Relativity and Quantum Field Theory

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Abstract Relativistic quantum field theories (RQFTs) are invariant under the action of the Poincaré group, the symmetry group of Minkowski spacetime. Non-relativistic quantum field theories (NQFTs) are invariant under the action of the symmetry group of a classical spacetime; i.e., a spacetime that minimally admits absolute spatial and temporal metrics. This essay is concerned with cashing out two implications of this basic difference. First, under a Received View, RQFTs do not admit particle interpretations. I will argue that the concept of particle that informs this view is motivated by non-relativistic intuitions associated with the structure of classical spacetimes, and hence should be abandoned. Second, the relations between RQFTs and NQFTs also suggest that routes to quantum gravity are more varied than is typically acknowledged. The second half of this essay is concerned with mapping out some of this conceptual space.

1 Introduction

The comparison of Minkowski spacetime with classical (i.e., non-relativistic) spacetimes has been fruitful in contemporary philosophy of spacetime in debates over the ontological nature of space and time (see e.g., Earman 1989, Chap. 2). In this essay, I extend this type of analysis to debates in the philosophy of quantum field theory. In particular, the distinction between Minkowski spacetime and classical spacetimes allows one to make a corresponding distinction between relativistic quantum field theories (RQFTs) and non-relativistic quantum field theories (NQFTs). This latter distinction is subsequently helpful, or so I shall argue, in clarifying the debate over whether or not RQFTs admit particle interpretations, and in investigating the conceptual space of possible extentions of RQFTs to include gravity.

Section 2 distinguishes between RQFTs and NQFTs in terms of the distinction between Minkowski spacetime and classical spacetimes. This distinction is then applied to an on-going debate over the ontology of QFTs. According to a Received View in this debate, RQFTs do not admit particle interpretations (Arageorgis et al. 2003; Fraser 2008; Halvorson and Clifton 2002). This view takes the existence of

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local number operators and a unique total number operator in the formulation of a QFT as necessary conditions for a particle interpretation of the theory. Given that formulations of RQFTs do not admit such objects, the Received View concludes that RQFTs cannot be given particle interpretations. I will argue that the existence of local and unique total number operators in a QFT requires the absolute temporal structure of a classical spacetime. Thus the Received View's concept of particle appears to be motivated by a non-relativistic concept of absolute time. The moral I draw is that the Received View's concept of particle is inappropriate for RQFTs.

No RQFT currently exists that consistently incorporates gravity. Section 3 reviews an example of an NQFT that does: Christian's (1997) Newtonian Quantum Gravity (NQG). NQG is an NQFT in (a version of) Newton-Cartan spacetime, the latter being an example of a curved classical spacetime. Part of the spacetime structure of NQG is dynamic and quantized, and its symmetry group is an extension of the non-relativistic Maxwell group. The latter entails that NQG is not plagued by the family of conceptual problems associated with unitarily inequivalent representations of the canonical (anti-) commutation relations, as are QFTs in curved Lorentzian spacetimes (Ruetsche 2002). In particular both local number operators and a unique total number operator are present in NQG, again due to the absolute temporal structure of classical spacetimes.

Using NQG as motivation, Sect. 4 undertakes the task of relating NQFTs, both in the presence and the absence of gravity, to RQFTs and to other theories, both of particles and fields, classical and quantum, in the presence and the absence of gravity. What emerges is a tentative map of the relations between some of the fundamental theories in physics, including the as-yet-to-be formulated, fully relativistic quantum theory of gravity (QG).

2 NQFTs and Particles

By an RQFT I will mean a quantum field theory invariant under the actions of the Poincaré group, the symmetry group of Minkowski spacetime. By an NQFT, I will mean a quantum field theory invariant under the actions of the symmetry group of a classical spacetime. Section 2.1 reviews the distinction between classical spacetimes and Minkowski spacetime. Section 2.2 indicates the significance this distinction has for the debate over particle interpretations of QFTs.

2.1 Classical Spacetimes vs. Minkowski Spacetime

Minkowski spacetime can be represented by a pair (M, η_{ab}), where M is a smooth 4-dim differentiable manifold and η_{ab} is a $(-1, 1, 1, 1)$ symmetric tensor field on M, the Minkowski metric, satisfying the compatibility condition $\nabla_a \eta_{ab} = 0$, for the derivative operator ∇_a associated with the connection on M. This condition determines a unique curvature tensor $R^a{}_{bcd}$, which vanishes, encoding spatiotemporal flatness. The isometry group of Minkowski spacetime, the Poincaré group, is generated by vector fields that Lie annihilate the Minkowski metric. Symbolically, we require $\pounds_x \eta^{ab} = 0$, where $\pounds_x$ is the Lie derivative associated with x^a. Intuitively, this means that the transformations between reference frames defined by the integral curves of the vector field x^a preserve the structure of the Minkowski metric. This structure famously entails that there is no unique way to separate time from space in Minkowski spacetime: any two observers moving inertially with respect to each other will disagree on the time interval between any two events, and on the spatial interval between any two events. In coordinate form, elements of the Poincaré group may be represented by transformations

$$x^\mu \to x^{\mu'} = \Lambda^\mu{}_\nu x^\nu + d^\mu \; (Poincaré) \tag{1}$$

where $\Lambda^\mu_\nu \in SL(2, \mathbb{C})$ is a pure Lorentz boost and $d^\mu \in \mathbb{R}^4$ is a spacetime translation.

In comparison, a classical spacetime is a spacetime that minimally admits absolute spatial and temporal metrics. More precisely, a classical spacetime may be represented by a tuple $(M,\ h^{ab},\ t_a,\ \nabla_a)$, where M is a differentiable manifold, h^{ab} is a (0, 1, 1, 1) symmetric tensor field on M identified as a spatial metric, t_a is a covariant vector field on M which induces a (1, 0, 0, 0) temporal metric $t_{ab} = t_a t_b$, and ∇_a is a derivative operator associated with a (non-unique) connection on M and compatible with the metrics in the sense $\nabla_c h^{ab} = \nabla_a t_b = 0$. The spatial and temporal metrics are also required to be orthogonal in the sense $h^{ab} t_b = 0$. These conditions allow M to be decomposed into instantaneous three-dimensional spacelike hypersurfaces parameterized by a global time function. The most general classical spacetime symmetry group is generated by vector fields x^a that Lie annihilate h^{ab} and t_a. Symbolically, we require $\pounds_x h^{ab} = \pounds_x t_a = 0$, and again, this means that the transformations between reference frames defined by the integral curves of the vector fields x^a preserve the structure of the absolute spatial and temporal metrics. This entails that in any classical spacetime, there is always a unique way to separate time from space: any two observers moving inertially with respect to each other will always agree on the time interval between any two events, and on the spatial interval between any two simultaneous events. In this sense, space and time are *absolute* in a classical spacetime.

On the other hand, the compatibility conditions in a classical spacetime do not determine a unique curvature tensor. Additional constraints on the curvature may be imposed, and such constraints define different types of classical spacetimes. Two examples include Neo-Newtonian spacetime, characterized by $R^a_{bcd} = 0$, encoding spatiotemporal flatness; and Maxwellian spacetime, characterized by $R^{ab}_{cd} = 0$, encoding a rotation standard (Bain 2004, pp. 348–352). The symmetries of Neo-Newtonian spacetime form the 10-parameter Galilei group (*Gal*) generated by vector fields x^a that Lie annihilate the spatial and temporal metrics, and the connection. Symbolically, $\pounds_x h^{ab} = \pounds_x t_a = \pounds_x \Gamma^a_{bc} = 0$ (where $\Gamma^a{}_{bc}$ is the connection defined by ∇_a), and in coordinate form,

$$\mathbf{x} \to \mathbf{x}' = R\mathbf{x} + \mathbf{v}t + \mathbf{a} \quad (Gal) \qquad (2)$$
$$t \to t' = t + b$$

where R is a constant orthogonal rotation matrix, $\mathbf{v}$, $\mathbf{a} \in \mathbb{R}^3$ are velocity boost and spatial translation vectors, and $b \in \mathbb{R}$ is a time translation. The symmetries of Maxwellian spacetime are given by the infinite dimensional Maxwell group (*Max*) generated by vector fields x^a that Lie annihilate the spatial and temporal metrics and the rotational part of the connection. Symbolically, $\pounds_x h^{ab} = \pounds_x t_a = \pounds_x \Gamma^{ab}_c = 0$ (where $\Gamma^{ab}_c = h^{bd}\Gamma^a_{bc}$). In coordinate form,

$$\mathbf{x} \to \mathbf{x}' = R\mathbf{x} + \mathrm{c}(t) \quad (Max) \qquad (3)$$
$$t \to t' = t + b$$

where R is a constant orthogonal rotation matrix, $\mathbf{c}(t) \in \mathbb{R}^3$ is a time-dependent spatial boost vector, and $b \in \mathbb{R}$ is a time translation. A quick and dirty distinction between Neo-Newtonian and Maxwellian spacetime can be given in terms of the way the absolute spatial slices are "rigged": In Neo-Newtonian spacetime, the rigging consists of "straight" trajectories, whereas in Maxwellian spacetime, it consists of "straight" and "curved" trajectories. More precisely, a Neo-Newtonian connection can distinguish between a straight and a curved trajectory, whereas a Maxwellian connection cannot. Both connections can, however, distinguish between straight and curved trajectories on the one hand, and "corkscrew" trajectories on the other; i.e., in both spacetimes, there is an absolute standard of rotation.

Now, just as there can be different types of classical spacetimes, there can be different types of NQFTs. A GQFT (Galilei-invariant Quantum Field Theory), for instance, is an NQFT invariant under *Gal* (Lévy-Leblond 1967), while an MQFT (Maxwell-invariant Quantum Field Theory) is an NQFT invariant under *Max*. A slight variant of the latter is Christian's (1997) Newtonian quantum gravity reviewed in Sect. 3 below.

2.2 Particle Interpretations

According to a Received View (Arageorgis et al. 2003; Fraser 2008; Halvorson and Clifton 2002), in order to admit a particle interpretation, a QFT must satisfy the following two conditions.

(a) The QFT must admit a Fock space formulation in which local number operators appear that can be interpreted as acting on a state of the system associated with a bounded region of spacetime and returning the number of particles in that region.
(b) The QFT must admit a unique Fock space formulation in which a total number operator appears that can be interpreted as acting on a state of the system and returning the total number of particles in that state.

Condition (a) is supposed to encode the essential particle characteristic of *localizability*: For a system of particles distributed over various regions of space, an adequate theory must be able to identify the number of particles located in each region.[1] Condition (b) is supposed to encode the essential particle characteristic of *countability*: For a system of particles distributed over various regions of space, an adequate theory must be able to identify a *unique* value for the total number of particles, counted over *all* regions. (Schematically, one would hope that a unique total number operator could be defined as the sum over all regions of spacetime of local number operators.)

One can now demonstrate that Conditions (a) and (b) fail in RQFTs. The Received View concludes that RQFTs do not admit particle interpretations. However, it can also be shown that Conditions (a) and (b) hold in NQFTs precisely because of the existence of an absolute temporal metric in classical spacetimes. The moral I draw is that Conditions (a) and (b) are motivated by a non-relativistic notion of time, and hence are inappropriate in the relativistic context. What should be offered in their place as conditions of adequacy for particle interpretations in the relativistic context is best left to another essay. The remainder of this section attempts to substantiate the moral.

2.2.1 Particles in RQFTs?

It is a fairly simple matter to demonstrate that RQFTs fail to satisfy Conditions (a) and (b). In general, Condition (b) is made problematic by the existence of unitarily inequivalent Fock space representations of the canonical (anti-) commutation relations (CCRs) of an RQFT.[2] To the extent that unitary equivalence is necessary for physical equivalence, this suggests that any given RQFT admits (uncountably) many different ways to parse particle talk, one for every unitary equivalence class of Fock space representations and their attendant total number operator. One is faced with a problem of which representation to privilege (Ruetsche 2002, pg. 359). This Problem of Privilege may appear to be solved in Minkowski spacetime by appeal to the time-like isometry subgroup of the Poincaré group. Intuitively, the time-like symmetries of Minkowski spacetime provide one with a way to "split" the frequencies of solutions to relativistic field equations, and thereby construct a one-particle state space on which a Fock space representation can then be built. One can then show that this method of constructing a Fock space representation is unique up to unitary equivalence.

This is made rigorous by a result due to Kay (1979). Let (S, σ, D_t) be a classical phase space, where S is the space of (well-behaved) solutions to a field equation, σ

[1] This follows the intuitions of Halvorson and Clifton (2002, pp. 17–18). This aspect of the Received View should thus be made distinct from concepts of localized particles that require the existence of position operators and/or localized states.

[2] This is due to the failure of the Stone-von Neumann theorem for theories with infinite degrees of freedom.

is a symplectic form on S, and $D_t : S \to S$ is a one-parameter group of linear maps that preserves σ and represents the evolution of the classical system in time. A one-particle structure over (S, σ, D_t) is a pair $(\mathcal{H}, U_t)$, where $\mathcal{H}$ is a Hilbert space and U_t is a weakly continuous one-parameter group of unitary operators on $\mathcal{H}$ with positive energy[3], such that there is a 1-1 real linear map $K : S \to \mathcal{H}$ with the following properties: (a) The (complex) range of K is dense in $\mathcal{H}$; (b) $2\text{Im}\langle Kf, Kg\rangle = \sigma(f, g)$ for all f, g S, where $\langle , \rangle$ is the inner product on $\mathcal{H}$; and (c) $D_t K = KU_t$. Kay (1979) proves that a one-particle structure associated with the classical Klein-Gordon field is unique up to unitary equivalence (similar results hold for the Dirac field). Thus, as Halvorson (2001, pg. 114) states, "… the choice of time evolution in the classical phase space suffices to determine uniquely the (first) quantization of the classical system."

However, if there is more than one choice of classical time evolution, there will be more than one choice of one-particle structure, and hence more than one unitary equivalence class of Fock space representations. Indeed, this occurs for classical fields defined over a portion of Minkowski spacetime referred to as the right Rindler wedge. The time-like isometry subgroup of the Poincaré group restricted to this portion admits two distinct time-like Killing vector fields, one associated with inertial reference frames and the other with accelerated frames. This gives rise to two unitarily inequivalent Fock space representations, the standard Minkowski representation, and the Rindler representation. This has suggested to some authors that inertial and accelerating observers will disagree over the particle content of an RQFT in Minkowski spacetime (see e.g., Wald 1994, Chap. 5). To such authors, then, the Problem of Privilege is not solved simply by appealing to Minkowski spacetime structure.[4]

Now suppose the Problem of Privilege could be solved to the satisfaction of all for non-interacting RQFTs in Minkowski spacetime. Haag's Theorem indicates that this would provide cold comfort for particle physicists engaged in experiments with what they take to be interacting particles. Under a reasonable assumption, Haag's Theorem entails that representations of the CCRs for both a non-interacting and an interacting RQFT cannot be constructed so that they are unitarily equivalent at a given time.[5] Provided, again, that unitary equivalence is a necessary condition for physical equivalence, this suggests that an interacting RQFT cannot be interpreted as consisting of a system of initially non-interacting particles that interact

[3] Such a U_t can be written $U_t = e^{itH}$ for H a positive operator.

[4] Arageorgis et al. (2003, pp. 180–181) argue that the Rindler representation is unphysical and hence, implicitly, that there is no Problem of Privilege for *physical* Fock space representations, appropriately construed, in Minkowski spacetime. They effectively argue that the time-like Killing vector field associated with accelerated frames in the right Rindler wedge should not count as a global way to "split the frequencies", in so far as it is not extendible to Minkowski spacetime as a whole.

[5] See, e.g., Earman and Fraser (2006, pg. 313). The reasonable assumption is that the representations admit unique Euclidean-invariant vacuum states. This assumption can be dropped by inserting a cut-off into the interacting RQFT and renormalizing the fields, but such tactics open up the host of conceptual problems afflicting renormalized field theories.

over a finite period of time, and then separate back into non-interacting states; a typical scenario for scattering experiments. More precisely, Haag's Theorem suggests that a Fock space representation of the CCRs of a non-interacting RQFT cannot be used to represent particle states in an interacting RQFT. One might then wonder if particle states might be represented more directly in an interacting RQFT by constructing an explicit Fock space representation of its CCRs, as opposed to piggy-backing on non-interacting representations. However, it is unclear if such a Fock space representation of the CCRs for an interacting RQFT is constructible (Fraser 2008).[6]

Thus is Condition (b) foiled in RQFTs, both non-interacting and interacting. Condition (a) is foiled in RQFTs by the consequences of the Reeh-Schlieder theorem. Briefly, the Reeh-Schlieder theorem entails that the vacuum state is *separating* for any local algebra of operators defined by an RQFT (Streater and Wightman 2000, pg. 138). This means that, given any bounded region of Minkowski spacetime, and any operator associated with that region (in the sense of being an element of the corresponding local operator algebra), if the operator annihilates the vacuum state, then it is identically zero. Now the annihilation operators that appear in Fock space formulations of QFTs are defined to annihilate the vacuum state and act non-trivially on other states. Thus separability of the vacuum state of an RQFT entails that there can be *no* annihilation operator associated with a bounded region of Minkowski spacetime; hence there can be no number operator associated with a bounded region of Minkowski spacetime. Thus "local" number operators in the sense of Condition (a) do not exist in RQFTs.

2.2.2 Particles in NQFTs?

In NQFTs, both free and interacting, Conditions (a) and (b) are satisfied, and one can argue that this is due to the presence of an absolute temporal metric in classical spacetimes. Consider Condition (b) first. What would guarantee uniqueness of a Fock space representation of the CCRs for a QFT is the presence of a unique global time function on the associated spacetime. This would provide a unique (up to unitary equivalence) means to construct a one-particle structure over the classical phase space. And such a unique global time function is only guaranteed in those spacetimes that admit an absolute temporal metric. To see this, note that the compatibility condition, $\nabla_a t_b = 0$, on the temporal metric of a classical spacetime entails t_a is closed, and thus locally exact. If M is topologically well-behaved (if, for instance, it is simply connected), then t_a is globally exact, and there exists a unique globally defined time function $t: M \to \mathbb{R}$ satisfying $t_a = \nabla_a t$. On the other hand, suppose there exists a unique global time function $t : M \to \mathbb{R}$. Then a temporal metric t_{ab} compatible with a connection ∇_a can be defined by $t_{ab} = (\nabla_a t)(\nabla_b t)$.

[6] Some authors have taken the moral of Haag's Theorem to be that (irreducible) representations of the CCRs are inappropriate for interacting RQFTs (Streater and Wightman 2000, pg. 101).

Thus there is no Problem of Privilege for non-interacting NQFTs. One can further demonstrate that Haag's theorem does not make trouble for interacting NQFTs, either. Haag's theorem entails the following necessary condition for the existence of an interacting quantum field unitarily equivalent to a free field: *Either the interaction polarizes the vacuum*[7] *or Poincaré-invariance does not hold* (Bain preprint). For non-relativistic quantum fields, the presence of an absolute temporal metric guarantees *both* the failure of Poincaré invariance *and* the failure of vacuum polarization. To see the latter, consider an interacting Hamiltonian $H = H_{free} + H_{int}$. Any representation of the symmetry group of a classical spacetime in which the time-translation generator is encoded in H will be unitarily equivalent (in the sense of satisfying the same commutation relations) to a representation in which the time-translation generator is encoded in H_{free}, provided that H_{int} is invariant under the group action. Thus if H_{free} annihilates the vacuum state, so will H. This does not hold true for the Lorentz group.[8]

An absolute temporal metric is also sufficient for Condition (a). While a version of the Reeh-Schlieder theorem can be proven in the NQFT context (Requardt 1982), it does not entail that the NQFT vacuum state is separating. Briefly, separability of the vacuum state for a local algebra $\mathfrak{R}(\mathcal{O})$ of operators associated with a region $\mathcal{O}$ of Minkowski spacetime is derived under the assumptions of vacuum cyclicity for $\mathfrak{R}(\mathcal{O})$ (guaranteed by the Reeh–Schlieder theorem), relativistic local commutativity, and the existence of a non-trivial causal complement of $\mathcal{O}$.[9] To extend this result to NQFTs, one must first replace relativistic local commutativity with its non-relativistic analogue.[10] This entails keeping track of the distinction between local algebras defined on *spatial* regions of spacetime, and those defined on *spatiotemporal* regions. Requardt's (1982) non-relativistic Reeh–Schlieder theorem only holds for the latter; but, due to the presence of an absolute temporal metric, spatiotemporal regions of classical spacetimes have trivial causal complements, and hence is separability denied.[11] On the other hand, the presence of a temporal metric also

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[7] Vacuum polarization occurs when an interacting Hamiltonian fails to annihilate the vacuum state of the free field.

[8] Lévy-Leblond (1967, pp. 160–161) makes this comparison explicit for the particular case of the Galilei group. Due to the presence of an absolute temporal metric in Neo-Newtonian spacetime (and classical spacetimes in general), the commutation relations that define the Galilei Lie algebra (and the Lie algebra of any classical spacetime symmetry group in general) are such that the generator of time-translations is independent of the other generators. In the commutation relations that define the Lorentz Lie algebra, the time-translation generator is mixed up with the other generators.

[9] Streater and Wightman (2000, pg. 139). Vacuum cyclicity for $\mathfrak{R}(\mathcal{O})$ requires that for any operator $A \in \mathfrak{R}(\mathcal{O})$, $A\Omega$ is dense in $\mathcal{H}$, where Ω is the vacuum state. Relativistic local commutativity requires that local fields ϕ, ψ commute, $[\phi(f), \psi(g)] = 0$, when the supports of the test functions f, g are spacelike separated. The causal complement of a region $\mathcal{O}$ of Minkowski spacetime consists of all points spacelike separated from points in $\mathcal{O}$.

[10] Namely, $[\phi(f), \psi(g)] = 0$, when the supports of the test functions f, g have zero temporal and non-zero spatial separation (Lévy-Leblond 1967, pg. 164).

[11] The causal complement of a spatiotemporal region of a classical spacetime may be identified with the set of all points with zero temporal separation and non-zero spatial separation from points

guarantees that the domain of dependence for an open spatial region $\mathcal{S}$ of a classical spacetimes is just $\mathcal{S}$, and this ensures that the differential operators that appear in the parabolic PDEs of NQFTs are not anti-local for such spatial regions.[12] This has the consequence that the vacuum is not cylic for algebras associated with spatial regions, and thus is separability denied in this case, too.

3 Newtonian Quantum Gravity

While no RQFT currently exists that consistently incorporates gravity, Christian (1997) has constructed an NQFT that does. Not only is it an explicit example of an interacting NQFT that satisfies Conditions (a) and (b) of the Received View's concept of particle, it also is an instance of an NQFT in a curved classical spacetime. As such, it can be compared with QFTs in curved Lorentzian (i.e., relativistic) spacetimes.[13] This comparison will suggest, in Sect. 4, ways of extending RQFTs to incorporate gravity. This section first reviews the distinction between two particular theories of Newtonian gravity in flat and curved classical spacetimes, and then considers how Christian quantizes a particular version of the latter.

The standard way the theory of classical Newtonian gravity is formulated is as a field theory set against the backdrop of flat Neo-Newtonian spacetime. Models in this formulation may be given by a 6-tuple $(M, h^{ab}, t_a, \nabla_a, \phi, \rho)$, where $(M, h^{ab}, t_a, \nabla_a)$ represents classical Neo-Newtonian spacetime, and ϕ and ρ are scalar fields on M that represent a Newtonian potential field and a mass density, respectively. These latter objects are required to satisfy the Poisson equation, and an equation of motion:

$$h^{ab} \nabla_a \nabla_b \phi = 4\pi G \rho \quad (\textit{Poisson equation}) \tag{4}$$

$$\xi^a \nabla_a \xi^b = -h_{ab} \nabla_a \phi \quad (\textit{equation of motion}) \tag{5}$$

where G is the Newtonian gravitational constant, and ξ^a is a tangent vector field for a timelike particle trajectory worldline that encodes its four-velocity.

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in the region. This assumes a prohibition on infinite causal propagations, but allows that finite causal propagations have no upper bound.

[12] The domain of dependence $D(\mathcal{O})$ of a region $\mathcal{O}$ of spacetime consists of points p for which any inextendible causal worldline through p intersects $\mathcal{O}$. A differential operator is said to be *anti-local* for a given region of spacetime just when a function and its transform under the operator can vanish in that region only if the function is identically zero. In classical spacetimes, for any open spatial region $\mathcal{S}$, $D(\mathcal{S})$ has no temporal extent. Thus if a solution ϕ to a well-posed PDE vanishes on $\mathcal{S}$, it vanishes on $D(\mathcal{S})$, but this does not guarantee that it vanishes on an open set in time. This blocks an inference to anti-locality by means of the Edge of the Wedge Theorem. One can further demonstrate that anti-locality of a differential operator entails cyclicity of the associated vacuum state. Segal and Goodman (1965) demonstrated this for the case of the Klein-Gordon operator, and subsequent authors have extended their results to cover operators associated with other relativistic field equations.

[13] A Lorentzian spacetime is a pair $(M, g_{\mu\nu})$, where M is a differentiable manifold and $g_{\mu\nu}$ is a metric defined on M with signature $(-1, 1, 1, 1)$.

One can also formulate Newtonian gravity by incorporating the gravitational potential field into the spacetime connection, and such theories are referred to as theories of Newton-Cartan gravity (NCG). Models of NCG eliminate the Newtonian gravitational potential, and may be given by $(M,\ h^{ab},\ t_a,\ \nabla_a,\ \rho)$. Here the objects $(M,\ h^{ab},\ t_a,\ \nabla_a)$ still represent a classical spacetime; in particular, the spatial and temporal metrics still satisfy orthogonality and compatibility constraints, and additional constraints may still be imposed on the curvature tensor defined by the derivative operator $_a$. But the Poisson equation (4) is now replaced with a generalized Poisson equation, and the equation of motion (5) is replaced with the geodesic equation:

$$R_{ab} = 4\pi G\rho t_a t_b \quad (\textit{generalized Poisson equation}) \tag{6}$$

$$\xi^a\ \nabla_a \xi^b = 0 \quad (\textit{equation of motion}) \tag{7}$$

where R_{ab} is the Ricci tensor defined, ultimately, by the derivative operator ∇_a. These changes enforce the principle of equivalence in NCG. Intuitively, the Newton-Cartan connection defined by (6) and (7) cannot distinguish "straight" inertial trajectories from "curved" gravitationally accelerated trajectories. In this sense, gravity is geometricized in NCG. Now there are different ways this geometrization procedure can be carried out, depending on additional constraints one might impose on the curvature tensor. Christian (1997) considers the following two constraints:

$$R^{[a\ c]}_{[b\ d]} = 0 \tag{8}$$

$$R^{ab}_{cd} = 0 \tag{9}$$

Let "strong NCG" refer to the theory of NCG that, in addition to the compatibility and orthogonality constraints of classical spacetimes, satisfies (6), (7), (8), (9), and call the classical spacetime associated with it strong Newton-Cartan spacetime. In strong Newton-Cartan spacetime, as in all classical spacetimes, there is a global time function that may be associated with absolute time, and there are globally defined spatial slices that may be interpreted as absolute space at an instant. And as with other examples of curved classical spacetimes, what is "curved" is the way these spatial slices are rigged together by the connection. Recall in Maxwellian spacetime, the rigging is determined by condition (9) above and consists of either "straight" or "curved" trajectories (a Maxwellian connection cannot tell these apart), but not "cork-screw" trajectories (there still is a standard of rotation). In strong Newton-Cartan spacetime, "curved" rigging is restricted to gravitationally accelerated trajectories, subject to the additional condition (8). More precisely, whereas the symmetries of Maxwellian spacetime are characterized by the Maxwell group (3), those of strong Newton-Cartan spacetime are characterized by an extension of the Maxwell group, and thus are slightly more constrained.[14]

[14] See, e.g., Bain (2004, pg. 372).

Christian (1997) demonstrates that Conditions (8) and (9) are sufficient to recast strong NCG as a constrained Hamiltonian system, and thus to quantize it. The reduced phase space (Christian 1997, pg. 4867) consists of variables encoding the matter degrees of freedom, and variables that encode the dynamical degrees of freedom of the strong NCG connection, which are identified as gravitational degrees of freedom. The matter variables are solutions to the Schrödinger equation in strong Newton-Cartan spacetime.[15] The connection variables take the form of extended Maxwell frames; i.e., rigid, non-rotating, gravitationally accelerating frames. This phase space has a nondegenerate symplectic structure, and a unique one-parameter family of time evolution maps (due to the absolute temporal metric of strong Newton-Cartan spacetime). Hence it admits a unique one-particle structure, and thus a unique Fock space representation of the CCRs. The result is Christian's Newtonian Quantum Theory of Gravity (NQG, hereafter), an interacting (extended) Maxwell-invariant QFT set in strong Newton Cartan spacetime.

NQG is a concrete example of an interacting NQFT that satisfies the Received View's necessary Conditions (a) and (b) for a particle interpretation. It is also an interacting NQFT that successfully incorporates gravity; in particular, the gravitational degrees of freedom in NQG are *both* fully dynamical and fully quantized. This is in stark contrast with attempts to incorporate gravity into RQFTs. For instance, the fact that the NQG gravitational degrees of freedom are fully *dynamical* distinguishes NQG from the program of QFTs in curved Lorentzian spacetimes. This program attempts to construct RQFTS that incorporate gravity by treating it classically as a manifestation of the curvature of spacetime. This is done by breaking the dynamical link between spacetime and matter forged in general relativity. The curved Lorentzian spacetime in such an RQFT is absolute in the sense that it has no dynamical degrees of freedom. In NQG, on the other hand, strong Newton-Cartan spacetime has quantized *dynamical* degrees of freedom; namely, those associated with the quantized strong Newton-Cartan connection. Intuitively, these quantized degrees of freedom are associated with the dynamical "rigging" of the absolute spatial slices. Moreover, as indicated above, NQG does not face the Problem of Privilege in determining a Fock space representation of the CCRs: the absolute temporal metric of strong Newton-Cartan spacetime decides the matter uniquely up to unitary equivalence. This is in contrast to QFTs in curved (Lorentzian) spacetimes in which

[15] Christian (1997, pg. 4855) refers to this as the Schrödinger-Kuchar equation after Kuchar (1980), who demonstrated that it can be quantized to produce a non-interacting Galilei-invariant NQFT in strong Newton-Cartan spacetime. Christian's NQG is an extention of Kuchar's non-interacting theory to one in which the quantized Schrödinger field interacts with a quantized strong Newton-Cartan connection field (thus Christian's NQG is a fully interacting NQFT). The key to this extention is Christian's construction of a Lagrangian density that produces not just the Schrödinger-Kuchar equation, but also the field equations of Strong NCG. In particular, all Lagrangian densities associated with NCG prior to Christian (1997) failed to recover the generalized Poisson equation (6).

there is not even a guarantee that the spacetime will admit time-like isometries in the first place.[16]

Finally, note that the fact that the NQG gravitational degrees of freedom are fully *quantized* distinguishes NQG from semi-classical approaches to incorporating gravity into RQFTs. These approaches attempt to include dynamical degrees of freedom associated with the gravitational field into an RQFT by replacing the stress-energy tensor in the Einstein equations with its expectation value with respect to quantized matter fields. In such approaches, one treats gravity classically (the metric is not quantized), but one quantizes the matter fields.

4 Intertheoretic Relations

NQG has suggested to Christian (1997, 2001) a novel route to formulating a fully relativistic quantum theory of gravity (QG, hereafter); namely, by relativizing NQG. This section reviews this strategy and expands on Christian's picture of intertheoretic relations associated with it. In particular, the existence of NQFTs suggests modifications to Christian's picture, which open up additional routes to QG. Since a full investigation of all such additional routes is beyond the scope of the current essay, this section will content itself with an initial explorative expedition.

To begin, Christian (1997, pg. 4847; 2001, pg. 307) views NQG as a means to fill a void in the "great dimensional monolith of physics". This is a diagrammatic representation of the relations between fundamental theories in physics. It takes the form of a cube with axes representing the Newtonian gravitational constant G, Planck's constant h, and the inverse speed of light $1/c$ (see Fig. 1).

The vertices of Christian's cube are meant to represent the following theories: classical mechanics (CM), special relativity (SR), general relativity (GR), Newton-Cartan gravity (NCG), Newtonian quantum gravity (NQG), Galilei-invariant quantum mechanics (GQM), relativistic quantum field theory (RQFT), and fully-relativistic quantum gravity (QG). Schematically, these theories can be described by their coordinates $(G, \ h, \ 1/c)$ in monolith space. GR, for instance, may be given the coordinates $(1, 0, 1)$, indicating that G and $1/c$ are "turned on", whereas h is "turned off". The cube thus entails that there are three distinct approaches to constructing QG: quantizing GR (epitomized in "background independent" approaches like loop quantum gravity); "turning on" gravity in an RQFT (epitomized in "background dependent" approaches like string theory); and the approach, novel to Christian (1997), of "relativizing" NQG.

[16] As Ruetsche (2002, pg. 361) notes, one way practitioners have attempted to address this problem is by becoming "algebraic imperialists" and elevating the status of the underlying abstract C^*-algebra over concrete Hilbert space realizations of it. (Doing so provides one access to notions of "physical equivalence" weaker than unitary equivalence.) This strategy is adopted by Christian (1997, pg. 4870) as a way of interpreting NQG, but this seems unnecessary, given that NQG does not face the problem of privilege in the first place.

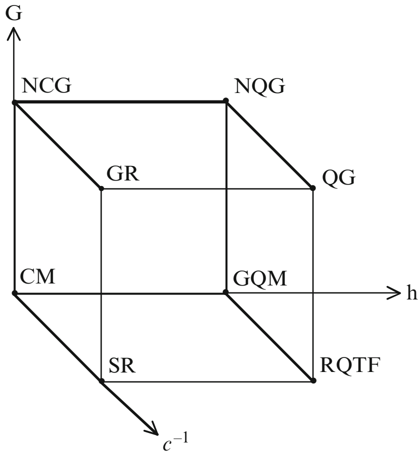

Fig. 1 Christian's (1997) dimensional monolith

To better understand Christian's monolith, and ways of extending it, requires understanding the nature of the limits that define the links in Fig. 1. Under closer inspection, multiple problems arise.

(a) First, the $1/c \rightarrow 0$ limit that "turns off" relativity might initially be thought of as a contraction of the Poincaré group to obtain the Galilei group (see e.g., Bacry and Lévy-Leblond 1968). However, more than one such limit can be taken for a given relativistic theory. Such limits depend in particular on the form of the dynamical equations of the theory. For instance, there are two distinct non-relativistic limits of the Maxwell equations (Holland and Brown 2003). Moreover, the $1/c \rightarrow 0$ link between GR and NCG cannot be described by a group contraction. On the one hand, the Poincaré group is not the symmetry group associated with GR (under one interpretation, the latter is Diff(M)). On the other hand, as Sect. 3 indicates, there is more than one version of NCG, depending on how the geometrization procedure is carried out. One of these versions can indeed be shown to be the $1/c \rightarrow 0$ limit of GR, but this version does not have the Galilei group as its symmetry group.[17]

(b) The $G \rightarrow 0$ limit might be associated simply with setting G to zero in the relevant dynamical equation (thus "turning off" gravity). But this would make the link between GR and SR problematic. Setting G to zero in the Einstein equations results in a Ricci-flat ($R_{ab} = 0$) Lorentzian spacetime, whereas Minkowski spacetime is spatiotemporally flat ($R^a{}_{bcd} = 0$). (Note that Ricci-flatness only entails spatiotemporal flatness in conformally flat (4-dim)

[17] This version can be referred to as "weak NCG" (Bain 2004, pg. 346). It differs from strong NCG by dropping Condition (8). Bain (2004, pg. 365) identifies the symmetry group of weak NCG with an extention of the Leibniz group, another classical spacetime symmetry group.

spacetimes, in which the Weyl tensor vanishes.) This problematizes the other $G \rightarrow 0$ links as well, in so far as there can be Ricci-flat classical spacetimes other than Neo-Newtonian spacetime, which, presumably, is the spacetime of CM and GQM.

(c) Finally, one might describe the $h \rightarrow 0$ limit as the inverse of quantization. But just how the quantization procedure should be characterized is far from settled. For instance, the quantization procedure that represents the link between SR and RQFT is not unique: For a theory of a classical relativistic field with infinite degrees of freedom, the failure of the Stone-von Neuman theorem entails that there are uncountably many unitarily inequivalent representations of the CCRs of the corresponding QFT. Furthermore, inequivalent quantizations are not only associated with systems with infinite degrees of freedom; they also arise for finite systems with topologically non-trivial state spaces.[18] This problematizes the link between CM and GQM, as well as the link between NCG and NQG (in the latter case, for topologically trivial gravitational fields, appeal to the unique global time function in classical spacetimes solves the Problem of Unique Quantization (viz., Privilege), as explained in Sect. 3).

In addition to these issues with the extant links in Christian's diagram, there also seems to be a deeper, structural problem. This problem manifests itself explicitly in the links between NQG and GQM, and RQFT and GQM:

1. First, Christian's NQG is an NQFT that incorporates gravity. Thus, one might expect that turning off gravity would result in an NQFT *sans* gravity. One might then wonder about the referent of "GQM": Is it meant to include infinite-dimensional non-relativistic quantum theories (viz, NQFTs) as well as finite-dimensional non-relativistic quantum theories (viz, non-relativistic quantum particle dynamics)? And moreover, it is not immediately clear that it should refer to a Galieli-invariant theory.
2. A second related concern involves the link between RQFT and GQM. The $1/c \rightarrow 0$ limit of an RQFT might be characterized by a contraction of the Poincaré group to yield the Galilei group, with the qualifications mentioned above. But this maneuver by itself does not take us from an RQFT to a theory of GQM, if we allow that the latter includes theories with finite degrees of freedom.

These concerns stem from the fact that NQFTs are missing from Christian's diagram. NQFTs may be thought of as appropriately qualified $1/c \rightarrow 0$ limits of RQFTs. Now suppose we relabel Christian's GQM as NQM (Non-relativistic Quantum Mechanics) and restrict its referent to finite-dimensional non-relativistic quantum theories of particle dynamics (i.e., finite theories of quantum particles invariant under the symmetry group of a classical spacetime). Then, for $N =$

[18] An example of such a system is a charged particle moving in a region external to an operating solenoid. Quantization of this system produces the Aharonov-Bohm effect (see e.g., Belot 1998, pg. 546).

degrees of freedom, NQMs may be thought of, schematically, as the "inverse thermodynamic" limit $N \to 0$ of NQFTs. This limit is intended to be applicable to quantum theories independently of classical theories, and vise-versa (i.e., it is intended to be "orthogonal" to the $h \to 0$ limit). So, for instance, it should also hold between a classical theory with an infinite number of degrees of freedom (a non-relativistic classical field theory, for instance), and a classical theory with finite degrees of freedom (a non-relativistic classical theory of particle dynamics, for instance). Whether such a limit can be precisely defined is a matter for another essay.[19] What it informally suggests is that Christian's cube should be replaced by a 4-dim hypercube with an additional axis representing degrees of freedom N. Suppressing the G-dimension, we then have the diagram in Fig. 2.

The vertices in Fig. 2 represent the following theories: non-relativistic classical particle mechanics (NCM), relativistic classical particle mechanics (RCM), non-relativistic classical field theory (NCFT), relativistic classical field theory (RCFT), non-relativistic quantum particle mechanics (NQM), relativistic quantum particle mechanics (RQM), non-relativistic quantum field theory (NQFT), and relativistic quantum field theory (RQFT). The distinctions here are between theories (classical and quantum, relativistic and non-relativistic) with infinite degrees of freedom, and theories (classical and quantum, relativistic and non-relativistic) with finite degrees of freedom.[20]

Theories in hypermonolith space are coodinatized by 4-tuples $(G, h, 1/c, N)$. There are now four distinct approaches to constructing relativistic QG: quantizing the classical field theory of GR, with coordinates (1, 0, 1, 1); "turning on" gravity in an RQFT with coordinates (0, 1, 1, 1); "relativizing" a non-relativistic QFT of gravity (such as Christian's NQG) with coordinates (1, 1, 0, 1); or "taking the thermodynamic limit" of a relativistic quantum particle theory of gravity with coordinates (1, 1, 1, 0). Just what the latter might involve requires further analysis.

As an example of how this investigation might proceed, consider how the eight $G \to 0$ links in the hypercube could be fleshed out (these all end in the vertices/theories that appear in Fig. 2). They may be divided into links in which gravity is turned off in a field theory, and links in which gravity is turned off in a particle theory.

1. (Non-relativistic classical field theory of gravity (1, 0, 0, 1)) → NCFT.
   An example of a theory with coordinates (1, 0, 0, 1) that produces an NCFT in the $G \to 0$ limit is asymptotically flat weak NCG. This is a version of

[19] Landsman (2007) discusses a rigorous way of defining an $N \to \infty$ limit that holds between a quantum system with N degrees of freedom and a classical system. The definition makes use of the C^*-algebra formulation of quantum and classical systems. This formalism also admits a rigorous definition of an $h \to 0$ limit, and Landsman notes that the former limit is a special case of the latter.

[20] For simplicity's sake, the former are identified as field theories and the latter as particle theories. This ignores field-theoretic systems on lattices (with finite degrees of freedom), as well as particle systems with infinitely many particles; and it also glosses over conceptual issues concerning the nature of a particle vis-à-vis a field; but nothing in the following hangs on this simplifying means of expediency.

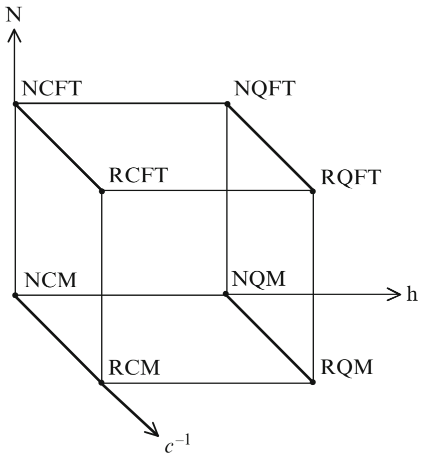

Fig. 2 Relations between theories in the absence of gravity

NCG that drops condition (9) in Sect. 3 above, and imposes asymptotic spatial flatness to enforce Galilei-invariance. Bain (2004, pg. 358) indicates that it is empirically equivalent to a version of (non-geometricized) Newtonian gravity in Neo-Newtonian spacetime in which an "island universe" boundary condition is imposed (namely, $\phi \rightarrow 0$ at spatial infinity). Hence turning off gravity in asymptotically flat weak NCG is equivalent to turning off gravity in (non-geometricized) Newtonian gravity in Neo-Newtonian spacetime under the island universe assumption, and this evidently yields a Galilei-invariant classical field theory in Neo-Newtonian spacetime.

2. (RCFT of gravity (1, 0, 1, 1)) → RCFT.
   GR is a theory with coordinates (1, 0, 1, 1). Turning off gravity in GR results in a field theory in a Ricci-flat Lorentzian spacetime (providing non-gravitational fields are present). This does not by itself guarantee the theory is Poincaré-invariant. To assure coherence here, one might additionally impose the requirement of conformal flatness (although whether this can be motivated on physical or other grounds remains to be seen). Alternatively, one might simply expand one's concept of a relativistic theory to include theories invariant under the symmetries of Lorentzian spacetimes in general.
3. (NQFT of gravity (1, 1, 0, 1)) → NQFT.
   NQG is an NQFT of gravity in strong Newton-Cartan spacetime. Evidently, turning off gravity yields an NQFT in a Ricci-flat classical spacetime satisfying conditions (8) and (9).
4. (RQFT of gravity (1, 1, 1, 1)) → RQFT.
   The expectation here is that the full-blown relativistic theory of quantum gravity will reproduce a relativistic quantum field theory in the limit of no gravity (just as it should produce GR in the classical limit).
   The remaining four links involve turning off gravity in a particle theory:

5. (Non-relativistic classical particle theory of gravity (1, 0, 0, 0)) → NCM.
6. (Non-relativistic quantum particle theory of gravity (1, 1, 0, 0)) → RCM.
7. (Relativistic classical particle theory of gravity (1, 0, 1, 0)) → NQM.
8. (Relativistic quantum particle theory of gravity (1, 1, 1, 0)) → RQM.

Whether examples of all the theories on the left hand side in links 5–8 can be identified is best left to another essay, with particular interest directed at an example of Link 8. Such an example, together with an appropriately formulated thermodynamic limit that links field theories with particle theories, would open up a fourth route to the elusive fully relativistic theory of quantum gravity.

5 Conclusion

This essay has used the distinction between Minkowski spacetime and classical spacetimes as a tool to probe two contemporary issues in philosophy of quantum field theory; namely, the debate over particle interpretations of RQFTs, and the status of approaches to a fully relativistic quantum theory of gravity. First, the distinction between Minkowski spacetime and classical spacetimes suggested a distinction between RQFTs and NQFTs which in turn suggested that the concept of particle that a Received View adopts in arguing against particle interpretations of RQFTs is motivated by a non-relativistic notion of absolute time. Second, the existence of NQFTs, and in particular, consistent NQFTs of gravity, also suggested that routes to fully relativistic quantum gravity are more varied than the current literature suggests.

Finally, a general moral can be drawn. The existence of NQFTs suggests that the distinction between relativistic and non-relativistic theories should not be couched in terms of Poincaré-invariance vs. Galilei-invariance. On the one hand, as is already evident in GR, a relativistic theory need not be Poincaré-invariant. On the other hand, as is evident in NQFTs, a non-relativistic theory need not be Galilei-invariant. The discussion in Sect. 4 of this essay suggests that a more appropriate distinction should be based on theories that are invariant under the symmetries of a Lorentzian spacetime vs. theories that are invariant under the symmetries of a classical spacetime.

References

Arageorgis, A., J. Earman, and L. Ruetsche (2003) ‘Fulling Non-uniquenes and the Unruh Effect: A Primer on Some Aspects of Quantum Field Theory’, *Philosophy of Science, 70*, 164–202.

Bain, J. (2010) ‘Quantum Field Theories in Classical Spacetimes and Particles’, forthcoming in *Studies in History and Philosophy of Modern Physics*.

Bain, J. (2004) ‘Theories of Newtonian Gravity and Empirical Indistinguishability’, *Studies in History and Philosophy of Modern Physics, 35*, 345–376.

Bacry, H. and J.-M. Lévy-Leblond (1968) 'Possible Kinematics', *Journal of Mathematical Physics, 9*, 1605–1614.
Belot, G. (1998) 'Understanding Electromagnetism', *British Journal for the Philosophy of Science, 49*, 531–555.
Christian, J. (2001) 'Why the Quantum Must Yield to Gravity'. In C. Callender and N. Huggett (eds.), *Physics Meets Philosophy at the Planck Scale* (pp. 204–338). Cambridge: Cambridge University Press.
Christian, J. (1997) 'Exactly Soluble Sector of Quantum Gravity, *Physical Review D*, *56*, 4844–4877.
Earman, J. (1989) *World Enough and Spacetime*, Cambridge: MIT.
Earman, J. and D. Fraser (2006) 'Haag's Theorem and its Implications for the Foundations of Quantum Field Theory', *Erkenntnis, 64*, 305–344.
Fraser, D. (2008) 'The Fate of "Particles" in Quantum Field Theories with Interactions', *Studies in History and Philosophy of Modern Physics, 39*, 841–859.
Halvorson, H. (2001) 'Reeh-Schlieder Defeats Newton-Wigner: On Alternative Localization Schemes in Relativistic Quantum Field Theory', *Philosophy of Science, 68*, 111–133.
Halvorson, H. and R. Clifton (2002) 'No Place for Particles in Relativistic Quantum Theories?', *Philosophy of Science, 69*, 1–28.
Holland, P. and H. Brown (2003) 'The Non-Relativistic Limits of the Maxwell and Dirac Equations: The Role of Galilean and Gauge Invariance', *Studies in History and Philosophy of Modern Physics, 34*, 161–187.
Kay, B. (1979) 'A Uniqueness Result in the Segal-Weinless Approach to Linear Bose Fields', *Journal of Mathematical Physics, 20*, 1712–1713.
Kuchar, K. (1980) 'Gravitation, Geometry, and Nonrelativistic Quantum Theory', *Physical Review D*, 22, 1285–1299.
Landsman, N. (2007) 'Between Classical and Quantum'. In J. Butterfield and J. Earman (eds.) *Handbook of the Philosophy of Physics*, Amsterdam: North-Holland, 417–554.
Lévy-Leblond, J.-M. (1967) 'Galilean Quantum Field Theories and a Ghostless Lee Model', *Communications in Mathematical Physics*, *4*, 157–176.
Requardt, M. (1982) 'Spectrum Condition, Analyticity, Reeh-Schlieder and Cluster Properties in Non-Relativistic Galilei-Invariant Quantum Theory', *Journal of Physics A*, *15*, 3715–3723.
Ruetsche, L. (2002) 'Interpreting Quantum Field Theory', *Philosophy of Science*, *69*, 348–378.
Segal, I. and R. Goodman (1965) 'Anti-locality of Certain Lorentz-Invariant Operators', *Journal of Mathematics and Mechanics, 14*, 629–638.
Streater, R. and A. Wightman (2000) *PCT, Spin and Statistics, and All That*, Princeton: Princeton University Press.
Wald, R. (1994) *Quantum Field Theory in Curved Spacetimes and Black Hole Thermodynamics*, Chicago: Chicago University Press.