Assignment #3: 2-Particle States and the Eigenvector-Eigenvalue Rule. Due Weds 2/16

1. (2pts.) Suppose a 2-particle system is in an entangled state represented by

$$|Q\rangle = \sqrt{1/2}\,|5\rangle_1|7\rangle_2 + \sqrt{1/2}\,|9\rangle_1|11\rangle_2$$

where $|x\rangle_1$ and $|y\rangle_2$ are eigenstates of position for Particle 1 and Particle 2, respectively. Let $X^{(1)}$ and $X^{(2)}$ be the single-particle position operators for Particles 1 and 2, respectively. Show that $|Q\rangle$ is not an eigenstate of the 2-particle operator $I^{(1)} \otimes X^{(2)}$ that represents the position of Particle 2. This indicates that, according to the Eigenvector-Eigenvalue Rule, Particle 2 has no well-defined position in the state represented by $|Q\rangle$.

(*Hint*: In class, we showed the exact same thing for the 2-particle operator $X^{(1)} \otimes I^{(2)}$.)

2. Suppose a 2-particle system is in a state represented by

$$|D\rangle = |q_7\rangle_1|t_{45}\rangle_2$$

which is an eigenvector of the product operator $Q^{(1)} \otimes T^{(2)}$ (with eigenvalues q_7, t_{45}).

(a) (2pts.) According to the Eigenvector-Eigenvalue Rule, can particles 1 and 2 be said to have definite values of the properties represented by $Q^{(1)}$ and $T^{(2)}$?

Now suppose we measure a property of Particle 2 represented by the operator $B^{(2)}$ and get the value b_{10}. Suppose, further, that eigenvectors of $B^{(2)}$ form a distinct basis for our state space than those of $T^{(2)}$. (Recall that this means that any eigenvector of $T^{(2)}$ can be expanded as a sum of eigenvectors of $B^{(2)}$.)

(b) (2pts.) What happens to $|D\rangle$ as a result of the $B^{(2)}$ measurement?

(c) (2pts.) According to the Eigenvector-Eigenvalue Rule, can Particle 1 be said to have a definite value of the property represented by $Q^{(1)}$ after the measurement? If so, what is it?

(d) (2pts.) According to the Eigenvector-Eigenvalue Rule, can Particle 2 be said to have a definite value of the property represented by $T^{(2)}$ after the measurement? If so, what is it?