

06. EPR & Bell Thought Experiments

1. EPR Thought Experiment
2. Bell Thought Experiment

How Should Superpositions be Interpreted? Part 1.

(A) Literally

QM description is complete; probabilities are *ontic*.

Sample Claim: The properties of a quantum system in a superposed state are *indeterminate* (do not possess values).

(B) Non-literally

QM description is incomplete; probabilities are *epistemic*.

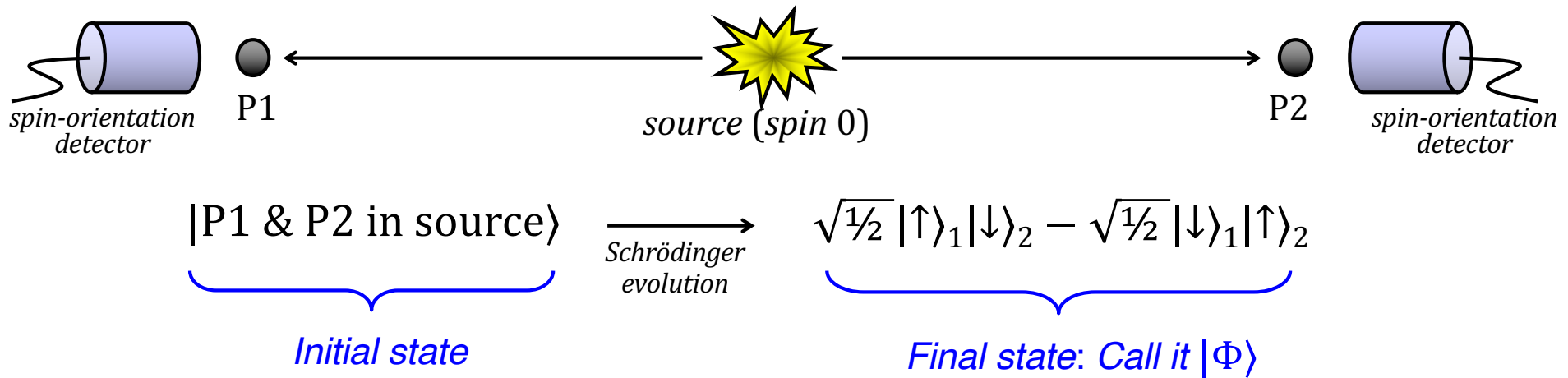
Sample Claim: The properties of a quantum system are *determinate* (possess values) at all times, even when the system is in a superposed state.

EPR pushes towards (B).

Bell pushes back.

1. EPR Thought Experiment

(Einstein, Podolsky, Rosen 1935)



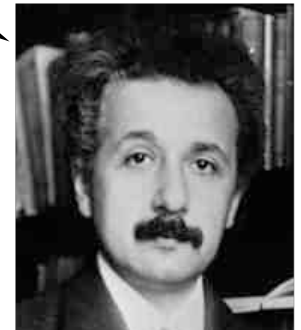
- Final state $|\Phi\rangle$ is an *entangled* 2-particle state.
- If measurement on P1 yields *spin-up*, then $|\Phi\rangle \xrightarrow{\text{collapse}} |\uparrow\rangle_1 |\downarrow\rangle_2$.

Suppose we interpret superpositions literally

- Before measurement, spin orientations of P1 and P2 in final state are both *indeterminate*.
- After a measurement of P1 that yields *spin-up*, P2 instantaneously has a *determinate* value of *spin-down*!

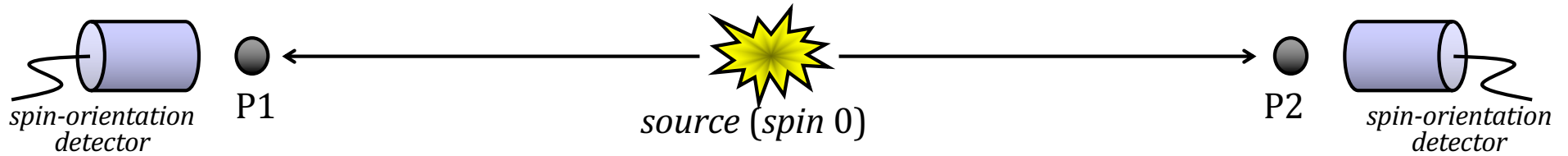
"Spooky action at a distance"!

This is the case no matter how far apart P1 and P2 have traveled!



1. EPR Thought Experiment

(Einstein, Podolsky, Rosen 1935)



Conclusion

- (i) *Either* adopt a literal interpretation of superpositions, and accept "Einstein non-locality".
- (ii) *Or* accept that QM is incomplete.

- "Einstein non-locality" = "spookly action at a distance".
- Einstein, Podolsky & Rosen pick (ii): Superpositions should not be interpreted literally; in particular, properties always have determinate values.

1. EPR Thought Experiment (Einstein, Podolsky, Rosen 1935)



Why is Einstein non-locality so spooky?

- P1 and P2 are in an entangled state and they are *correlated*:
 - *The value of spin that P1 possesses depends on the value of spin that P2 possesses.*
- What explains this correlation?
 - *The correlation is instantaneous: When P1 is found to have a value of spin, P2 instantaneously has the opposite value.*
 - *And we cannot explain this in terms of a causal signal that P1 might have sent to P2 (since by assumption causal signals don't travel instantaneously).*

So: Einstein non-locality occurs when two systems are correlated and the correlation cannot be explained by a direct cause that travels from one system to the other.

But what about a "common cause"?

2. Bell Thought Experiment (Bell 1964)

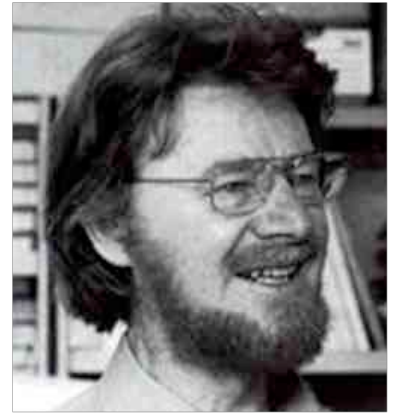
- If QM is incomplete, then perhaps a "*Hidden Variables*" description of quantum states and properties is possible in which properties are always determinate (possess values) at all times.
- Can we compare QM to such a Hidden Variables Theory?

Yes!

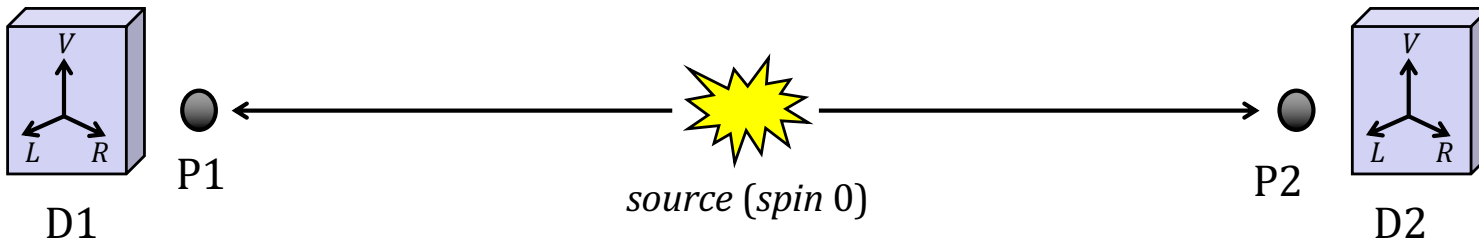
- And: The predictions about certain correlations that QM makes are confirmed by experiment, while those that a Hidden Variables Theory makes are not.

Moreover:

- The QM correlations cannot be explained by a *direct cause* (they violate "Einstein locality").
- The QM correlations cannot even be explained by a *common cause* (they violate "Bell locality")!



John Stewart Bell
(1928-1990)



Set-Up:

- D1 and D2 measure spin along one of three axes (V , R , L) oriented at 120° with respect to each other.
- D1 and D2 are set so that they do not measure spin along the same axis.

Question: What is the probability that P1 and P2 have *different* spin orientations (one spin-up and the other spin-down)?

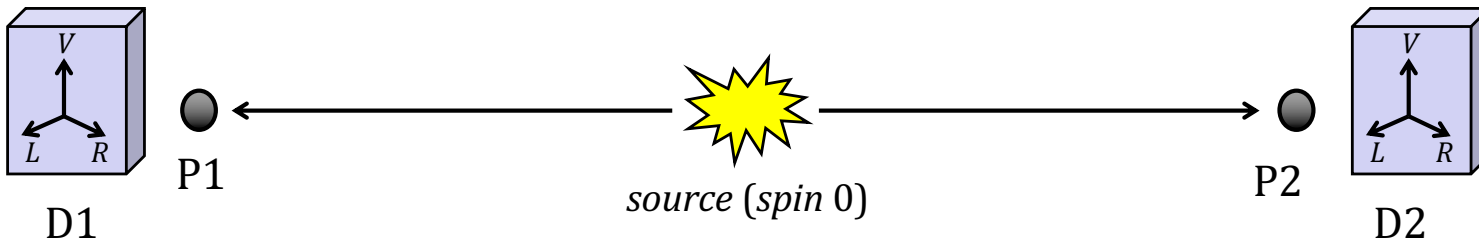
Method 1 (Literal QM)

Properties do not have definite values before measurement.

Method 2 (Hidden Variables)

- *Determinateness*: Properties always have values.
- *Einstein Locality*: No spooky action-at-a-distance.
- *Bell Locality*: Measurement outcomes are determined by source (common cause).

- Bell (1964): Methods 1 and 2 make different predictions!
- Freedman & Clauser (1972): Experiments confirm Method 1's predictions!



Method 1 (Literal QM): One pre-measurement state, three ways of writing it:

$$|P1 \text{ \& } P2 \text{ in source}\rangle \xrightarrow{\text{Schrödinger evolution}} \sqrt{1/2} |\uparrow_V\rangle_1 |\downarrow_V\rangle_2 - \sqrt{1/2} |\downarrow_V\rangle_1 |\uparrow_V\rangle_2 \quad (1)$$

$$\text{OR} \quad \sqrt{1/2} |\uparrow_R\rangle_1 |\downarrow_R\rangle_2 - \sqrt{1/2} |\downarrow_R\rangle_1 |\uparrow_R\rangle_2 \quad (2)$$

$$\text{OR} \quad \sqrt{1/2} |\uparrow_L\rangle_1 |\downarrow_L\rangle_2 - \sqrt{1/2} |\downarrow_L\rangle_1 |\uparrow_L\rangle_2 \quad (3)$$

Source doesn't
determine spin
values! ("Bell
non-locality")

Technical Result: How to relate states for spins along different axes z, z'

$$|\uparrow_z\rangle = \cos(\theta/2) |\uparrow_{z'}\rangle + \sin(\theta/2) |\downarrow_{z'}\rangle, \quad \theta = \text{angle between } z \text{ and } z'$$

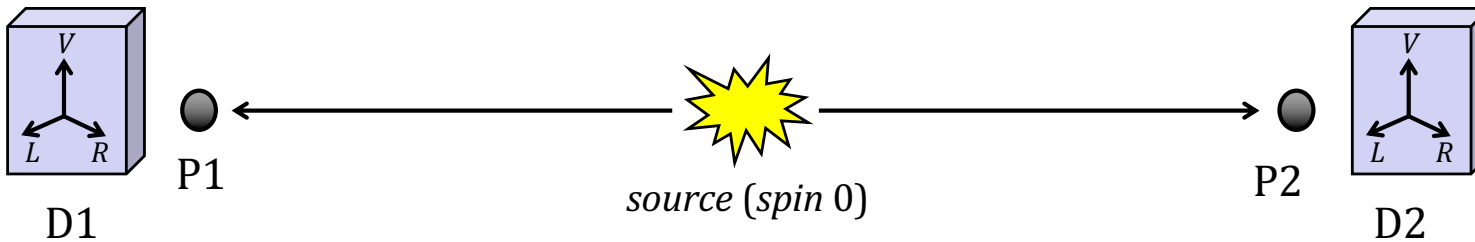
EX: $|\uparrow_R\rangle_2 = \cos(120^\circ/2) |\uparrow_V\rangle_2 + \sin(120^\circ/2) |\downarrow_V\rangle_2$

Claim: $\Pr(P2 \uparrow_V, \text{ given } P1 \downarrow_R) = 1/4$

Proof: $\Pr(P2 \uparrow_V, \text{ given } P1 \downarrow_R) = \Pr(P2 \uparrow_V, \text{ given } P2 \uparrow_R)$ by (2): If $P1$ is \downarrow_R , then $P2$ must be \uparrow_R
 $= |\cos(120^\circ/2)|^2 = 1/4$ by the technical result

Claim extends to general case:

$$\Pr(P1 \text{ and } P2 \text{ have different spin orientations}) = 1/4$$



Method 2 (Hidden Variables): 8 possible pre-measurement states:

$$\begin{array}{cccc}
 \left[\begin{array}{l} (\uparrow_V \uparrow_L \uparrow_R)_1 \\ (\downarrow_V \downarrow_L \downarrow_R)_2 \end{array} \right]_I & \left[\begin{array}{l} (\uparrow_V \uparrow_L \downarrow_R)_1 \\ (\downarrow_V \downarrow_L \uparrow_R)_2 \end{array} \right]_{II} & \left[\begin{array}{l} (\downarrow_V \downarrow_L \downarrow_R)_1 \\ (\uparrow_V \uparrow_L \uparrow_R)_2 \end{array} \right]_{III} & \left[\begin{array}{l} (\downarrow_V \downarrow_L \uparrow_R)_1 \\ (\uparrow_V \uparrow_L \downarrow_R)_2 \end{array} \right]_{IV} \\
 \left[\begin{array}{l} (\downarrow_V \uparrow_L \uparrow_R)_1 \\ (\uparrow_V \downarrow_L \downarrow_R)_2 \end{array} \right]_V & \left[\begin{array}{l} (\uparrow_V \downarrow_L \downarrow_R)_1 \\ (\downarrow_V \uparrow_L \uparrow_R)_2 \end{array} \right]_{VI} & \left[\begin{array}{l} (\uparrow_V \downarrow_L \uparrow_R)_1 \\ (\downarrow_V \uparrow_L \downarrow_R)_2 \end{array} \right]_{VII} & \left[\begin{array}{l} (\downarrow_V \uparrow_L \downarrow_R)_1 \\ (\uparrow_V \downarrow_L \uparrow_R)_2 \end{array} \right]_{VIII}
 \end{array}$$

Source
determines
spin values!
("Bell locality")

Device settings		States							
D1	D2	I	II	III	IV	V	VI	VII	VIII
V	L	↑↓	↑↓	↓↑	↓↑	↓↓	↑↑	↑↑	↓↓
V	R	↑↓	↑↑	↓↑	↓↓	↓↓	↑↑	↑↓	↓↑
L	V	↑↓	↑↓	↓↑	↓↑	↑↑	↓↓	↓↓	↑↑
L	R	↑↓	↑↑	↓↑	↓↓	↑↓	↓↑	↓↓	↑↑
R	V	↑↓	↓↓	↓↑	↑↑	↑↑	↓↓	↑↓	↓↑
R	L	↑↓	↓↓	↓↑	↑↑	↑↓	↓↑	↑↑	↓↓
Prob different spin orientation		1	1/3	1	1/3	1/3	1/3	1/3	1/3

Measurement of
one particle does
not determine
value of other!
("Einstein locality")

$$\Pr(P1 \text{ and } P2 \text{ have different spin orientations}) \geq 1/3$$

Recap

- Literal QM Prediction:

$$\Pr(P1 \text{ and } P2 \text{ have different spin orientations}) = 1/4$$

- Hidden Variables Prediction:

$$\Pr(P1 \text{ and } P2 \text{ have different spin orientations}) \geq 1/3$$

Literal QM says

In 1 out of 4 trials, on average, the spin orientations of P1 and P2 will differ.

Hidden Variables says

At the least, in 1 out of 3 trials, on average, the spin orientations of P1 and P2 will differ.

Do many trials...

...result is always Literal QM prediction!

↖ *There are correlations in nature that violate Einstein locality and Bell locality (no direct cause or common cause explanations)!*

Current Options

Value Definiteness (VD)

The properties of a quantum system are determinate (possess values) at all times, even when the system is in a superposed state.

- EPR say: Either QM is incomplete, or QM violates Einstein non-locality.

- Options for advocates of completeness:

(1) Local Hidden Variables Theory based on VD.

 **Bell says: NO!**
Conflicts with experiment!

- But what about:

(2) Non-local Hidden Variables Theory based on VD.

- In particular: Is Einstein non-locality really so "spooky"?

Why Einstein Non-Locality Isn't All That Spooky

Recall: EPR state is represented by

$$|A\rangle = \sqrt{1/2} |\uparrow\rangle_1 |\downarrow\rangle_2 - \sqrt{1/2} |\downarrow\rangle_1 |\uparrow\rangle_2$$

- If the outcome of a spin measurement on P1 is *spin-up*, then

$$|A\rangle \xrightarrow{\text{collapse}} |\uparrow\rangle_1 |\downarrow\rangle_2$$

- So: The outcome of a spin measurement on P2 will be *spin-down*.
- And: If the outcome of a spin measurement on P1 is *spin-down*, then the outcome of a spin measurement on P2 will be *spin-up*.

What this means

- The outcome of a measurement on P2 depends non-locally on the outcome of a measurement on P1 (and *vice-versa*).
- But: The outcome of a measurement on P2 does *not* depend on whether or not a measurement was performed on P1.

Check:

1. Suppose a spin measurement is done on P2.
 - Then $\Pr(P2 \text{ spin-up}) = \frac{1}{2}$ and $\Pr(P2 \text{ spin-down}) = \frac{1}{2}$.
 2. Suppose a spin measurement is done on P1 and then another is done on P2.
 - Then $\Pr(P1 \text{ spin-up}) = \frac{1}{2}$ and $\Pr(P1 \text{ spin-down}) = \frac{1}{2}$.
 - If P1 does have *spin-up*, then P2 will have *spin-down*.
 - If P1 does have *spin-down*, then P2 will have *spin-up*.
- Thus: The outcome of a measurement on P2 is *equally likely* to be *spin-up* or *spin-down*, *regardless* of whether or not a measurement was performed on P1!
 - Upshot: Einstein non-locality of outcome dependence can't be used to send signals.

Ex: If we measure P2 *here* to have *spin-down*, then we know P1 *over there* has *spin-up*.

- But we *don't* know if P1 was already found to have *spin-up*: We don't know if P2's having *spin-down here* is a consequence of someone over there measuring P1 to have *spin-up*.

- So: Einstein non-locality doesn't violate a prohibition on faster-than-light signalling that can be associated with Special Relativity.