

Assignment #2. The Principles of QM.

1. (4pt.) Suppose eigenvectors of *Hardness* and *Color* are given by the following column vectors:

$$|hard\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |soft\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |black\rangle = \begin{pmatrix} \sqrt{1/2} \\ \sqrt{1/2} \end{pmatrix} \quad |white\rangle = \begin{pmatrix} \sqrt{1/2} \\ -\sqrt{1/2} \end{pmatrix}$$

- (a) Show that $|hard\rangle$ and $|soft\rangle$ are orthonormal; and that $|black\rangle$ and $|white\rangle$ are orthonormal.
(*Hint*: "orhtonormal" means *both* perpendicular *and* unit length.)
- (b) Show that $|white\rangle = \sqrt{1/2}|hard\rangle - \sqrt{1/2}|soft\rangle$
- (c) Show that $|hard\rangle = \sqrt{1/2}|black\rangle + \sqrt{1/2}|white\rangle$
- (d) Show that $|soft\rangle = \sqrt{1/2}|black\rangle - \sqrt{1/2}|white\rangle$

2. (4pt.) Suppose the *Hardness* and *Color* operators are given by the following matrices:

$$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- (a) Show that $|black\rangle$ is not an eigenvector of H .
- (b) Show that $|hard\rangle$ and $|soft\rangle$ are not eigenvectors of C .
- (c) What does (a) together with the *Eigenvector/Eigenvalue Rule* say about a *black* electron with respect to its *Hardness*?
- (d) What does (b) together with the *Eigenvector/Eigenvalue Rule* say about an electron with a definite value of *Hardness* with respect to its *Color*?

3. (2pt.) Suppose you have a *soft* electron.

- (a) How would you expand its state in order to measure its *Color*?
- (b) According to the *Born Rule*, what is the probability that a measurement of its *Color* would return the value *black*?
What is the probability that a measurement of its *Color* would return the value *white*?
- (c) Suppose you measure the *Color* of your *soft* electron and it turns out to be *black*. What does the *Projection Postulate* say its state is after the measurement? What is the probability that a second *Color* measurement would return the value *black*?