases in which the "cross terms" with coefficients d_{12}, d_{21}, *etc*, all vanish. And these bases will be unique just when all the remaining coefficients are different from each other.

Why do we want bases in which the cross terms vanish?

- Because these are the bases associated with *post-measurement* systems.

<u>*Example*</u>: Composite system of *Hardness* measuring device *m* and *black* electron *e*.

- A basis for *m-e* system:

  $\{|\text{"}hard\text{"}\rangle_m|hard\rangle_e, |\text{"}hard\text{"}\rangle_m|soft\rangle_e, |\text{"}soft\text{"}\rangle_m|hard\rangle_e, |\text{"}soft\text{"}\rangle_m|soft\rangle_e\}$

- General expansion of a state $|Q\rangle$ in this basis is:

  $|Q\rangle = a|\text{"}hard\text{"}\rangle_m|hard\rangle_e + c|\text{"}hard\text{"}\rangle_m|soft\rangle_e + d|\text{"}soft\text{"}\rangle_m|hard\rangle_e + b|\text{"}soft\text{"}\rangle_m|soft\rangle_e$

- If this basis is *biorthogonal*, then $c = d = 0$, and we have:

  $|Q\rangle = a|\text{"}hard\text{"}\rangle_m|hard\rangle_e + b|\text{"}soft\text{"}\rangle_m|soft\rangle_e$

- This is just the post-measurement state of our composite system!
- We could avoid the Projection Postulate if we assume that the properties associated with these bases vectors (pointing to "*hard*" or "*soft*" for *m*, being *hard* or *soft* for *e*) are always determinate.

- <u>*So*</u>: KHD just *stipulates* that properties associated with biorthogonal expansion bases are always determinate.

KHD Rules (replace Projection Postulate):

Rule 1: For any physical system S that is composed of two subsystems S_1 and S_2, there are some properties for which S always possesses values. To identify them:

(i) Expand the state vector $|Q\rangle$ for S in its biorthogonal decomposition:

$$|Q\rangle = c_{11}|a_1\rangle|b_1\rangle + c_{22}|a_2\rangle|b_2\rangle + \cdots + c_{NN}|a_N\rangle|b_N\rangle$$

(ii) The biorthogonal basis states $|a_1\rangle, \ldots, |a_N\rangle$, and $|b_1\rangle, \ldots, |b_N\rangle$ are the eigenvectors of the determinate properties, call them A and B.

(iii) The subsystems S_1 and S_2 can be said to have determinate values for the properties A and B, so identified.

Rule 2: (*Born Rule*) If S is in the state $|Q\rangle$, then the probability that S_1 has the value a_i of the property A is ${c_{ii}}^2$, and the probability that S_2 has the value b_i of the property B is ${c_{ii}}^2$.

Why this is helpful:

- Suppose a system is in a state represented by

$$|Q\rangle = a|\text{"}hard\text{"}\rangle_m |hard\rangle_e + b|\text{"}soft\text{"}\rangle_m |soft\rangle_e \quad (a \neq b)$$

- *A Literal Interpretation says*: This is a state in which e can't be said to have the *Hardness* property, and m can't be said to be indicating "*hard*" or "*soft*".
- *KHD says*: This is a state in which e does have a definite value of *Hardness*, and m is definitely pointing to either "*hard*" or "*soft*" (even though we don't know what *Hardness* value e has, and we don't know where m is pointing).

Essential Characteristics of Modal Interpretations

(A) Rejection of *Eigenvector/Eigenvalue Rule.*

(B) Rejection of *Projection Postulate.*

(C) Probabilities are epistemic.

3. Three Problems with KHD

1. Non-uniqueness of biorthogonal expansions.

- Consider the biorthogonal expansion $|Q\rangle = c_{11}|a_1\rangle|b_1\rangle + \cdots + c_{NN}|a_N\rangle|b_N\rangle$.
- The *Biorthog Decomp Theorem* says this expansion is *unique*, provided that $|c_{11}| \neq |c_{11}| \neq \ldots \neq |c_{NN}|$.
- If this does not hold; *i.e.*, if any of the expansion coefficients are equal, then there will be other biorthogonal expansions of $|Q\rangle$, in fact *infinitely* many.

Ex: $|Q\rangle = \sqrt{½}\,|"hard"\rangle_m|hard\rangle_e + \sqrt{½}\,|"soft"\rangle_m|soft\rangle_e$

$= \sqrt{½}\,|"black"\rangle_m|black\rangle_e + \sqrt{½}\,|"white"\rangle_m|white\rangle_e$

$= etc.$

- In such cases, *KHD Rule 1* will say that infinitely many properties will have definite values at any given time, and this violates the *KS* Theorem!

2. Dynamics for determinate properties.

- All modal interpretations (not just *KHD*) say that, at any given time, a physical system possesses the values of some subset of properties (in addition to, and including those given by the *EE Rule*).

- Let Det_t be this set of determinate properties at time t.
  - This set can change from moment to moment!
  - In other words, Det_t may be different from $Det_{t'}$ for $t \neq t'$.

> *Ex*: In the *KHD* version, Det_t depends on the component states of the composite system, and these component states may change over time.

- *So*: All modal interpretations need to tell us how Det_t changes over time.
  - They need to give us a *dynamics* for the determinate properties.
  - But KHD does not specify this.

> *Ex*: Bohmian Mechanics can be considered as a modal interpretation in which the property dynamics (for the position property) is given by Bohm's Equation.

3. Imperfect Measurements.

Claim: The post-measurement states that *KHD* identifies represent *ideal* perfect measurements. For *actual* imperfect measurements, *KHD* does not pick out the right post-measurement properties.

- *KHD* seemed to work for the post-measurement state:

  $|Q\rangle = a|\text{"}hard\text{"}\rangle_m|hard\rangle_e + b|\text{"}soft\text{"}\rangle_m|soft\rangle_e$ (suppose $a \neq b$)

- This is in the form of a biorthogonal expansion, so *KHD* says:

  The electron has a definite value of Hardness.

- *But*: The Schrödinger-evolved post-measurement state will *really* be:

$$|J\rangle = c|\text{"}hard\text{"}\rangle_m|hard\rangle_e + d|\text{"}soft\text{"}\rangle_m|soft\rangle_e + \underbrace{f|\text{"}hard\text{"}\rangle_m|soft\rangle_e + g|\text{"}soft\text{"}\rangle_m|hard\rangle_e}$$

Error terms! Represent the fact that real measuring devices will never perfectly correlate pointers with Hardness property. For realistic measuring devices, f and g can be made very small, but they will never vanish.

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- $|J\rangle$ has a biorthogonal expansion (guaranteed by the *Biorthog Decomp Theorem*), but it will *not* be the one that *KHD* cites:

  $|J\rangle = k|w\rangle_m|grump\rangle_e + l|w'\rangle_m|gromp\rangle_e$

- *Grump* and *gromp* are values of *some* property (they are eigenvectors of *some* operator), but not *Hardness*.
- So the *KHD Rule 1* entails that, after a *Hardness* measurement, the electron is either *grump* or *gromp*, and not either *hard* or *soft*.

12b. Quantum Logic

1. Motivation

- When a physical system is in a state represented by a superposition, we can't use classical logic to describe the properties it possesses.
- Consider the state $|Q\rangle = a|\text{"}hard\text{"}\rangle_m|hard\rangle_e + b|\text{"}soft\text{"}\rangle_m|soft\rangle_e$.

Recall: Under a literal interpretation, an electron in this state:
(a) Can't be said to be *hard*.
(b) Can't be said to be *soft*.
(c) Can't be said to be both *hard* and *soft*.
(d) Can't be said to be neither *hard* nor *soft*.

- Perhaps to make sense of such superposed states, we need to change our logic!

Goal: To develop a *quantum* logic that will allow us to say meaningful things about the properties of states in superpositions.

2. Classical Properties and Classical Logic

- Classical mechanics represents properties in a certain way (functions on a phase space), and this way has a structure that is identical to the structure of classical logic.
- Quantum mechanics represents properties in a different way (operators on a Hilbert space), so the structure of QM properties is different from that of CM properties and classical logic.

The Logic of Classical Mechanics (CM)

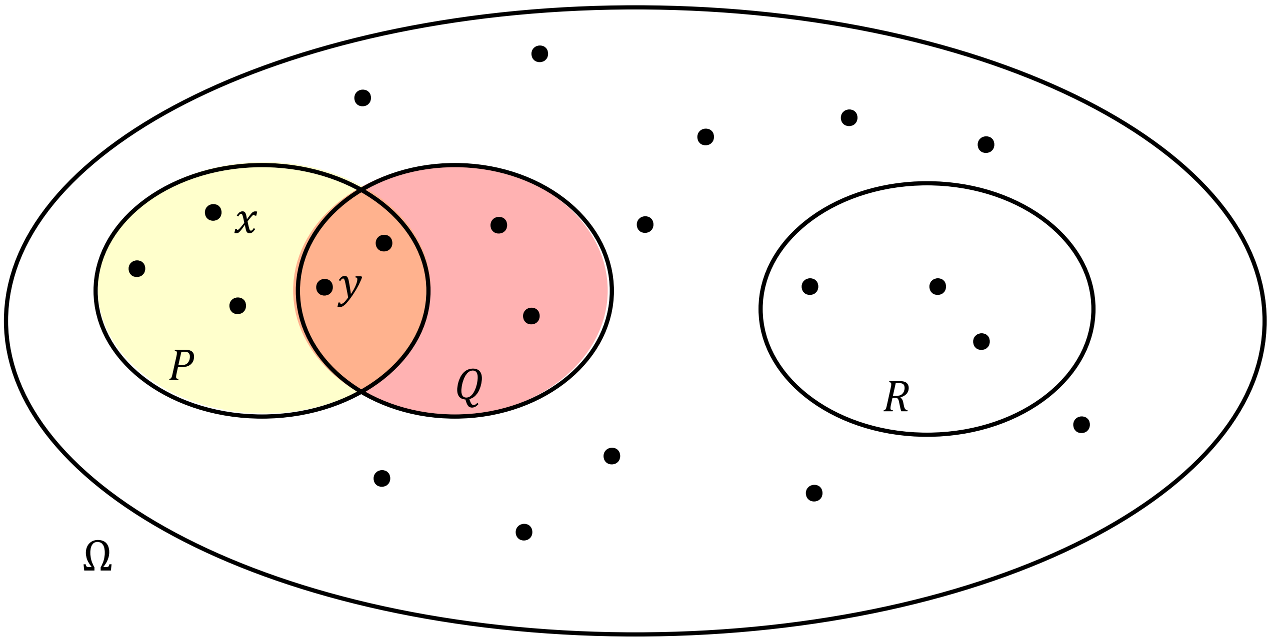
- *Recall*: *CM state space* = phase space (set of points)

  CM states = points

  CM properties = functions

- Consider the property, "The value of property A is a".

  - In *QM*, this property can be represented by a *projection operator*.
  - In *CM*, this property is represented by a *subset* of phase space; namely, the collection of all phase space points that represent states in which the value of property A is a.

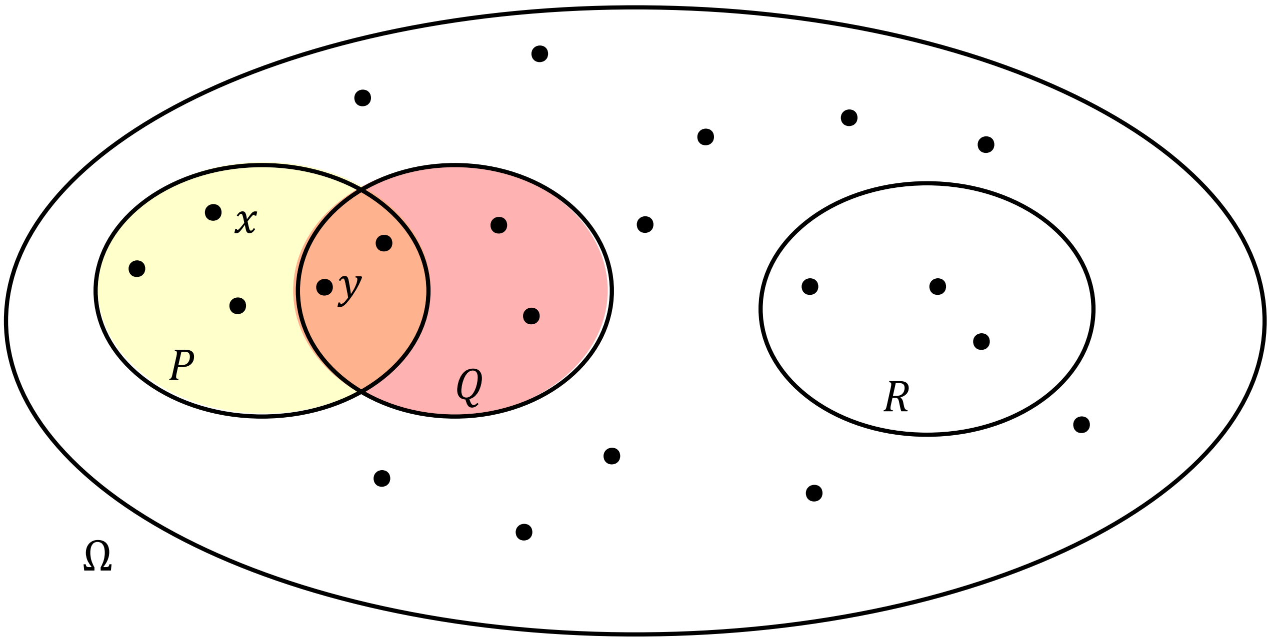


Phase space Ω and three subsets: P, Q, R.

- Let P represent the property "The value of property A is a".

All points in P represent states in which the value of property A is a.

- Let Q represent the property "The value of property B is b".
- The intersection $P \cap Q$ represents the property
  "The value of property A is a ***and*** the value of property B is b".
- The union $P \cup Q$ represents the property
  "The value of property A is a ***or*** the value of property B is b".
- The complement $\neg P$ represents the property
  "The value of property A is ***not*** a".



Phase space Ω and three subsets: P, Q, R.

- *Claim*: The structure of sets under $\cap$, $\cup$ and $\neg$ is the same as the structure of classical sentential logic with connectives $\wedge_C$, $\vee_C$ and $\neg_C$.

Let the set P represent the sentence $p =$ "The value of property A is a."
Let the set Q represent the sentence $q =$ "The value of property B is b."
Then:
- $P \cap Q$ represents $p \wedge_C q$.
- $P \cup Q$ represents $p \vee_C q$.
- $\neg P$ represents $\neg_C p$.

set operation	*classical logic connective*
$\cap$ (*intersection*)	$\wedge_C$ (*and*)
$\cup$ (*union*)	$\vee_C$ (*or*)
$\neg$ (*complement*)	$\neg_C$ (*not*)

- A collection of sets with $\cap$, $\cup$, $\neg$ defined on it and a collection of sentences with $\wedge_C$, $\vee_C$, $\neg_C$ defined on it are both representations of a *Boolean algebra*.
  - *CM properties, collections of sets, and classical logic all have the same Boolean algebraic structure.*
- *So*: Classical logic = the logic of the structure of *CM* properties.

  An empirical approach to logic!

- Why do we use classical logic to describe the world?
  - *Because of the way classical physics describes the world.*
- This suggests that, when the physics changes, so should the logic!

"Even logic must give way to physics."

The Logic of Quantum Mechanics

- *Recall*: *QM state space* $=$ Hilbert space $\mathcal{H}$

  QM states $=$ vectors

  QM properties $=$ operators

- Consider the property, "The value of property A is a".

  - Represented by a projection operator $P_{|a\rangle}$.
  - $P_{|a\rangle}$ projects any vector onto the 1-dim *subspace* of $\mathcal{H}$ (*i.e.*, ray) defined by the eigenvector $|a\rangle$ of A with eigenvalue a.
  - *So*: In *QM*, properties of the type "The value of property X is x" are represented by *subspaces* (and not *subsets*).

3. The Structure of Quantum Properties

Def. A *subspace* of a Hilbert space $\mathcal{H}$ is a subset of $\mathcal{H}$ closed under vector addition and scalar multiplication.

- *This means*: A subspace is just a part of $\mathcal{H}$ that is itself a vector space.
- There is a 1-1 correspondence between projection operators and subspaces.

Subspaces are related by 3 operations:

$\cap$ (*intersection*)	$V \cap W = \{$all vectors in both V and $W\}$
$\oplus$ (*linear span*)	$V \oplus W = \{$all *linear combinations* of vectors from V and $W\}$
$\perp$ (*orthocomplement*)	$V^{\perp} = \{$all vectors that are *orthogonal* to vectors in $V\}$

- If V and W are both 1-dim, then $V \oplus W$ is a 2-dim subspace; i.e., a plane containing all vectors of the form $a|v\rangle + b|w\rangle$, where $|v\rangle \in V$ and $|w\rangle \in W$.
- $V \oplus W$ corresponds to the projection operator $P_{V \oplus W} = P_{|v\rangle} + P_{|w\rangle}$.
- If V is 1-dim and W is 2-dim, then $V \oplus W$ is a 3-dim subspace; *etc*.

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- *Why linear span replaces union*: The *union* of two subspaces is not in general a subspace.

- Suppose V, W are both 1-dim subspaces of $\mathcal{H}$.
- Then $V \cup W$ is the set of all vectors in both V and W.
- *This set is not a subspace*: The sum of two vectors from V and W may not itself be in $V \cup W$ (it may point in a direction other than the directions defined by V and W)

The structure of *QM* properties is given by the *subspace structure* of a Hilbert space (as opposed to the *subset structure* of a phase space).

- Important property of the subspace structure: *It is not distributive!*

Claim: For any subspaces V, W, X, of $\mathcal{H}$, it is not in general the case that

$$X \cap (V \oplus W) = (X \cap V) \oplus (X \cap W)$$

Proof:

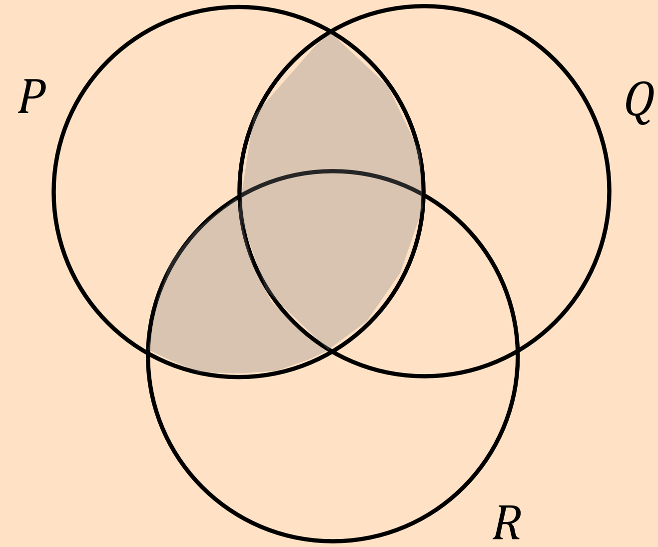
- Suppose V, W and X are subspaces of $\mathcal{H}$ and suppose X is a subspace of $V \oplus W$ such that X is neither a subset of V nor a subset of W.
- This means: Any vector $|x\rangle$ in X can be written as $|x\rangle = a|v\rangle + b|w\rangle$, with $|v\rangle \in V$, $|w\rangle \in W$, and a, b nonzero.
- Then $X \cap (V \oplus W) = X$.
- But $(X \cap V) \oplus (X \cap W) = 0 \oplus 0 = 0$.

- Since Boolean algebras are distributive, this means that the subspace structure of QM properties is not a Boolean algebra.
  - *So it really is different from the subset structure of CM properties and the structure of classical logic (which are Boolean)!*

Boolean algebras are distributive

Set theory example:

$$P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$$



Classical logic example: $p \wedge_C (q \vee_C r) \equiv (p \wedge_C q) \vee_C (p \wedge_C r)$

p	q	r	$q \vee_C r$	$p \wedge_C (q \vee_C r)$	$(p \wedge_C q)$	$(p \wedge_C r)$	$(p \wedge_C q) \vee_C (p \wedge_C r)$
T	T	T	T	**T**	T	T	**T**
T	T	F	T	**T**	T	F	**T**
T	F	T	T	**T**	F	T	**F**
T	F	F	F	**F**	F	F	**F**
F	T	T	T	**F**	F	F	**F**
F	T	F	T	**F**	F	F	**F**
F	F	T	T	**F**	F	F	**F**
F	F	F	F	**F**	F	F	**F**

Now: Construct a *non-Boolean* quantum logic based on the following correspondences:

subspace operation		*quantum logic connective*	
$\cap$	(*intersection*)	$\wedge_Q$	(*and*)
$\oplus$	(*linear span*)	$\vee_Q$	(*or*)
$\perp$	(*orthocomplement*)	$\neg_Q$	(*not*)

- Let the subspace V represent the sentence $v =$ "The value of property A is a."
- Let the subspace W represent the sentence $w =$ "The value of property B is b."
- Let the subspace X represent the sentence $x =$ "The value of property C is c."

- $V \cap W$ represents "The value of property A is a and the value of property B is b" (or "$v \wedge_Q w$").
- $V \oplus W$ represents "The value of property A is a or the value of property C is c" (or "$v \vee_Q w$").
- $V^\perp$ represents "The value of property A is not a" (or "$\neg_Q v$").

Why this is supposed to help: We can now claim that, as a matter of QL (Quantum Logic):

(1) "A has a definite value" is a *QL tautology* (always a true statement), for *all* properties A.

Why?

["A has a definite value."] *means* ["The value of property A is a_1, *or* the value of property A is a_2, *or* ..., *or* the value of property A is a_N."]

which means ["The state of the system lies in $V_1 \oplus \cdots \oplus V_N$."]

where V_1, ..., V_N are the 1-dim subspaces spanned respectively by the eigenvectors $|a_1\rangle$, ..., $|a_N\rangle$ of A.

- *Now note*: $V_1 \oplus \cdots \oplus V_N = \mathcal{H}$, and it's always true that the state of a system lies in its state space $\mathcal{H}$.
- *So*: As a matter of *QL*, all properties *always* have definite values at all times, *even properties of measuring devices in superposed states!*

(2) Statements about incompatible properties possessing simultaneous values are contradictory (always false).

<u>*Why?*</u> Suppose A and B are incompatible properties (*i.e.*, *Hardness* and *Color*).

$\left[\text{"Property } A \text{ has a value and property } B \text{ has a value."}\right]$ *means* $\left[\text{"(The value of } A \text{ is } a_1 \textit{ and } \text{the value of } B \text{ is } b_1) \textit{ or } (\text{the value of } A \text{ is } a_1 \textit{ and } \text{the value of } B \text{ is } b_2) \textit{ or } \ldots \textit{ or } (\text{the value of } A \text{ is } a_2 \textit{ and } \text{the value of } B \text{ is } b_1) \textit{ or } \ldots \textit{ or } (\text{the value of } A \text{ is } a_N \textit{ and } \text{the value of } B \text{ is } b_N)."\right]$

which means $\left[\text{"The state of the system lies in } (V_1 \cap W_1) \oplus (V_1 \cap W_2) \oplus \cdots \oplus (V_2 \cap W_1) \oplus \cdots \oplus (V_N \cap W_N)."\right]$

- <u>*Note*</u>: Since V_i and W_j are disjoint for any i, j, all the intersection terms are the empty subspace $\emptyset$ (contains no vectors), and we're left with $\emptyset \oplus \emptyset \oplus \cdots \oplus \emptyset = \emptyset$.
- <u>*But*</u>: The state of the system is *somewhere* in $\mathcal{H}$. So the statement that it is "nowhere" (*i.e.*, in the empty subspace) is always false.

Essential Characteristics of QL Interpretation

(A) Rejects *Eigenvector/Eigenvalue Rule*

(B) Rejects of *Projection Postulate*

(C) Probabilities are epistemic

- All 3 characteristics are a result of the QL claim that all properties have determinate values at all times.

Major Problem: If QL says all properties of a system have definite values at all times, this gets around the Measurement Problem, but it then runs up against the Kochen-Specker Theorem!

How QL can get around the KS Theorem:

- First show that, according to QL, to say that every property always has a value is not to say that there is always a value that every property has:

- Let $V_1, V_2, ..., V_N$ and $W_1, W_2, ..., W_N$ be the 1-dim subspaces spanned by the eigenvectors $|a_1\rangle, |a_2\rangle, ..., |a_N\rangle$ and $|b_1\rangle, |b_2\rangle, ..., |b_N\rangle$ of two operators A, B.
- Then $W_i \cap (V_1 \oplus V_2 \oplus \cdots \oplus V_N)$ represents the sentence:

  "The value of property B is b_i *and* property A has a definite value." $(*)$

- And $(W_i \cap V_1) \oplus (W_i \cap V_2) \oplus \cdots \oplus (W_i \cap V_N)$ represents the sentence:

  "(The value of B is b_i *and* the value of A is a_1) *or*
  (the value of B is b_i *and* the value of A is a_2) *or* ... *or* $(**)$
  (the value of B is b_i *and* the value of A is a_N)."

- *Which means*: "The value of B is b_i *and* the value of A lies in $\{a_1, a_2, ..., a_N\}$."
- *Which means*: "The value of B is b_i *and* there is a value that A has."
- *Now*: $W_i \cap (V_1 \oplus V_2 \oplus \cdots \oplus V_N) \neq (W_i \cap V_1) \oplus (W_i \cap V_2) \oplus \cdots \oplus (W_i \cap V_N)$.
- *So*: The sentences $(*)$ and $(**)$ do not mean the same thing!
- *Thus*: To say that property A has a definite value is not to say that there is some definite value $(a_1, a_2, ..., a_N)$ it has!

- *Next*: Define the notion of a "disjunctive property":

> **Def.** A *disjunctive property* is a property that possesses a *disjunction* (a_1 or a_2 or a_3 or ...) of individual values, any one of which the property cannot be said to possess.

- *Now Claim*: All quantum properties are disjunctive properties!

> *How this gets around the KS Theorem*:
>
> - *KS says*: A quantum property may fail to possess a value at a given time.
> - *QL agrees and says*: While a quantum property may fail to possess any given value at a given time, it always possesses a disjunction of all of its values at all times.

- *Lingering Concern*:
    - Under this view, QL is motivated by the desire to view properties realistically.
    - Does the notion of a disjunctive property really provide us with an adequate notion of property realism?