Moore: Intro

Topics
I. Paradoxes of the Infinitely Small
II. Paradoxes of the Infinitely Big
III. Paradoxes of the One and the Many
IV. Paradoxes of Thought about the Infinite

The Infinite -- Two clusters of concepts:

- boundlessness
- endlessness
- unlimitedness
- immeasurability
- eternity
- that which is greater than any assignable quantity

- *"negative" characteristics*
- *potentiality*
- *mathematical*

- completeness
- wholeness
- unity
- universality
- absoluteness
- perfection
- self-sufficiency
- autonomy

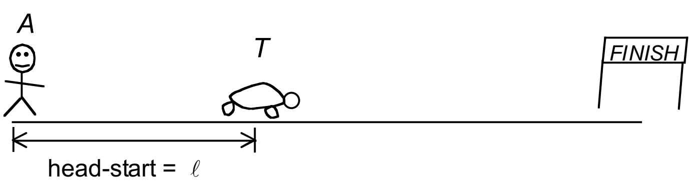
- *"positive" characteristics*
- *actuality*
- *metaphysical/theological*

Paradoxes

1. Paradoxes of the infinitely small
2. Paradoxes of the infinitely big
3. Paradoxes of the one and the many
4. Paradoxes of thought about the infinite

I. Paradoxes of the Infinitely Small

Ex. Achilles and the Tortoise



A runs at speed $v = \ell/t$

T runs at speed $v/2 = (\ell/2)/t$

Claim: Achilles will never overtake the tortoise.

Proof:
(1) To overtake *T*, *A* must first travel ℓ, which takes him time t.
(2) In time t, *T* travels $\ell/2$.
(3) To travel $\ell/2$, *A* needs further time $t/2$.
(4) In time $t/2$, *T* travels $\ell/4$.
(5) To travel $\ell/4$, *A* needs even more time, $t/4$. Etc...

In general:

The distance between *A* and *T* at any given moment after the start of the race is finite (even though it's approaching 0).

And:

To travel a finite distance at finite speed requires a finite amount of time.

important Euclidean assumption : (A line segment is infinitely divisible)

II. Paradoxes of the Infinitely Big

Ex. 1. The Paradox of the Even Numbers

Claim: There are as many even natural numbers as there are natural numbers.

natural numbers = non-negative whole numbers (0, 1, 2, ..)

Aside: Two sets have the same number of members just when there is a 1-1 correspondence between their members.

Proof: { 0 , 1 , 2 , 3 , 4 , ………… , n , ……….}

↕ ↕ ↕ ↕ ↕ ↕

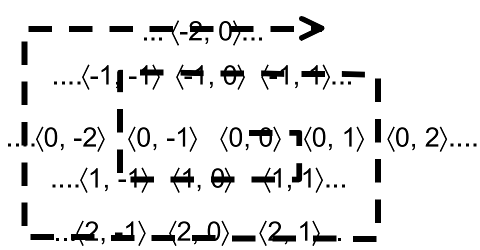
{ 0 , 2 , 4 , 6 , 8 , ………… , 2n , ……….}

Ex. 2. The Paradox of the Pairs

Claim: There are as many pairs of whole numbers as there are natural numbers.

Consequence: There are as many rational numbers as natural numbers: a rational number is given by a pair of whole numbers.

Proof:



Path assigns every pair to one natural number

Ex. 3. The Paradox of Two Guys in Heaven and Hell

For all past eternity:

Mr. *A* in heaven
Mr. *B* in hell
} except for one day each year when they switch (Christmas Day, say)

Claim: Mr. *B* has spent just as much time in heaven as Mr. *A*.

Proof: Since 01/01/03, say:

A-days in heaven		*B*-days in heaven
12/31/02	⟷	12/25/02
12/30/02	⟷	12/25/01
12/29/02	⟷	12/25/00
12/28/02	⟷	12/25/99
⋮		⋮

III. Paradoxes of the One and the Many

In general: Can a collection of infinitely many things be considered a single thing?

Set Theory: What exactly is a set?

Cantor: "a many which allows itself to be thought of as a one"

Are there infinite sets? ← We will return to this question later.

IV. Paradoxes of Thought About the Infinite

Moore: In general, is the concept of infinity coherent? Yes and No.

Yes
- want to be able to say there are infinitely many natural numbers
- want to be able to say the world "includes" everything (infinitely inclusive)

No
- prior paradoxes

Moore's suggestion: Admit concept of infinite, but acknowledge that we as finite beings cannot come to know it. → How can we grasp the ungraspable? → But we do know it!