06. EPR & Bell Thought Experiments

How Should Superpositions be Interpreted? Part 1.

(A) Literally

QM description is complete; probabilities are *ontic*.

> *Sample Claim*: The properties of a quantum system in a superposed state are *indeterminate* (do not possess values).

(B) Non-literally

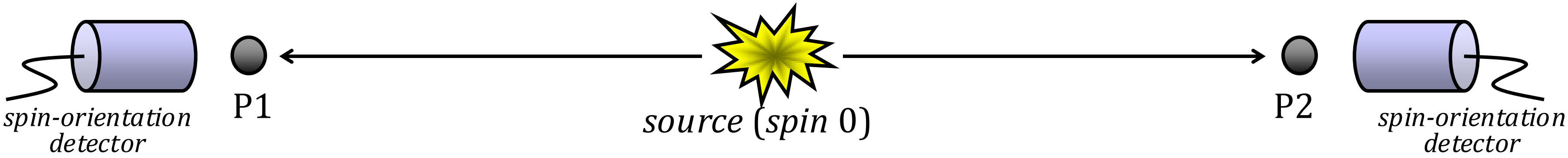
QM description is incomplete; probabilities are *epistemic*.

> *Sample Claim*: The properties of a quantum system are *determinate* (possess values) at all times, even when the system is in a superposed state.

EPR pushes towards (B).

Bell pushes back.

1. EPR Thought Experiment (Einstein, Podolsky, Rosen 1935)

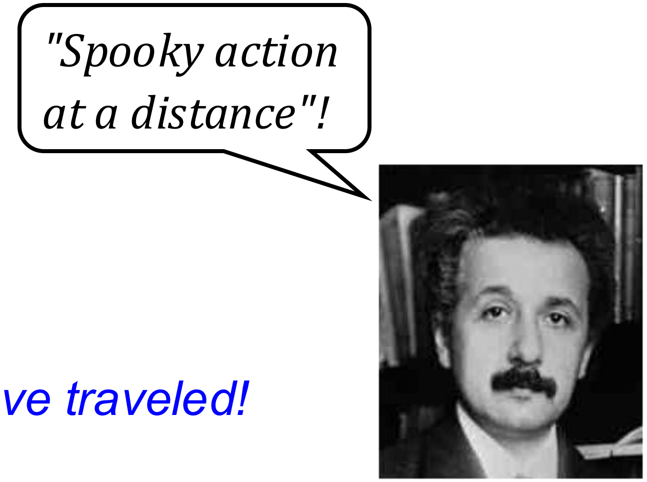


$$\underbrace{|\text{P1 \& P2 in source}\rangle}_{\textit{Initial state}} \xrightarrow[\textit{evolution}]{\textit{Schrödinger}} \underbrace{\sqrt{1/2}\,|\uparrow\rangle_1|\downarrow\rangle_2 - \sqrt{1/2}\,|\downarrow\rangle_1|\uparrow\rangle_2}_{\textit{Final state: Call it } |\Phi\rangle}$$

- Final state $|\Phi\rangle$ is an *entangled* 2-particle state.
- If measurement on P1 yields *spin-up*, then $|\Phi\rangle \xrightarrow[\textit{collapse}]{} |\uparrow\rangle_1|\downarrow\rangle_2$.

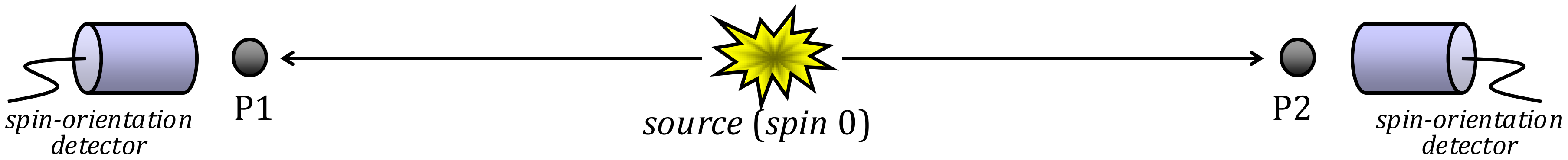
Suppose we interpret superpositions literally

- Before measurement, spin orientations of P1 and P2 in final state are both *indeterminate*.
- After a measurement of P1 that yields *spin-up*, P2 instantaneously has a *determinate* value of *spin-down*!



This is the case no matter how far apart P1 and P2 have traveled!

1. EPR Thought Experiment (Einstein, Podolsky, Rosen 1935)



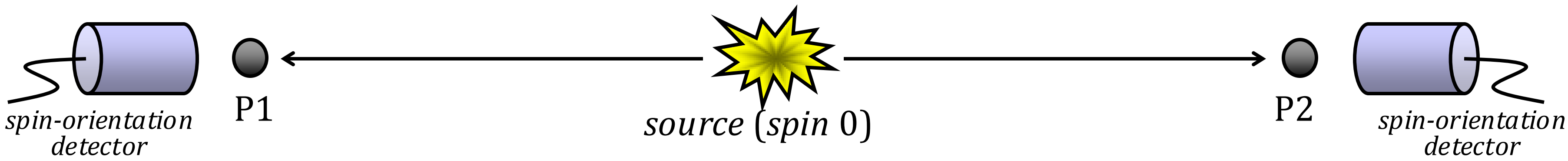
<u>*Conclusion*</u>

(i) *Either* adopt a literal interpretation of superpositions, and accept "Einstein non-locality".

(ii) *Or* accept that QM is incomplete.

- "Einstein non-locality" = "spooky action at a distance".
- Einstein, Podolsky & Rosen pick (ii): Superpositions should not be interpreted literally; in particular, properties always have determinate values.

1. EPR Thought Experiment (Einstein, Podolsky, Rosen 1935)



<u>*Why is Einstein non-locality so spooky?*</u>

- P1 and P2 are in an entangled state and they are *correlated*:
  - *The value of spin that P1 possesses depends on the value of spin that P2 possesses.*

- What explains this correlation?
  - *The correlation is instantaneous: When P1 is found to have a value of spin, P2 instantaneously has the opposite value.*
  - *And we cannot explain this in terms of a causal signal that P1 might have sent to P2 (since by assumption causal signals don't travel instantaneously).*

<u>*So*</u>: Einstein non-locality occurs when two systems are correlated and the correlation cannot be explained by a direct cause that travels from one system to the other.

But what about a "common cause"?

2. Bell Thought Experiment (Bell 1964)

John Stewart Bell
(1928-1990)

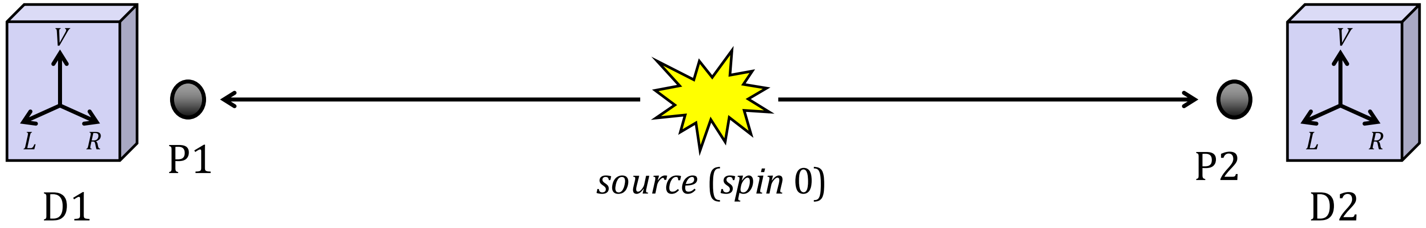
- If *QM* is incomplete, then perhaps a "*Hidden Variables*" description of quantum states and properties is possible in which properties are always determinate (possess values) at all times.
- Can we compare *QM* to such a Hidden Variables Theory?

Yes!

- *<u>And</u>*: The predictions about certain correlations that QM makes are confirmed by experiment, while those that a Hidden Variables Theory makes are not.

<u>Moreover</u>:

- The QM correlations cannot be explained by a *direct cause* (they violate "Einstein locality").
- The QM correlations cannot even be explained by a *common cause* (they violate "Bell locality")!



Set-Up:

- D1 and D2 measure spin along one of three axes (V, R, L) oriented at 120° with respect to each other.
- D1 and D2 are set so that they do not measure spin along the same axis.

Question: What is the probability that P1 and P2 have *different* spin orientations (one spin-up and the other spin-down)?

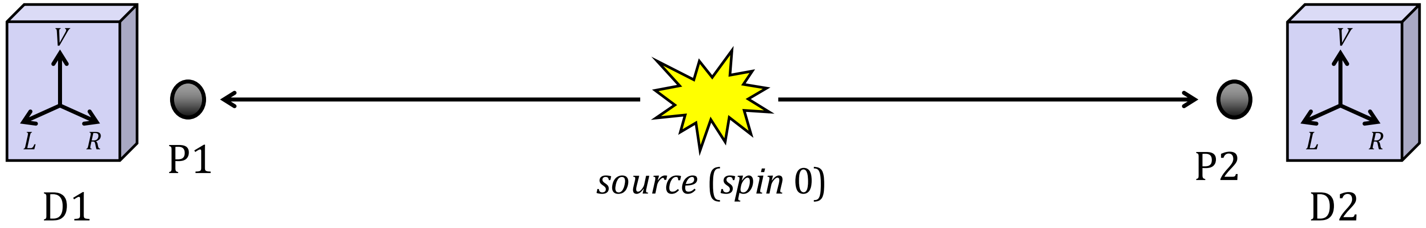
Method 1 (Literal QM)

Properties do not have definite values before measurement.

Method 2 (Hidden Variables)

- *Determinateness*: Properties always have values.
- *Einstein Locality*: No spooky action-at-a-distance.
- *Bell Locality*: Measurement outcomes are determined by source (common cause).

- Bell (1964): Methods 1 and 2 make different predictions!
- Freedman & Clauser (1972): Experiments confirm Method 1's predictions!



Method 1 (Literal QM): One pre-measurement state, three ways of writing it:

$$|\text{P1 \& P2 in source}\rangle \xrightarrow[\textit{evolution}]{\textit{Schrödinger}} \sqrt{½}\,|\uparrow_V\rangle_1|\downarrow_V\rangle_2 - \sqrt{½}\,|\downarrow_V\rangle_1|\uparrow_V\rangle_2 \quad (1)$$

$$\text{OR} \quad \sqrt{½}\,|\uparrow_R\rangle_1|\downarrow_R\rangle_2 - \sqrt{½}\,|\downarrow_R\rangle_1|\uparrow_R\rangle_2 \quad (2)$$

$$\text{OR} \quad \sqrt{½}\,|\uparrow_L\rangle_1|\downarrow_L\rangle_2 - \sqrt{½}\,|\downarrow_L\rangle_1|\uparrow_L\rangle_2 \quad (3)$$

Source doesn't determine spin values! ("Bell non-locality")

Technical Result: How to relate states for spins along different axes z, z'

$$|\uparrow_z\rangle = \cos(\theta/2)|\uparrow_{z'}\rangle + \sin(\theta/2)|\downarrow_{z'}\rangle, \quad \theta = \textit{angle between } z \textit{ and } z'$$

Ex: $|\uparrow_R\rangle_2 = \cos(120°/2)|\uparrow_V\rangle_2 + \sin(120°/2)|\downarrow_V\rangle_2$

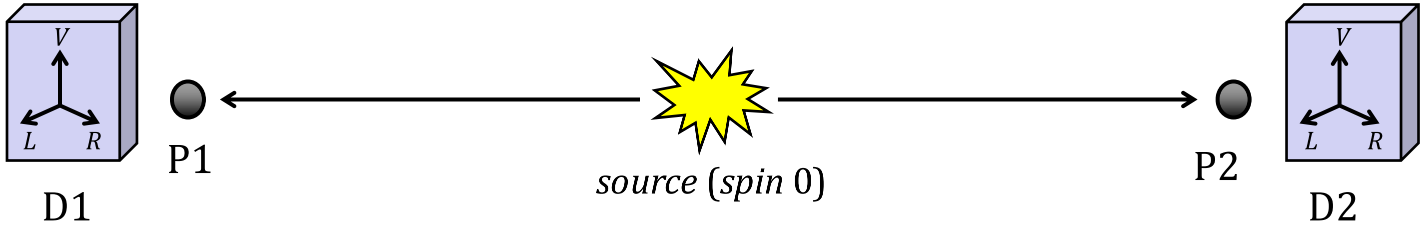
Claim: $\Pr(P2 \uparrow_V, \textit{given } P1 \downarrow_R) = 1/4$

Proof: $\Pr(P2 \uparrow_V, \textit{given } P1 \downarrow_R) = \Pr(P2 \uparrow_V, \textit{given } P2 \uparrow_R)$ *by (2): If P1 is* $\downarrow_R$*, then P2 must be* $\uparrow_R$

$= |\cos(120°/2)|^2 = 1/4$ *by the technical result*

Claim extends to general case:

$$\Pr(\textit{P1 and P2 have different spin orientations}) = 1/4$$



__Method 2 (Hidden Variables)__: 8 possible pre-measurement states:

$$\begin{bmatrix}(\uparrow_V\uparrow_L\uparrow_R)_1\\(\downarrow_V\downarrow_L\downarrow_R)_2\end{bmatrix}_{\mathrm{I}}\quad\begin{bmatrix}(\uparrow_V\uparrow_L\downarrow_R)_1\\(\downarrow_V\downarrow_L\uparrow_R)_2\end{bmatrix}_{\mathrm{II}}\quad\begin{bmatrix}(\downarrow_V\downarrow_L\downarrow_R)_1\\(\uparrow_V\uparrow_L\uparrow_R)_2\end{bmatrix}_{\mathrm{III}}\quad\begin{bmatrix}(\downarrow_V\downarrow_L\uparrow_R)_1\\(\uparrow_V\uparrow_L\downarrow_R)_2\end{bmatrix}_{\mathrm{IV}}$$

$$\begin{bmatrix}(\downarrow_V\uparrow_L\uparrow_R)_1\\(\uparrow_V\downarrow_L\downarrow_R)_2\end{bmatrix}_{\mathrm{V}}\quad\begin{bmatrix}(\uparrow_V\downarrow_L\downarrow_R)_1\\(\downarrow_V\uparrow_L\uparrow_R)_2\end{bmatrix}_{\mathrm{VI}}\quad\begin{bmatrix}(\uparrow_V\downarrow_L\uparrow_R)_1\\(\downarrow_V\uparrow_L\downarrow_R)_2\end{bmatrix}_{\mathrm{VII}}\quad\begin{bmatrix}(\downarrow_V\uparrow_L\downarrow_R)_1\\(\uparrow_V\downarrow_L\uparrow_R)_2\end{bmatrix}_{\mathrm{VIII}}$$

Source determines spin values! ("Bell locality")

Device settings		*States*							
D1	D2	I	II	III	IV	V	VI	VII	VIII
V	*L*	↑↓	↑↓	↓↑	↓↑	↓↓	↑↑	↑↑	↓↓
V	*R*	↑↓	↑↑	↓↑	↓↓	↓↓	↑↑	↑↓	↓↑
L	*V*	↑↓	↑↓	↓↑	↓↑	↑↑	↓↓	↓↓	↑↑
L	*R*	↑↓	↑↑	↓↑	↓↓	↑↓	↓↑	↓↓	↑↑
R	*V*	↑↓	↓↓	↓↑	↑↑	↑↑	↓↓	↑↓	↓↑
R	*L*	↑↓	↓↓	↓↑	↑↑	↑↓	↓↑	↑↑	↓↓
Prob different spin orientation		1	1/3	1	1/3	1/3	1/3	1/3	1/3

Measurement of one particle does not determine value of other! ("Einstein locality")

$$\Pr(\textit{P1 and P2 have different spin orientations}) \geq 1/3$$

Recap

- Literal QM Prediction:

$$\Pr(\textit{P1 and P2 have different spin orientations}) = 1/4$$

- Hidden Variables Prediction:

$$\Pr(\textit{P1 and P2 have different spin orientations}) \geq 1/3$$

Literal QM says

In 1 out of 4 trials, on average, the spin orientations of P1 and P2 will differ.

Hidden Variables says

At the least, in 1 out of 3 trials, on average, the spin orientations of P1 and P2 will differ.

Do many trials...

...result is always Literal QM prediction!

There are correlations in nature that violate Einstein locality and Bell locality (no direct cause or common cause explanations)!

Current Options

> *Value Definiteness (VD)*
>
> *The properties of a quantum system are determinate (possess values) at all times, even when the system is in a superposed state.*

- *EPR say*: *Either* QM is incomplete, or QM violates Einstein non-locality.

- Options for advocates of completeness:

  (1) Local Hidden Variables Theory based on VD.

  Bell says: ***NO!***

  Conflicts with experiment!

- But what about:

  (2) Non-local Hidden Variables Theory based on VD.

  - *In particular*: Is Einstein non-locality really so "spooky"?

Why Einstein Non-Locality Isn't All That Spooky

Recall: *EPR* state is represented by

$$|A\rangle = \sqrt{½}\,|\!\uparrow\rangle_1|\!\downarrow\rangle_2 - \sqrt{½}\,|\!\downarrow\rangle_1|\!\uparrow\rangle_2$$

- If the outcome of a spin measurement on P1 is *spin-up*, then

$$|A\rangle \xrightarrow[\textit{collapse}]{} |\!\uparrow\rangle_1|\!\downarrow\rangle_2$$

- *So*: The outcome of a spin measurement on P2 will be *spin-down*.
- *And*: If the outcome of a spin measurement on P1 is *spin-down*, then the outcome of a spin measurement on P2 will be *spin-up*.

What this means

- The outcome of a measurement on P2 depends non-locally on the outcome of a measurement on P1 (and *vice-versa*).
- *But*: The outcome of a measurement on P2 does *not* depend on whether or not a measurement was performed on P1.

Check:

1. Suppose a spin measurement is done on P2.
   - Then Pr(*P2 spin-up*) = ½ and Pr(*P2 spin-down*) = ½.
2. Suppose a spin measurement is done on P1 and then another is done on P2.
   - Then Pr(P1 *spin-up*) = ½ and Pr(P1 *spin-down*) = ½.
   - If P1 does have *spin-up*, then P2 will have *spin-down*.
   - If P1 does have *spin-down*, then P2 will have *spin-up*.

- *Thus*: The outcome of a measurement on P2 is *equally likely* to be *spin-up* or *spin-down*, *regardless* of whether or not a measurement was performed on P1!
- *Upshot*: Einstein non-locality of outcome dependence can't be used to send signals.

> *Ex*: If we measure P2 *here* to have *spin-down*, then we know P1 *over there* has *spin-up*.
>
> - But we *don't* know if P1 was already found to have *spin-up*: We don't know if P2's having *spin-down here* is a consequence of someone over there measuring P1 to have *spin-up*.

- *So*: Einstein non-locality doesn't violate a prohibition on faster-than-light signalling that can be associated with Special Relativity.