09. Collapse and GRW

Recall: There are two ways a quantum state can change:

1. In absence of measurement, states change *via Schrödinger dynamics*:

$$|\psi(t_1)\rangle \xrightarrow[\substack{\textit{Schrödinger}\\ \textit{evolution}}]{} |\psi(t_2)\rangle$$

2. In presence of measurement, states change *via* the *Projection Postulate*:

When a measurement of property B is made on a state $|\psi\rangle = a_1|b_1\rangle + \cdots + a_N|b_N\rangle$ expanded in the eigenvector basis of B with result b_i, then $|\psi\rangle$ collapses to $|b_i\rangle$:

$$|\psi\rangle \xrightarrow[\textit{collapse}]{} |b_i\rangle$$

Problems:

- What is a *measurement*?
- *When* is the Projection Postulate supposed to take over from the Schrödinger dynamics?

Typical Attempts at Responses

1. *When a conscious observer looks at a measuring device*.

   Consequence: Dualism--2 fundamental types of physical systems.

   - *Purely physical systems*: Always evolve *via* Schrödinger dynamics.
   - *Conscious systems*: Interact with physical systems in certain situations to cause collapse *via* Projection Postulate.

   Mystery-mongering!

   - *What are conscious systems?*
   - *How are they distinct from physical systems?*

Eugene Wigner
(1902-1995)

2. *When a macroscopic system interacts with a microscopic system*.

   Consequence: Dualism--2 fundamental types of physical systems.

   - *Microscopic systems*: Always evolve *via* Schrödinger dynamics.
   - *Macroscopic systems*: Interact with microscopic systems in certain situations to cause collapse *via* Projection Postulate.

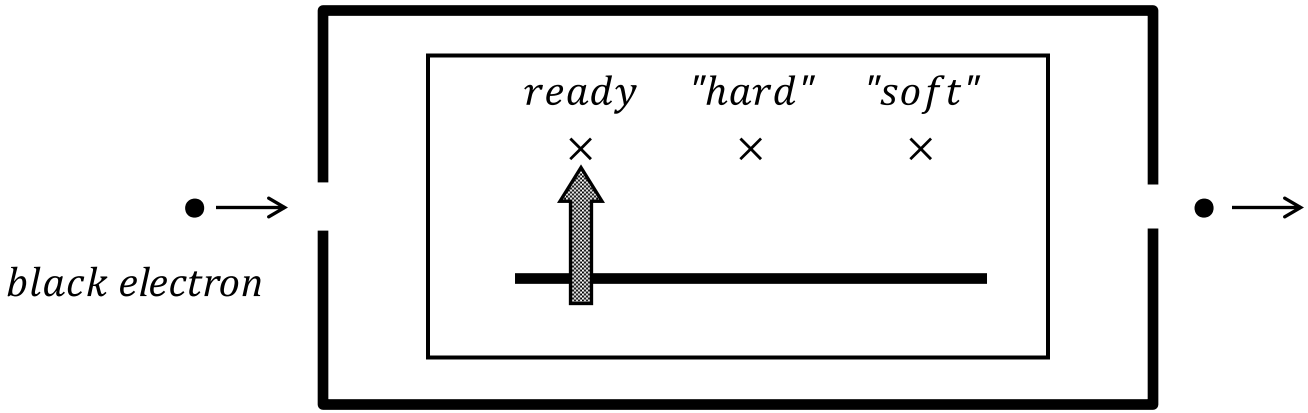
   Mystery-mongering by another name!

   - *What are microscopic systems?*
   - *How are they distinct from macroscopic systems?*

1. Theories of Collapse

<u>*Can experiments determine when a collapse occurs?*</u>

In principle, Yes!
But in practice, No!



A bipartite system m-e

<u>*Let initial state at t_i be*</u>:
$\sqrt{½}\,(|ready\rangle_m|hard\rangle_e + |ready\rangle_m|soft\rangle_e)$

What is final state at $t_f > t_i$?

<u>*Theory 1*</u>: Collapse occurs at t_f.
<u>*So*</u>: At t_f, state will be $|\mathrm{T1}\rangle =$ *either* $|"hard"\rangle_m|hard\rangle_e$ *or* $|"soft"\rangle_m|soft\rangle_e$, each with prob = ½.

<u>*Theory 2*</u>: Collapse occurs *after* t_f.
<u>*So*</u>: At t_f, state will be $|\mathrm{T2}\rangle =$
$\sqrt{½}\,(|"hard"\rangle_m|hard\rangle_e + |"soft"\rangle_m|soft\rangle_e)$

- Suppose the *m-e* system in state $|\mathrm{T2}\rangle$ has a measureable bipartite ("2-system") property that it doesn't have in either of the forms of $|\mathrm{T1}\rangle$.
  - *This would let us distinguish between Theory 1 and Theory 2.*

Task: Find a bipartite property of the m-e system in the state $|\text{T2}\rangle$ that is not possessed by the m-e system in either of the forms of $|\text{T1}\rangle$.

- Recall Lecture 4's *Hardness* H^e and *Color* C^e operators that act on states of e:

$$H^e|hard\rangle_e = +1|hard\rangle_e \qquad C^e|black\rangle_e = +1|black\rangle_e$$

$$H^e|soft\rangle_e = -1|soft\rangle_e \qquad C^e|white\rangle_e = -1|white\rangle_e$$

- Now define *"Hardness"* H^m and *"Color"* C^m operators that act on states of m by:

$$H^m|ready\rangle_m = 0|ready\rangle_m \qquad C^m|ready\rangle_m = 0|ready\rangle_m$$

$$H^m|\text{"}hard\text{"}\rangle_m = +1|\text{"}hard\text{"}\rangle_m \qquad C^m|\text{"}black\text{"}\rangle_m = +1|\text{"}black\text{"}\rangle_m$$

$$H^m|\text{"}soft\text{"}\rangle_m = -1|\text{"}soft\text{"}\rangle_m \qquad C^m|\text{"}white\text{"}\rangle_m = -1|\text{"}white\text{"}\rangle_m$$

- H^m represents the property of m of pointing to either $ready$, $\text{"}hard\text{"}$, or $\text{"}soft\text{"}$.
- $|ready\rangle_m$, $|\text{"}hard\text{"}\rangle_m$, $|\text{"}soft\text{"}\rangle_m$ are states of m that are eigenvectors of H^m.
- C^m represents the property of m of pointing to either $ready$, $\text{"}black\text{"}$, or $\text{"}white\text{"}$.
- $|ready\rangle_m$, $|\text{"}black\text{"}\rangle_m$, $|\text{"}white\text{"}\rangle_m$ are states of m that are eigenvectors of C^m.

Aside: How to represent properties and states of m using column vectors and matrices

$$|ready\rangle_m = \begin{pmatrix}0\\0\\0\end{pmatrix},\quad |"hard"\rangle_m = \begin{pmatrix}0\\1\\0\end{pmatrix},\quad |"soft"\rangle_m = \begin{pmatrix}0\\0\\1\end{pmatrix},$$

$$|"black"\rangle_m = \begin{pmatrix}0\\\sqrt{1/2}\\\sqrt{1/2}\end{pmatrix},\quad |"white"\rangle_m = \begin{pmatrix}0\\\sqrt{1/2}\\-\sqrt{1/2}\end{pmatrix}$$

$$H^m = \begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}\qquad C^m = \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

This entails:

$$|"hard"\rangle_m = \sqrt{1/2}\,(|"black"\rangle_m + |"white"\rangle_m)\quad |"black"\rangle_m = \sqrt{1/2}\,(|"hard"\rangle_m + |"soft"\rangle_m)$$

$$|"soft"\rangle_m = \sqrt{1/2}\,(|"black"\rangle_m - |"white"\rangle_m)\quad |"white"\rangle_m = \sqrt{1/2}\,(|"hard"\rangle_m - |"soft"\rangle_m)$$

And:

$$H^m|ready\rangle_m = 0|ready\rangle_m \qquad C^m|ready\rangle_m = 0|ready\rangle_m$$

$$H^m|"hard"\rangle_m = +1|"hard"\rangle_m \qquad C^m|"black"\rangle_m = +1|"black"\rangle_m$$

$$H^m|"soft"\rangle_m = -1|"soft"\rangle_m \qquad C^m|"white"\rangle_m = -1|"white"\rangle_m$$

Task: Find a bipartite property of the *m*-*e* system in the state $|T2\rangle$ that is not possessed by the *m*-*e* system in either of the forms of $|T1\rangle$.

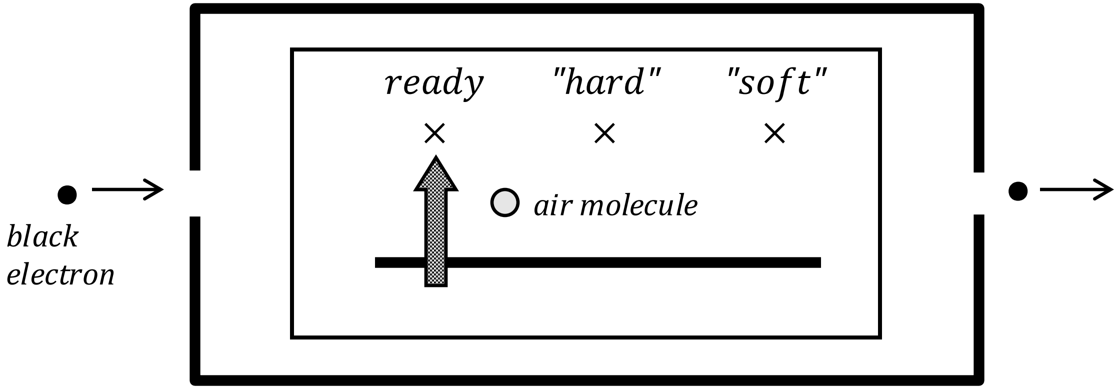
- The bipartite operator $C^m \otimes I^e$ represents *m*'s *"Color"* in the *m*-*e* system.
- The bipartite operator $I^m \otimes C^e$ represents *e*'s *Color* in the *m*-*e* system.
- *And*: The bipartite operator $(C^m \otimes I^e) - (I^m \otimes C^e)$ represents another bipartite property of the *m*-*e* system, call it (*"Color"* − *Color*).

Claim: $|T2\rangle$ is an eigenstate of $(C^m \otimes I^e) - (I^m \otimes C^e)$ with eigenvalue 0, but $|T1\rangle$ in *either* form is *not!*

- *So*: *In principle*, at time t_1 we can measure the bipartite property *"Color"* − *Color*.
  - If Theory 2 is correct, the value should *always* be 0.
  - If Theory 1 is correct, measurements should yield values other than 0.

But this method fails in practice...

What happens if there is a *single air molecule* in the measuring device?



A tripartite system m-e-a!

- Let $|center\rangle_a$ represent state of air molecule located under *"hard"* pointer-reading.
- Let $|right\rangle_a$ represent state of air molecule located under *"soft"* pointer-reading.

Theory 1: Collapse occurs at t_f.

So: At t_f, state will be *either* $|\text{"}hard\text{"}\rangle_m|hard\rangle_e|center\rangle_a$ *or* $|\text{"}soft\text{"}\rangle_m|soft\rangle_e|right\rangle_a$, each with prob = ½.

Theory 2: Collapse occurs *after* t_f.

So: At t_f, state will be $\sqrt{½}\,(|\text{"}hard\text{"}\rangle_m|hard\rangle_e|center\rangle_a + |\text{"}soft\text{"}\rangle_m|soft\rangle_e|right\rangle_a)$.

- These predictions are not eigenstates of (*"Color"* − *Color*)!
- To tell them apart, we need a more complicated, *tripartite* property of the *m*-*e*-*a* system.
- If there are other microscopic subsystems in *m*, we will need more complicated multi-partite properties.

In practice there will be trillions of microscopic subsystems...

2. The GRW Interpretation

Ghirardi, Rimini, Weber (1986)

- *Motivation*: Most (all?) properties can be correlated with the *position* of a pointer in an appropriate measuring device.
- We can maintain Option A1 if we *stipulate* that (for whatever reason) superpositions of *position eigenstates* collapse with high probability.

> *GRW Dynamical Law*
> During any time interval, there is a non-zero probability that the state of an elementary particle will collapse to a position eigenstate.

- Suppose $|\psi\rangle$ represents the state of an elementary particle.
- Expand $|\psi\rangle$ in a basis of eigenvectors of position: $|\psi\rangle = a_1|x_1\rangle + \cdots + a_N|x_N\rangle$.
- *The GRW Collapse Theory says*:
  - $|\psi\rangle$ generally evolves *via* the Schrödinger dynamics.
  - But there is a non-zero chance that $|\psi\rangle$ will collapse to a position eigenstate.
  - If this occurs, the probability of it ending up in the specific position eigenstate $|x_i\rangle$ is given by the Born Rule:

$$Pr(\textit{particle is located at position } x_i \textit{ in state } |\psi\rangle) = |\langle\psi|x_i\rangle|^2 = |\psi(x_i)|^2$$

<u>Why this helps:</u>

Consider the entangled composite state of pointer particles and electron:

$$\sqrt{½}\,\big(\underbrace{|x_a\rangle_1|x_a\rangle_2|x_a\rangle_3\cdots}_{}\big)|hard\rangle_e + \sqrt{½}\,\big(\underbrace{|x_b\rangle_1|x_b\rangle_2|x_b\rangle_3\cdots}_{}\big)|soft\rangle_e$$

Position states of pointer particles when pointer is located at the position, call it x_a, that registers "hard".

Position states of pointer particles when pointer is located at the position, call it x_b, that registers "soft".

- Particle 1 is in a superposition of being located at x_a and x_b, particle 2 is is a superposition of being located at x_a and x_b, *etc*.
- If just *one* of all of these (trillions of) pointer particles has a definite location, then according to the Projection Postulate, the composite state will collapse to the term containing that position eigenstate.
- <u>So</u>: Even if the probability of collapse in the GRW Law is *extremely small*, since any measuring device has trillions of particles, any superposition containing them will have an extremely likely chance of collapsing.

<u>Upshot</u>: By modifying the Schrödinger dynamics with the GRW Law we guarantee that pointers in measuring devices will always have well-defined post-measurement positions.

The GRW Law in terms of Wavefunctions:

- Let $|\psi\rangle = a_1|x_1\rangle + \cdots + a_N|x_N\rangle$ represent the state of an elementary particle expanded in a basis of eigenvectors of position.
- The corresponding *position wavefunction* is defined by $\psi(x) = \langle\psi|x\rangle$.
- $\psi(x)$ encodes all the values of the expansion coefficients. *Ex*: $\psi(x_i) = a_i$.

- *Now*: Represent *state vector* collapse: $|\psi\rangle \xrightarrow[collapse]{} |x_i\rangle$

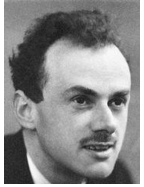
  as *wavefunction* collapse: $\psi(x) \xrightarrow[collapse]{} \psi(x_i)$

  general form of wavefunction for all position eigenstates

  specific form of wave-function for state $|x_i\rangle$

- Mathematically, wavefunction collapse is accomplished by multiplying the general wavefunction $\psi(x)$ by a *Dirac delta function* $\delta(x-x_i)$:

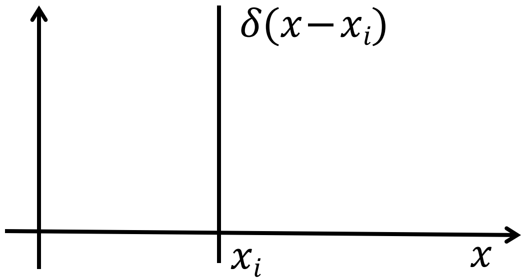
$$"\psi(x_i) = \delta(x-x_i)\psi(x)"$$



Paul Dirac
(1902-1984)

The Dirac Delta Function

- Think of $\delta(x - x_i)$ as a position *eigenfunction*.
- It is an infinite spike exactly at x_i, and zero everywhere else:



$\delta(x - x')$ is defined by the following:

1. $\delta(x - x') = 0$, when $x \neq x'$.
2. $\delta(x - x') = \infty$, when $x = x'$.
3. $\int \delta(x - x')dx' = 1$.
4. $\int \delta(x - x')f(x)dx' = f(x')$, for any $f(x)$.

Not a legitimate function! (Rather, a "distribution".)

So instead of "$\psi(x_i) = \delta(x - x_i)\psi(x)$", we should write $\psi(x_i) = \int \delta(x - x_i)\psi(x)dx$.

Initial Problem:

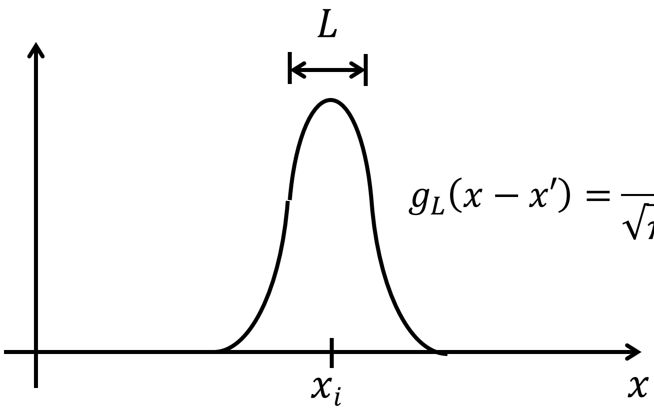
- If our state is represented by an eigenvector of position, then according to the EE Rule, this means that it has a *definite value* of position (namely, x_i).
- *So*: Properties *incompatible* with position will be *maximally* indeterminate.

$$\begin{pmatrix} \textit{Uncertainty in} \\ \textit{position} = 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \textit{Uncertainty in} \\ \textit{momentum} = \infty! \end{pmatrix}$$

- When a GRW collapse occurs (and we get an exact value of position), we could potentially have violations of conservation of momentum and energy!

Technical Solution

- For GRW collapses, instead of multiplying the wavefunction by a Dirac delta function $\delta(x-x_i)$, use a *Gaussian* (*i.e.*, Bell-shaped) function $g_L(x-x_i)$ spread out a finite width L about x_i.



$$g_L(x-x') = \frac{1}{\sqrt{\pi L^2}} e^{-(x-x')/L^2}$$

The Gaussian function *is* a legitimate function.
- Centered at x'.
- Width $= L$.
- Maximum height $= \frac{1}{\sqrt{\pi L^2}}$
- Unit area.
- $\lim_{L\to 0} g_L(x-x') = \delta(x-x')$

- Can choose L so that the uncertainty in momentum is effectively cut-off.
- *The price*: We've had to smear the position about x_i.

Essential Characteristics of GRW Collapse Theory:

1. Modifies Schrödinger dynamics with GRW Law.
2. Keeps Projection Postulate.
3. Introduces two new constants of nature:
   (a) Probability per time per particle of collapse.
   (b) Width *L* of Gaussian position eigenfunctions.

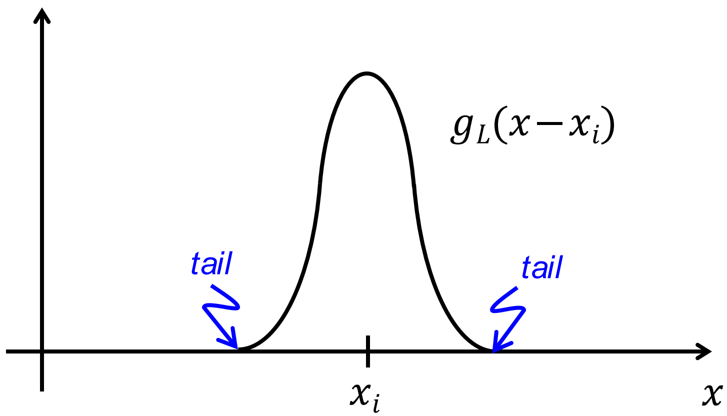
Why #2?

- *Recall*: The Projection Postulate says, when a system aquires a definite value of a property, then the state of the system collapses to the corresponding value-state.
- The GRW Law guarantees that a macroscopic pointer device will always have a definite value of position; the Projection Postulate then entails that the entangled state of the pointer device and the physical system it is measuring will collapse to that term that contains this pointer device value-state.

Problems

1. Wavefunction Tails

- We just saw that GRW needs Gaussian position functions $g_L(x-x_i)$ in order to avoid violations of energy/momentum conservation.
- *But*: $g_L(x-x_i)$ is *never* zero. It has *non-vanishing tails*.



- The tails asymptotically approach the x-axis, but never reach it, no matter how far away from x_i you get.
- $g_L(x-x_i) \neq 0$ for all finite values of x.

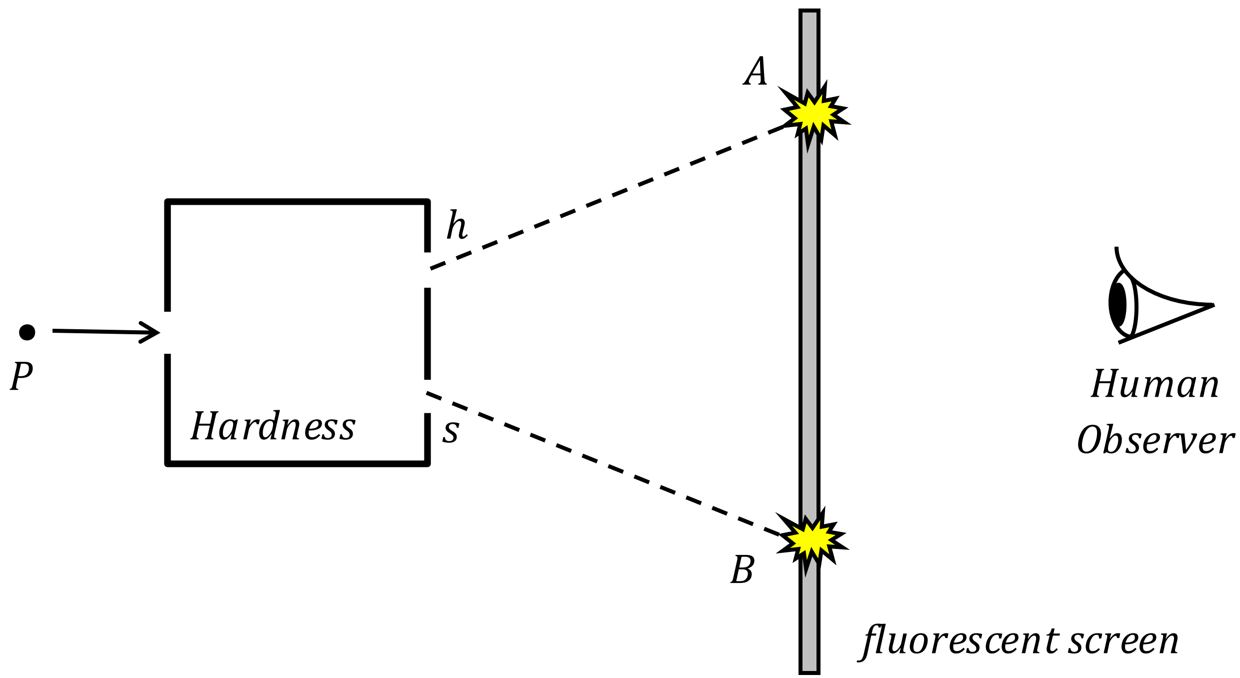
- *So*: Even after a GRW collapse, an elementary particle is *still* in a superposition of position eigenvectors.
  - *There is still a non-zero probability of finding it at some location $x \neq x_i$.*
- *Thus*: According to the EE Rule, it *still* has no definite value of position.
  - *When GRW agreed to use Gaussians instead of Dirac delta functions, they gave up exact position collapses.*

2. Positionless Measurements

- GRW assume that the position property is *fundamental*, in-so-far as all other properties must be correlated with the positions of pointers in measuring devices in order to measure them.

Is this correct?

- *Ex*: Measuring a particle's *Hardness* by means of a fluorescent screen.
  - To measure *Hardness* of *P*, insert it into *Hardness* box.
  - If it's $hard$, it will exit at h and impact screen at A.
  - If it's $soft$, it will exit at s and impact screen at B.



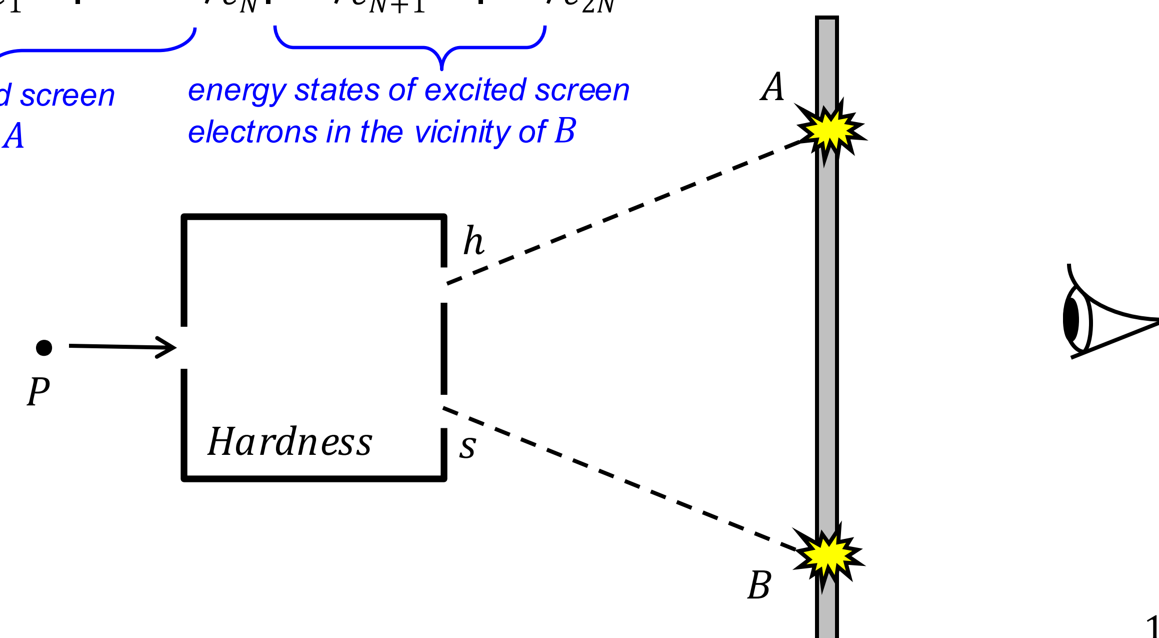
Claim: At no point in this process is the *Hardness* of the particle correlated with the *position* of a pointer (or any sort of measuring device).

- The impact of P at A or B correlates its *Hardness* with the *energies* (and not positions) of the electrons in the atoms of the fluorescent screen.
- *Suppose*: P is initially *black*.
- *Then*: Just after impact, its state can be represented by:

$$\sqrt{1/2}\,|hard, x = A\rangle_P \overbrace{|ex\rangle_{e_1}\cdots|ex\rangle_{e_N}}^{\text{energy states of excited screen electrons in the vicinity of } A} \overbrace{|unex\rangle_{e_{N+1}}\cdots|unex\rangle_{e_{2N}}}^{\text{energy states of unexcited screen electrons in the vicinity of } B}$$

$$+ \sqrt{1/2}\,|soft, x = B\rangle_P \underbrace{|unex\rangle_{e_1}\cdots|unex\rangle_{e_N}}_{\text{energy states of unexcited screen electrons in the vicinity of } A} \underbrace{|ex\rangle_{e_{N+1}}\cdots|ex\rangle_{e_{2N}}}_{\text{energy states of excited screen electrons in the vicinity of } B}$$

A GRW collapse of any of the energy states of the electrons onto a position eigenstate will not cause the state of P to collapse into one or the other of these terms.



What about the photons that the electrons emit?

- *Don't they experience GRW collapses?*

- *No!*

- The GRW Collapse Law doesn't apply to relativistic objects (like photons).
- *Moreover*: There can be just a few photons released on impact to record the *Hardness* measurement (the human eye can discern photons at very low intensities).
- The GRW Collapse Law only guarantees that one of a *large number* (trillions!) of elementary particles will collapse for real times.

- *So*: To address this problem, GRW advocates will have to push back the process of measurement, perhaps to brain states in the human observer, to a point at which they can say the positions of *something* (brain neuron states?) get correlated with the *Hardness* property of *P*.

3. Microscopic Measurements

- Due to the *randomness* of the GRW collapse, collapses will only occur in real times for *macrosopic* measuring devices (with trillions of elementary particles).
- *But*: What about the possibility of *microsopic* measuring devices?
  - *These would not be expected to have GRW collapses in real times.*
  - *If they could be correlated with macroscopic measuring devices, we would have the measurement problem all over again!*