

Assignment #2. The Principles of QM.

1. (4pt.) Suppose eigenvectors of *Hardness* and *Color* are given by the following column vectors:

$$|\text{hard}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\text{soft}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |\text{black}\rangle = \begin{pmatrix} \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \end{pmatrix} \quad |\text{white}\rangle = \begin{pmatrix} \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{2}} \end{pmatrix}$$

(a) Show that $|\text{hard}\rangle$ and $|\text{soft}\rangle$ are orthonormal; and that $|\text{black}\rangle$ and $|\text{white}\rangle$ are orthonormal.

(*Hint*: "orthonormal" means *both* perpendicular *and* unit length.)

(b) Show that $|\text{white}\rangle = \sqrt{\frac{1}{2}}|\text{hard}\rangle - \sqrt{\frac{1}{2}}|\text{soft}\rangle$
 (c) Show that $|\text{hard}\rangle = \sqrt{\frac{1}{2}}|\text{black}\rangle + \sqrt{\frac{1}{2}}|\text{white}\rangle$
 (d) Show that $|\text{soft}\rangle = \sqrt{\frac{1}{2}}|\text{black}\rangle - \sqrt{\frac{1}{2}}|\text{white}\rangle$

2. (4pt.) Suppose the *Hardness* and *Color* operators are given by the following matrices:

$$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(a) Show that $|\text{black}\rangle$ is not an eigenvector of H .
 (b) Show that $|\text{hard}\rangle$ and $|\text{soft}\rangle$ are not eigenvectors of C .
 (c) What does (a) together with the *Eigenvector/Eigenvalue Rule* say about a *black* electron with respect to its *Hardness*?
 (d) What does (b) together with the *Eigenvector/Eigenvalue Rule* say about an electron with a definite value of *Hardness* with respect to its *Color*?

3. (2pt.) Suppose you have a *soft* electron.

(a) How would you expand its state in order to measure its *Color*?
 (b) According to the *Born Rule*, what is the probability that a measurement of its *Color* would return the value *black*? What is the probability that a measurement of its *Color* would return the value *white*?
 (c) Suppose you measure the *Color* of your *soft* electron and it turns out to be *black*. What does the *Projection Postulate* say its state is after the measurement? What is the probability that a second *Color* measurement would return the value *black*?