

**Assignment #3: 2-Particle States and the Eigenvector-Eigenvalue Rule.**

1. (2pts.) Suppose a 2-particle system is in an entangled state represented by

$$|Q\rangle = \sqrt{\frac{1}{2}}|5\rangle_1|7\rangle_2 + \sqrt{\frac{1}{2}}|9\rangle_1|11\rangle_2$$

where  $|x\rangle_1$  and  $|y\rangle_2$  are eigenstates of position for Particle 1 and Particle 2, respectively. Let  $X^{(1)}$  and  $X^{(2)}$  be the single-particle position operators for Particles 1 and 2, respectively. Show that  $|Q\rangle$  is not an eigenstate of the 2-particle operator  $I^{(1)} \otimes X^{(2)}$  that represents the position of Particle 2. This indicates that, according to the Eigenvector-Eigenvalue Rule, Particle 2 has no well-defined position in the state represented by  $|Q\rangle$ .

(*Hint:* In class, we showed the exact same thing for the 2-particle operator  $X^{(1)} \otimes I^{(2)}$ .)

2. Suppose a 2-particle system is in a state represented by

$$|D\rangle = |q_7\rangle_1|t_{45}\rangle_2$$

which is an eigenvector of the product operator  $Q^{(1)} \otimes T^{(2)}$  (with eigenvalues  $q_7, t_{45}$ ).

(a) (2pts.) According to the Eigenvector-Eigenvalue Rule, can particles 1 and 2 be said to have definite values of the properties represented by  $Q^{(1)}$  and  $T^{(2)}$ ?

Now suppose we measure a property of Particle 2 represented by the operator  $B^{(2)}$  and get the value  $b_{10}$ . Suppose, further, that eigenvectors of  $B^{(2)}$  form a distinct basis for our state space than those of  $T^{(2)}$ . (Recall that this means that any eigenvector of  $T^{(2)}$  can be expanded as a sum of eigenvectors of  $B^{(2)}$ .)

(b) (2pts.) What happens to  $|D\rangle$  as a result of the  $B^{(2)}$  measurement?

(c) (2pts.) According to the Eigenvector-Eigenvalue Rule, can Particle 1 be said to have a definite value of the property represented by  $Q^{(1)}$  after the measurement? If so, what is it?

(d) (2pts.) According to the Eigenvector-Eigenvalue Rule, can Particle 2 be said to have a definite value of the property represented by  $T^{(2)}$  after the measurement? If so, what is it?