11. Bohm's Theory (Bohmian Mechanics)

Motivation: Replace Hilbert *state space* of QM with one that is more classical *and* reproduces QM predictions.

David Bohm
(1917-1992)

1. Principles of Bohmian Mechanics

I. *States*: The state of a physical system is given by *both* a wave function ψ *and* particle positions.

- *Recall*: In QM, a state is given *entirely* by a wave function ψ.
- In *classical mechanics*, a state is given by *positions* x and *momenta* p:
  - 1 particle needs 6 numbers: $(x, y, z; p_x, p_y, p_z) = 1$ point in 6-dim phase space.
  - N particles need $6N$ numbers: $(x_1, y_1, z_1; p_x^1, p_y^1, p_z^1; \ldots; x_N, y_N, z_N; p_x^N, p_y^N, p_z^N) = 1$ point in $6N$-dim phase space.

Examples of configuration space (state space of positions):

- $q = (x, y, z)$
- $q' = (x', y', z')$

3-dim configuration space for 1 particle

- $Q = (x_1, y_1, z_1, \ldots, x_N, y_N, z_N)$
- $Q' = (x_1', y_1', z_1', \ldots, x_N', y_N', z_N')$

3N-dim configuration space for N particles

Now add specification of ψ for each point in configuration space and get the state space of BM!

II. *Wave Function Dynamics*: The wave function associated with a state evolves according to the *Schrödinger dynamics*:

$$\psi(Q_i, t_i) \xrightarrow[\text{Schrödinger evolution}]{t_i \to t_f} \psi(Q_f, t_f)$$

III. *Particle Dynamics*: Particle velocities are determined by *Bohm's Equation*:

$$\vec{V}_i[\psi(Q)] = \frac{\overrightarrow{dq_i}}{dt} = \frac{\hbar}{m_i} \mathrm{Im}\left(\frac{\psi^* \vec{\partial}_i \psi}{\psi^* \psi}\right)\Bigg|_Q = \frac{\vec{J}_i}{\rho}$$

The *velocity* $\vec{V}_i$ of the ith particle, located at $q_i = (x_i, y_i, z_i)$ ...

... is a function of it's mass m_i and the N-particle wave function $\psi(Q)$, which depends on the positions Q of all the N particles...

... where $\vec{J}_i = (\hbar/m_i)\mathrm{Im}(\psi^* \vec{\partial}_i \psi)$ is the "probability current" and $\rho = \psi^* \psi = |\psi|^2$ is the "probability density".

IV. *The Distribution (or Statistical) Postulate*: At some time t_0, particle positions are given by a probability defined by the wave function at t_0:

$$\Pr(\textit{particle positions are } Q \textit{ at time } t_0) = |\psi(Q, t_0)|^2$$

This entails BM is empirically indistinguishable from QM in the sense that BM reproduces all the QM probability predictions!

Why?

- *QM says*: (*Born Rule*) The probabilities for particle positions at *any* time t are given by $|\psi(Q, t)|^2$.
- *BM says exactly the same thing, because:*
  - The probability density $\rho = |\psi|^2$ is *conserved* by the Schrödinger equation (ρ satisfies the *equation of continuity*: $\partial\rho/\partial t + \vec{\nabla} \cdot \vec{J} = 0$).
  - *So*: If at time t_0, the probabilities are given by $|\psi(Q, t_0)|^2$ (the BM *Distribution Postulate*), then at any future (or past) time t, the probabilities will be given by $|\psi(Q, t)|^2$ (*Born Rule*).

What Principles II, III, and IV are saying:

The point Q (representing positions of all N particles at any given time) moves about configuration space by being "guided" by the wave function ψ*!*

$$Q \xrightarrow[\substack{\textit{particle} \\ \textit{dynamics via } \psi}]{t_i \to t_f} Q'$$

- *One interpretation*: The particles are swept along by the probability current defined by ψ (just like charges that are swept along in an electrical current).

- *Recall 2-slit experiment*: Are the electrons really particles that are being guided by some force that makes them impact the screen in an interference pattern? (Bohm's Theory = "Pilot Wave" theory.)

- *But*: This analogy is not perfect: ψ is a function on configuration space ($6N$-dim for N particles), not physical space (3-dim Euclidean space).
- *So*: ψ literally *isn't* a physical force (like an electric field).
- *But*: Maybe it encodes properties of a physical force.

Characteristics of Bohmian Mechanics

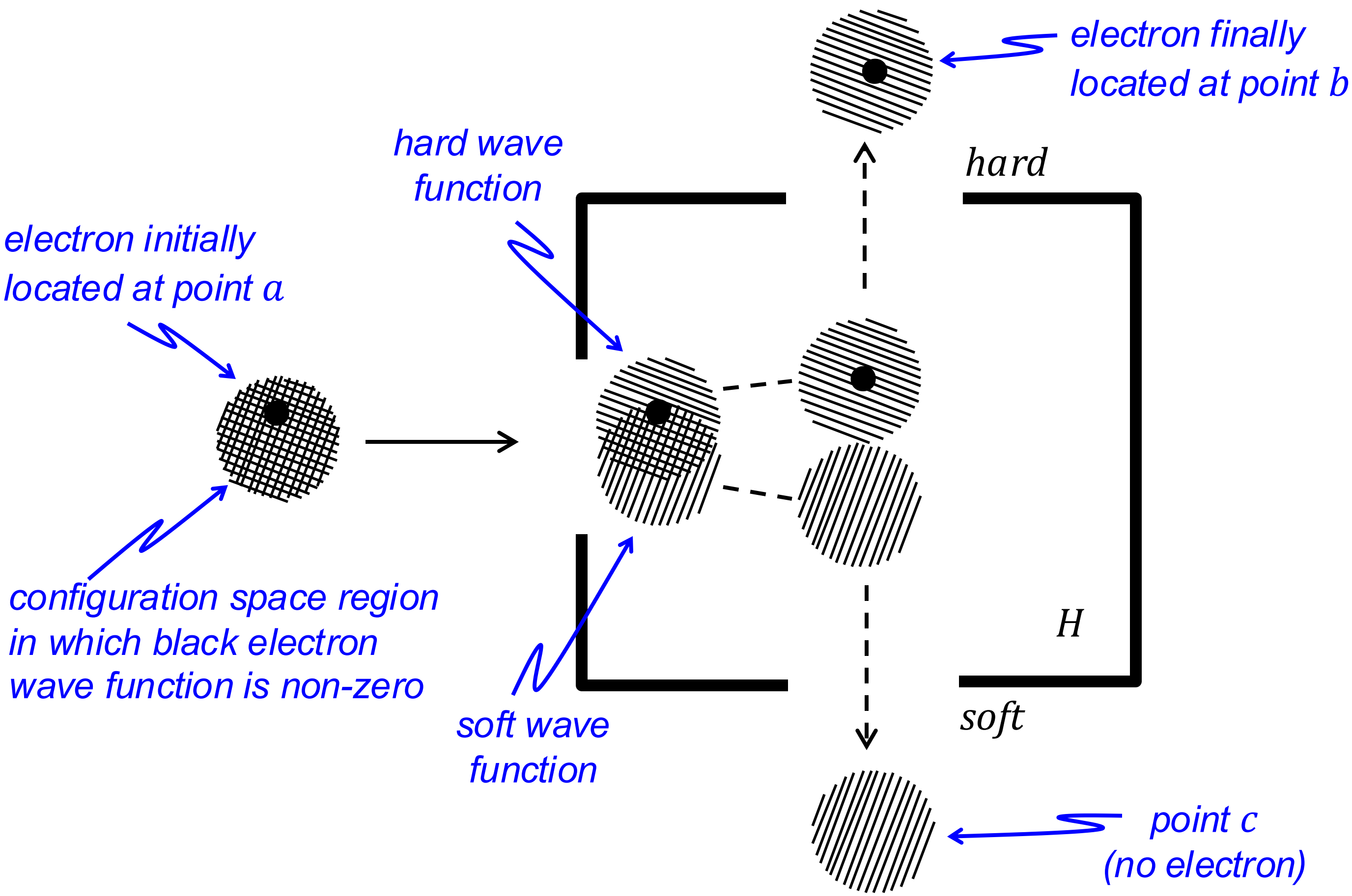
(A) Positions of particles are always determinate. (Particles always have definite positions.)

(B) Positions evolve completely deterministically. (Any initial position state Q evolves to a *unique* final position state Q'.)

(C) BM reproduces the same probability predictions as QM.

But: In BM, probabilities are *epistemic*! Particles *always* have definite positions, and BM probabilities just reflect our ignorance as to what they are.

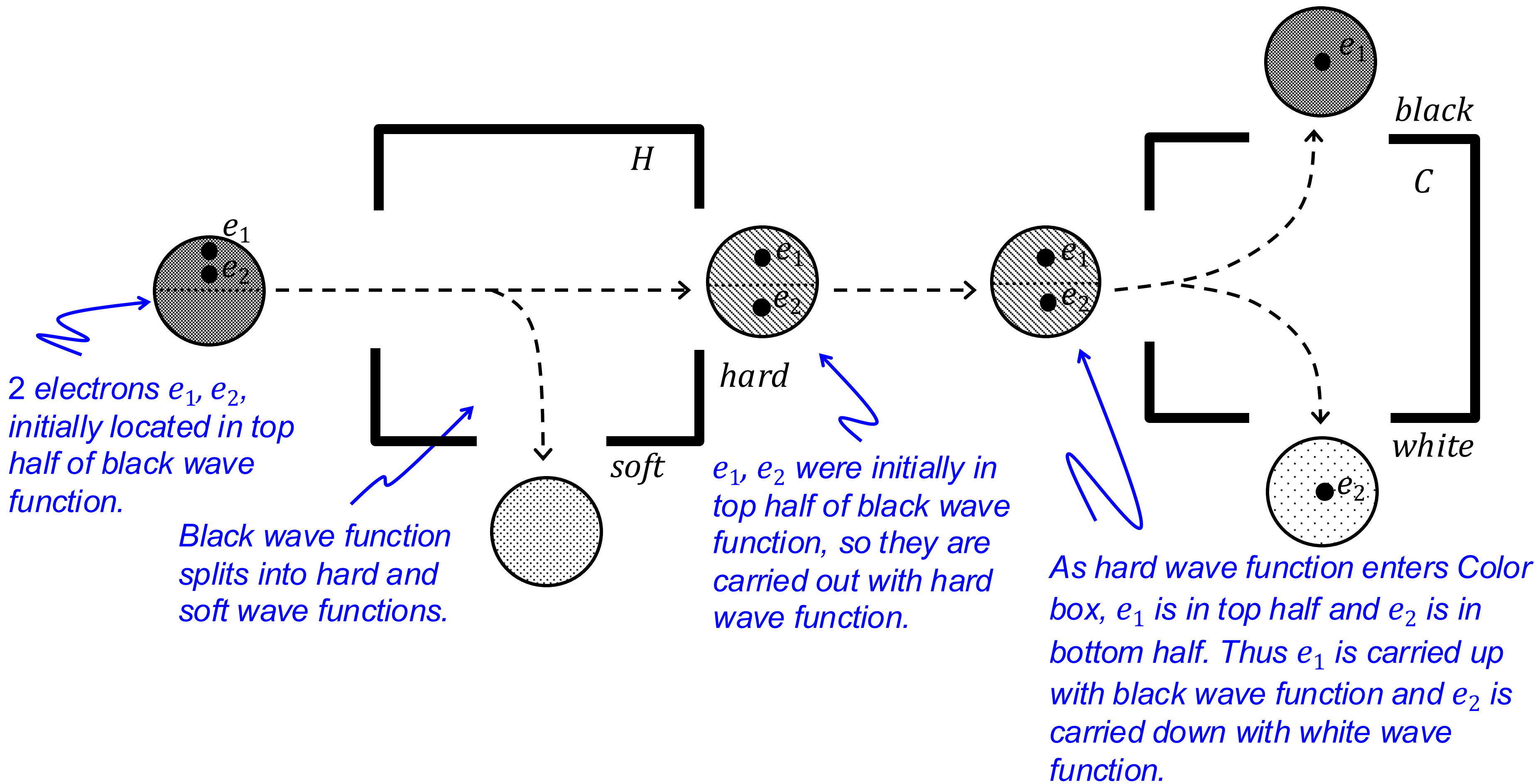
2. Example: Measuring the Hardness of a black electron



- Inside *Hardness* box, *black* wave function "splits" into *soft* and *hard* wave functions.
- Depending on where electron is initially located, it will either be carried up with the *hard* wave function, or down with the *soft* wave function.
- An initial position in upper half of the *black* wave function entails it gets carried up.
- $|black\rangle|\psi_a(x)\rangle \longrightarrow \sqrt{½}\,|hard\rangle|\psi_b(x)\rangle + \sqrt{½}\,|soft\rangle|\psi_c(x)\rangle$

Now: Start with *black* electron. First measure *Hardness*, then *Color*.

- *QM*: Pr(*black*) = Pr(*white*) = ½.
- *BM*: Electron's initial location determines what its final *Color* value will be:



- If a *black* electron is initially located in the top half of the *black* wave function, it has a 50/50 chance of being either in the upper top half or the lower top half.
  - *So*: It has a 50/50 chance of emerging as a *black* electron out of the *Color* box.

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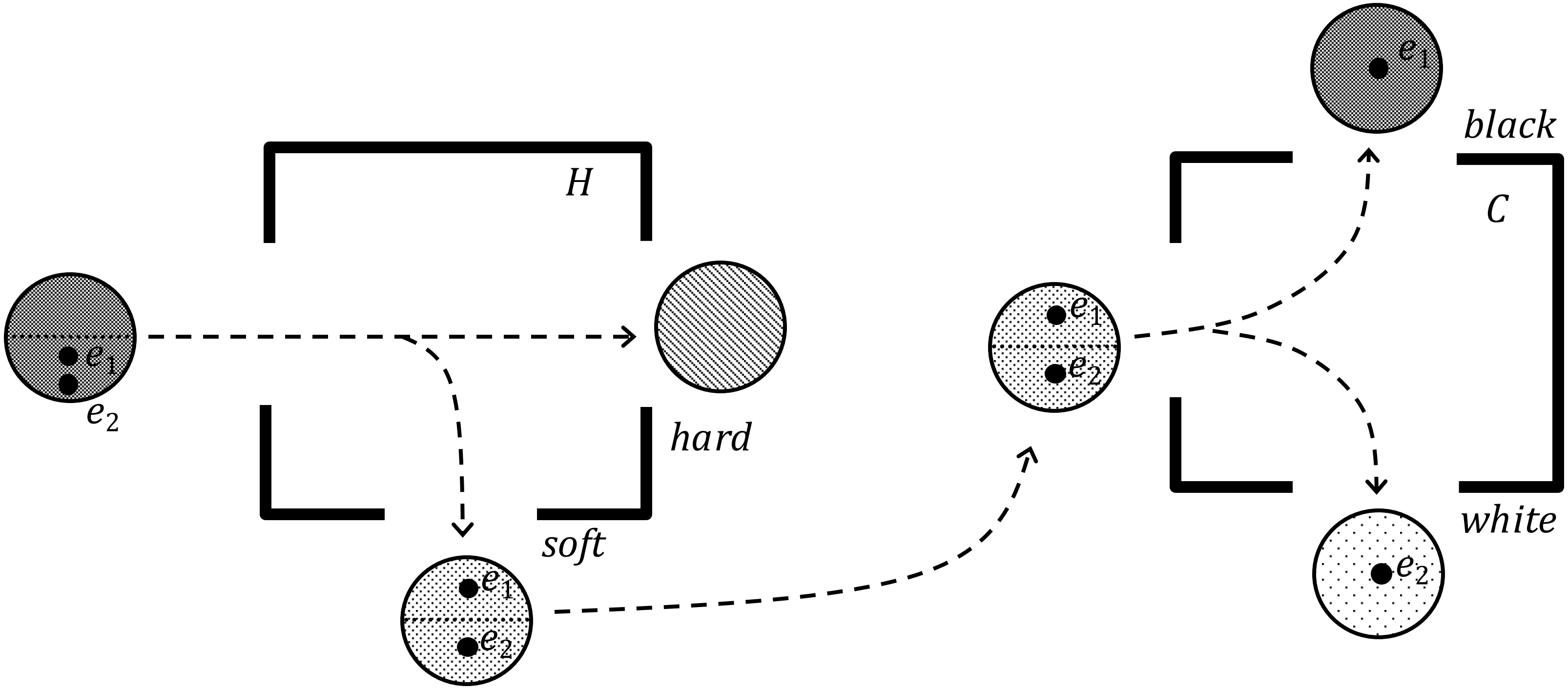
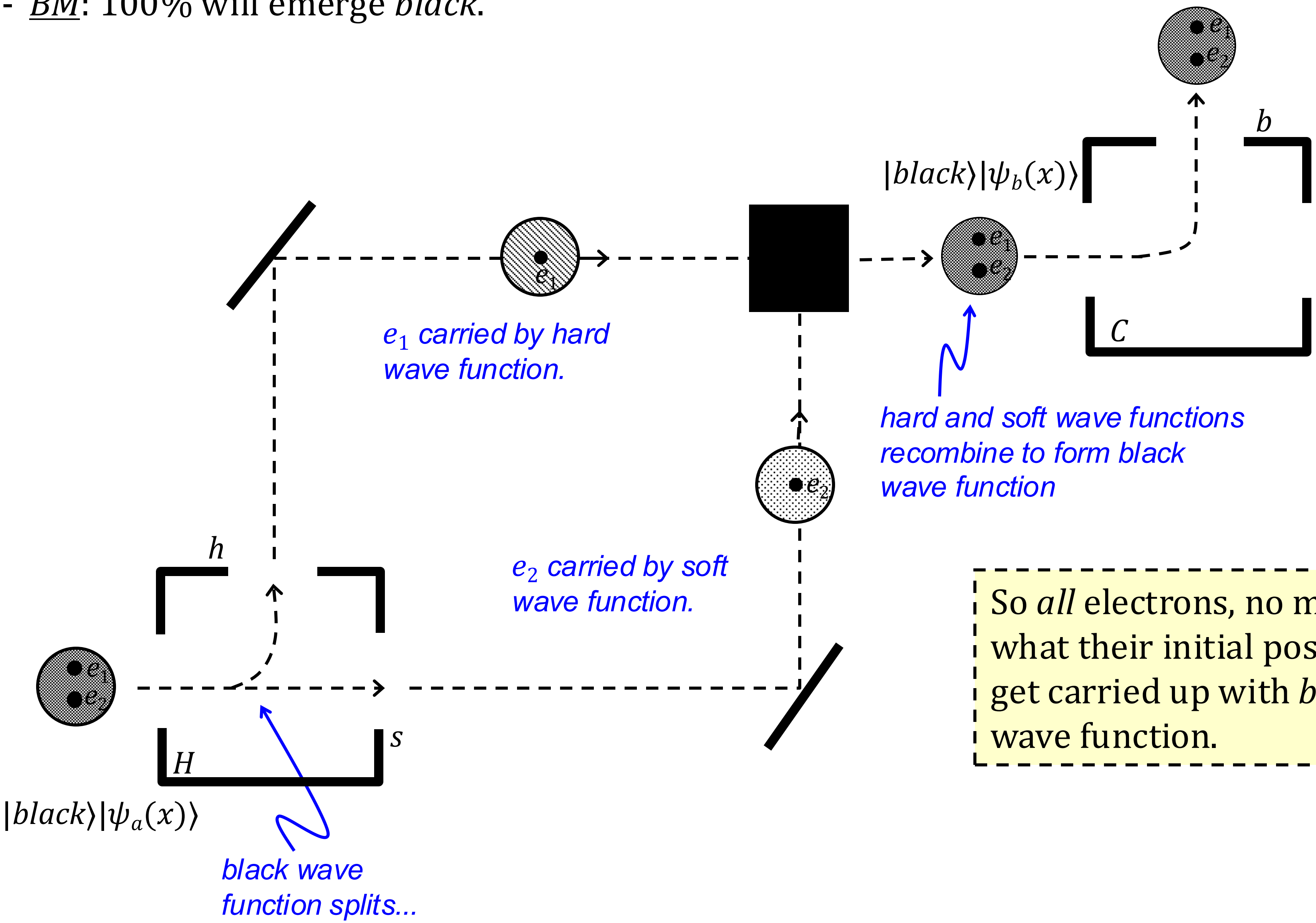


- If a *black* electron is initially located in the bottom half of the *black* wave function, it has a 50/50 chance of being either in the upper bottom half or the lower bottom half.
  - *So*: It has a 50/50 chance of emerging as a *black* electron out of the *Color* box.

Thus: There is a 50/50 chance of the black electron being black after the Color measurement, if all we know is that it is initially located somewhere in the black wave function.

Now: Send *black* electrons through a 2-path device, *without barrier*.

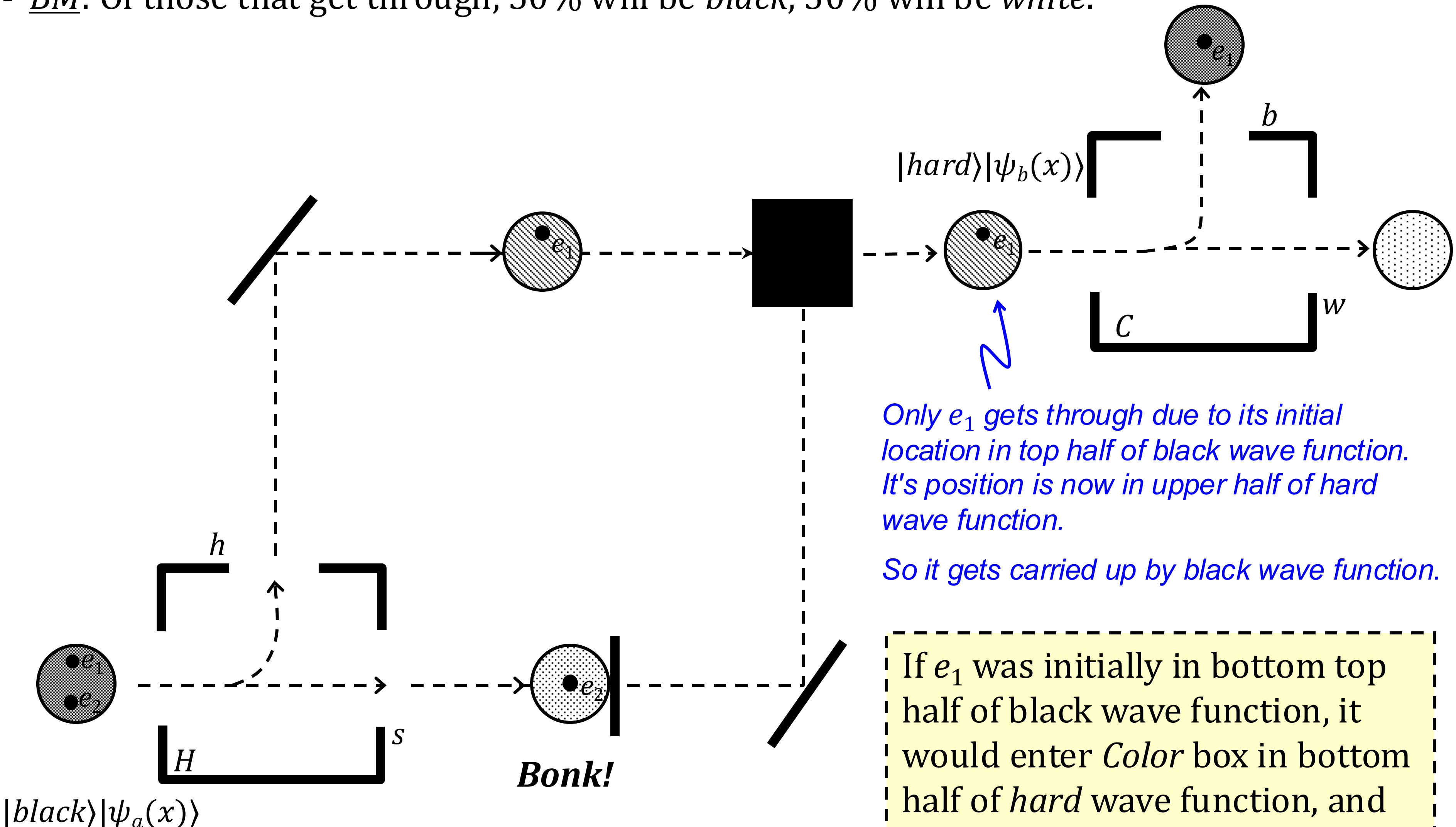
- *QM*: 100% will emerge *black*.
- *BM*: 100% will emerge *black*.



So *all* electrons, no matter what their initial positions, get carried up with *black* wave function.

Now: Send *black* electrons through a 2-path device, *with barrier*.

- *QM*: Of those that get through, 50% will be *black*, 50% will be *white*.
- *BM*: Of those that get through, 50% will be *black*, 50% will be *white*.



Only e_1 gets through due to its initial location in top half of black wave function. It's position is now in upper half of hard wave function.

So it gets carried up by black wave function.

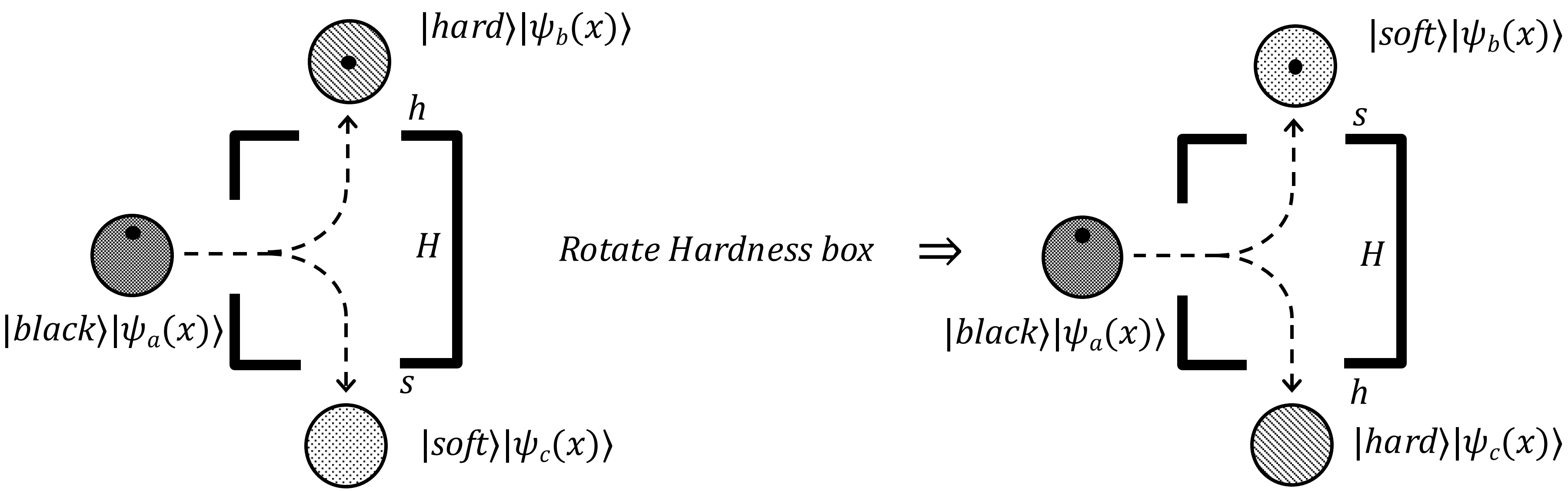
If e_1 was initially in bottom top half of black wave function, it would enter *Color* box in bottom half of *hard* wave function, and exit as a *white* electron!

Suppose e_1 is in upper top half and e_2 is in lower half of black wave function.

3. Contextual Properties

- A property is *intrinsic* just when, whether or not a physical system possesses it does not depend on how it is measured.
- A property is *contextual* just when, whether or not a physical system possesses it depends on how it is measured.

- In BM, position is an intrinsic property; all others are contextual.
- *Ex*: In BM, *Hardness* is a contextual property.



- Electron starts out in same initial location.

Depending on how it is measured, it's Hardness value will be either hard or soft.

4. Locality

- *<u>Consider</u>*: 2 electrons in an entangled state (e_1 at point a, e_2 at point f):

$$\sqrt{½}|hard\rangle_1|\psi_a(x)\rangle_1|soft\rangle_2|\psi_f(x)\rangle_2 - \sqrt{½}\,|soft\rangle_1|\psi_a(x)\rangle_1|hard\rangle_2|\psi_f(x)\rangle_2$$

- Now measure *Hardness* of e_1, with result:

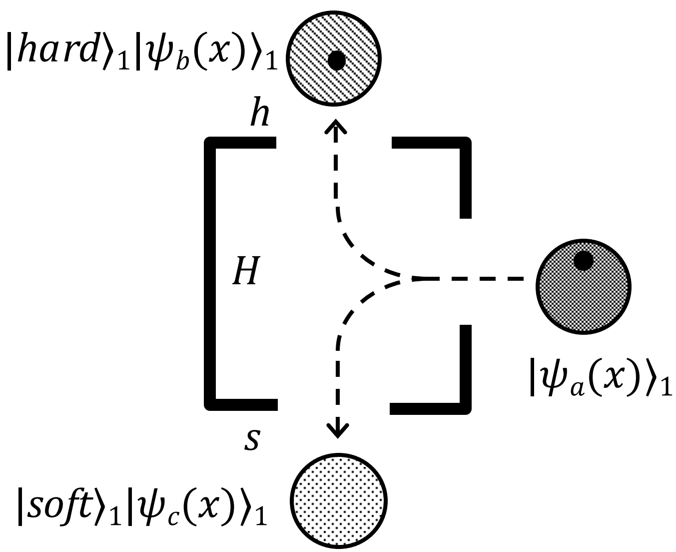
"effectively" zero

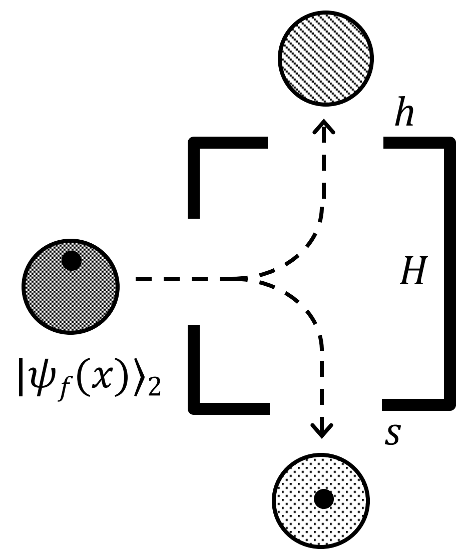
$$\sqrt{½}|hard\rangle_1|\psi_b(x)\rangle_1|soft\rangle_2|\psi_f(x)\rangle_2 - \sqrt{½}\,|soft\rangle_1|\psi_c(x)\rangle_1|hard\rangle_2|\psi_f(x)\rangle_2$$

- Now measure *Hardness* of e_2:

e_2 is carried down through soft exit (only soft wave function acts on it)!

If e_1 had not been measured, then e_2 would have come out hard!

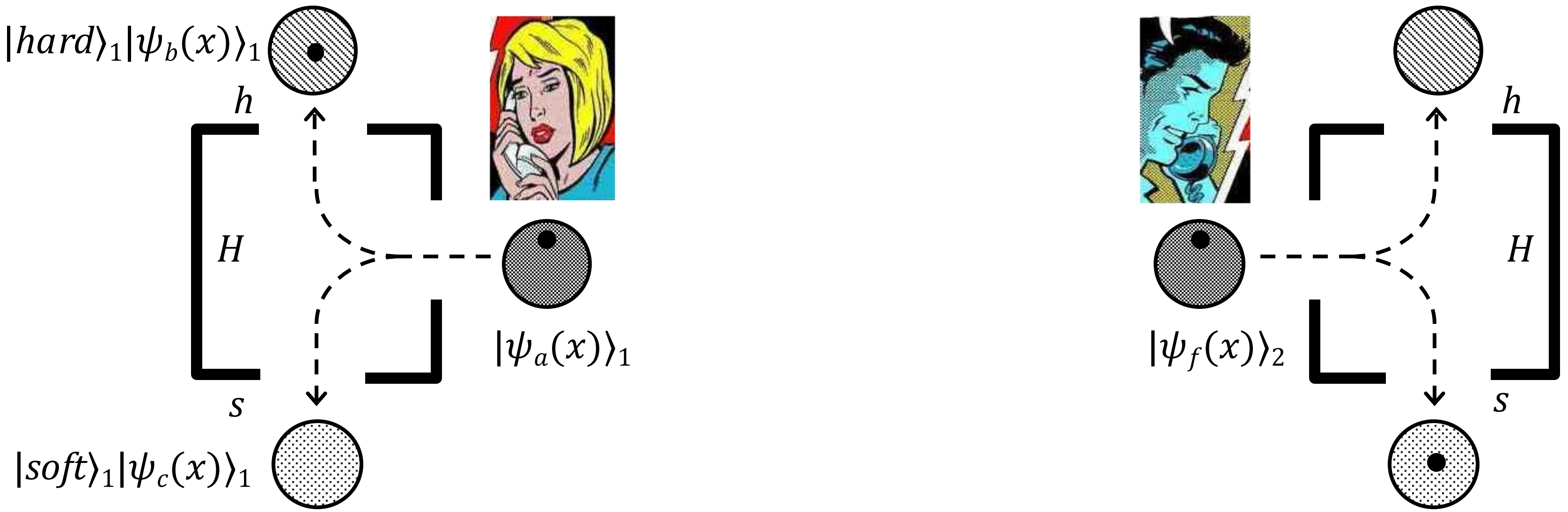




In Bohm's Theory, electrons *always* have a definite position, and the final position of e_2 is determined by the final position of e_1.

- <u>*Suppose*</u>: Alice and e_1 are very far from Bob and e_2.
- <u>*Suppose*</u>: Bob knows the initial positions of e_1 and e_2, and he gets the strange result that e_2 came out *soft* (when it should have come out *hard*, given it's initial location).
- <u>*Then*</u>: Bob knows that Alice way over there must have used a *hard*-side up *Hardness* box to measure e_1!

This allows Bob and Alice to send instantaneous signals to each other!

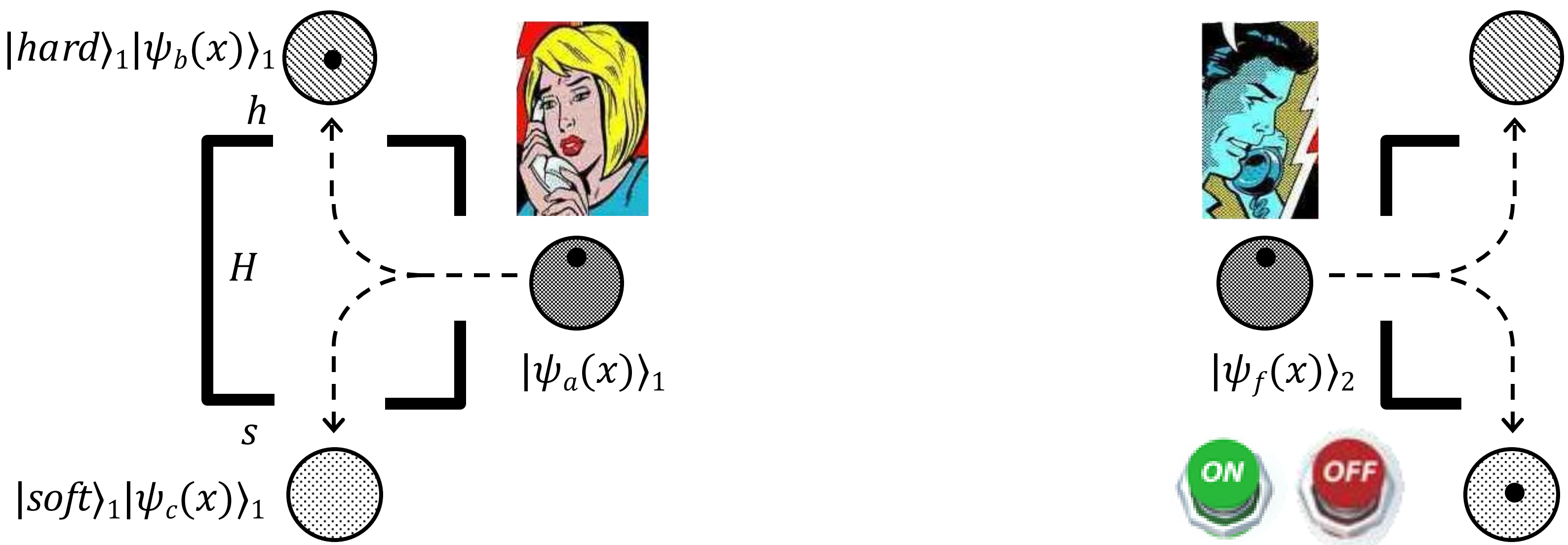


How to send an instantaneous message in BM:

- *Suppose*: Alice desires to send Bob a message instructing him to push either Button *A* or Button *B* at some future time *t*.
- They share initial positions of their e_1 and e_2 and agree to the following:

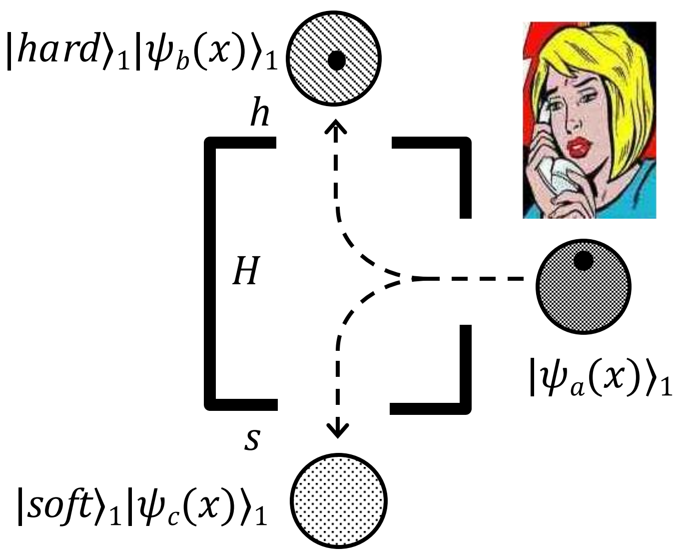
> - If Alice wants Bob to push Button *A*, then before *t* she orients her *Hardness* box so that a *Hardness* measurement will yield the value *hard*.
> - If Alice wants Bob to push Button *B*, then before *t* she orients her *Hardness* box so that a *Hardness* measurement will yield the value *soft*.

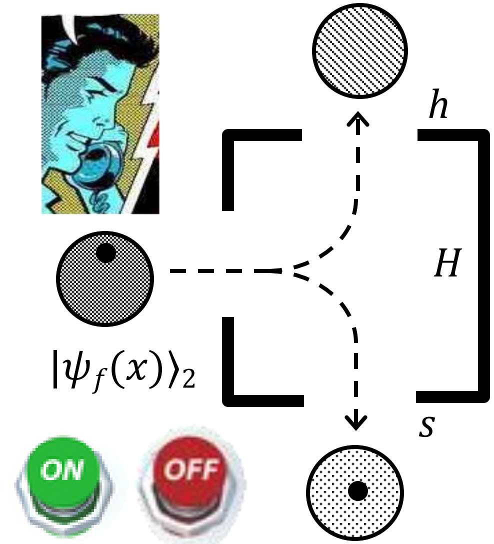
- At *t*, Bob measures his electron: This will tell him what the outcome of Alice's measurement was, and hence which Button she wants him to push!



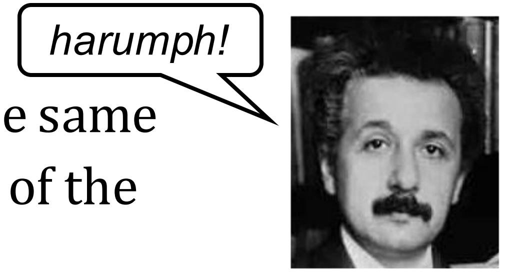
QM vs BM on instantaneous messaging:

- *Under a literal interpretation of QM*:
  - The outcome of an e_2 measurement depends non-locally on the outcome of an e_1 measurement.
  - *But*: The outcome of an e_2 measurement does *not* depend on whether or not an e_1 measurement was done.
- *In BM*:
  - The outcome of an e_2 measurement *does* depend on whether or not an e_1 measurement was done.





Does BM violate Special Relativity?



- In Special Relativity, the simultaneity of distant events in the same inertial reference frame is relative: there is *no* absolute fact of the matter which occurs before the other.
- In BM, there *is* a fact of the matter (a "privileged" reference frame that determines the simultaneity of distant events).

Why? Because, *if* they can instantaneous message, Alice and Bob will always agree on the order of their measurements.

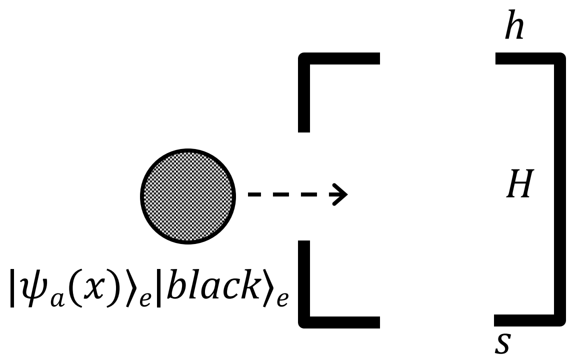
- *So*: BM will violate special relativity, *unless it can explain why the privileged reference frame is in principle unobservable*.

> *Claim*: For any given measurement set-up, the initial positions of particles can *never* be known in BM. *All* that can be known is the wave function.

- *Thus*: *In practice*, instantaneous signaling is *not* possible in BM.
- *So*: *In practice*, the privileged simultaneity frame cannot be determined.
- *And so*: *In practice*, BM does not violate Special Relativity.

Why initial particle positions can never be known in BM

- Consider measuring the *Hardness* of a *black* electron *e*:



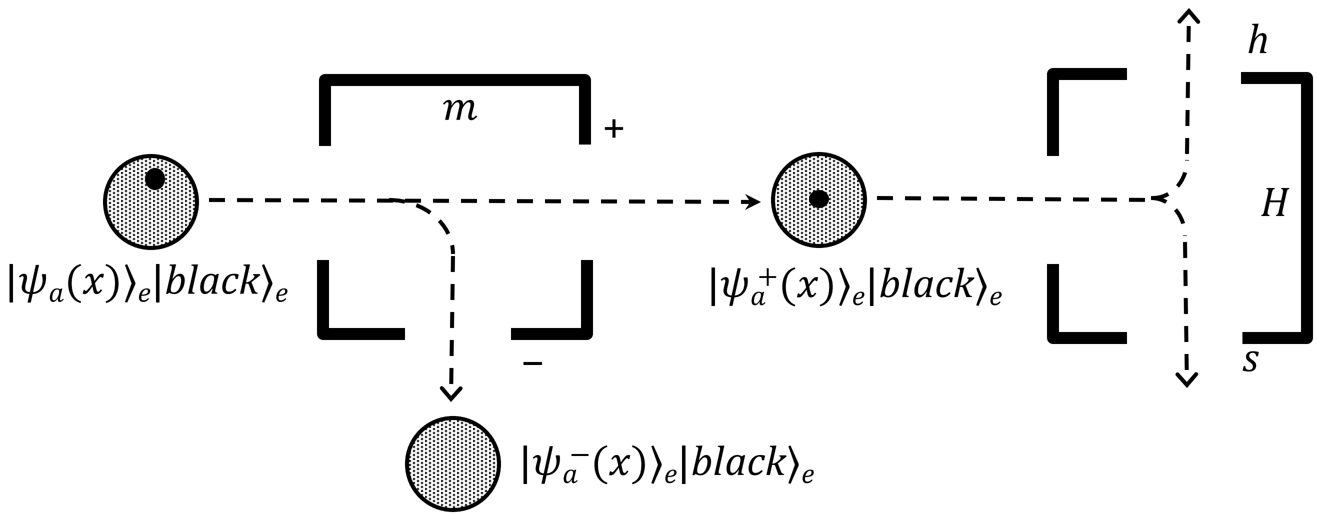
- *If* we could determine *e*'s initial position, then we could predict with certainty which exit it will take:
  - Initially in upper half, then *hard* exit.
  - Initially in lower half, then *soft* exit.
- *So*: How could we determine initial position?
- *Problem*: According to *BM*, any attempt will change the pre-*Hardness* measurement wave function, and so affect all subsequent measurements!

<u>*Suppose*</u>: Before measuring *Hardness* of *e*, we measure its position:

$$|ready\rangle_m|\psi_a(x)\rangle_e|black\rangle_e \rightarrow \sqrt{½}\,|+\rangle_m|\psi_a^+(x)\rangle_e|soft\rangle_e + \sqrt{½}\,|-\rangle_m|\psi_a^-(x)\rangle_e|black\rangle_e$$

- If *e* is measured to be in the *upper-half* of $\psi_a(x)$, then it's (effective) wave function is now $\psi_a^+(x)$.

- This will not allow us to predict how it will move through a *Hardness* device:



- If *e* is initially in upper-half of $\psi_a(x)$, then it will emerge from *m* as $\psi_a^+(x)$.
- <u>*But*</u>: To predict where it will emerge from *H*, we need to know if it's in the *upper-half* or *lower-half* of $\psi_a^+(x)$!
- And to measure this is to disrupt the wave function again!