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Quantum field theories in classical spacetimes and particles

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ABSTRACT

According to a Received View, relativistic quantum field theories (RQFTs) do not admit particle interpretations. This view requires that particles be localizable and countable, and that these characteristics be given mathematical expression in the forms of local and unique total number operators. Various results (the Reeh-Schlieder theorem, the Unruh Effect, Haag's theorem) then indicate that formulations of RQFTs do not support such operators. These results, however, do not hold for non-relativistic QFTs. I argue that this is due to the absolute structure of the classical spacetimes associated with such theories. This suggests that the intuitions that underlie the Received View's choice of mathematical representations of localizability and countability are non-relativistic. Thus, to the extent that such intuitions are inappropriate in the relativistic context, they should be abandoned when it comes to interpreting RQFTs.

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1. Introduction

According to a Received View in the philosophy of quantum field theory, relativistic quantum field theories (RQFTs) do not admit particle interpretations (Arageorgis, Earman, & Ruetsche, 2003; Clifton & Halvorson, 2001; Fraser, 2006, 2008; Halvorson & Clifton, 2002; Malament, 1996). This view requires that particles be localizable and countable, and that these characteristics be given mathematical expression in the forms of local and unique total number operators. But for RQFTs, the Reeh-Schlieder theorem entails local number operators do not exist. And while a total number operator is guaranteed to exist for non-interacting RQFTs, its uniqueness has been called into question. Moreover, Haag's theorem suggests total number operators for interacting RQFTs do not exist. Thus, since the mathematical representations of particles are not defined in these theories, these theories cannot be said to be about particles.

This essay argues against the Received View. I will claim that the Received View's concept of particle is informed by non-relativistic representations of localizability and countability. The way I will argue for this is by first making a distinction between relativistic QFTs and non-relativistic QFTs in terms of spacetime structure. I will then argue that it is the presence of absolute spacetime structure associated with non-relativistic QFTs that allows them to support the Received View's concept of particle, whereas it is the lack of this absolute structure in RQFTs that is the reason why these theories do not support the Received View's concept of particle. This suggests that the intuitions that underlie the Received View's treatment of particles are non-relativistic, and to the extent that such intuitions are inappropriate in the relativistic context, they should be abandoned when it comes to interpreting RQFTs.

Section 2 begins by summarizing the Received View's concept of particle and the argument it mounts against particle interpretations of RQFTs. Section 3 makes the distinction between RQFTs and NQFTs in terms of the spacetime symmetries they admit. Section 4 makes this distinction more precise by comparing the Wightman axioms for RQFTs with the Lévy-Leblond axioms for a particular family of NQFTs; namely, Galilei-invariant QFTs (GQFTs). This comparison suggests extensions of the Lévy-Leblond axioms to include NQFTs in general. The axiomatic formalism gives the Received View a rigorous way to define its concept of particle, and a rigorous way to argue against particle interpretations of RQFTs. Section 5 reviews the technical details of this argument involving the Reeh-Schlieder theorem, the Unruh Effect, and Haag's theorem. Here I mount my response by indicating the extent to which the absolute structure of classical spacetimes allows NQFTs to avoid the associated conceptual problems. I conclude by considering options that interpreters of quantum field theory are faced with, given that my argument against the Received View goes through.

2. The Received View

According to the Received View, there are two essential characteristics of a particle. The first is *localizability*. Of any particle, we should be able to say that it is *Here*, *Now*, as opposed to *There*, *Now*. The Received View translates this intuition into

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mathematics by means of the requirement that any QFT that talks about particles must admit a Fock space formulation in which *local number operators* occur. Intuitively, if our physical system consists of eight particles distributed over space, so that, for instance, three are in region $\mathcal{R}$ and five are in region $\mathcal{R}'$, then the mathematical formulation of the theory that describes this system must admit number operators associated with each region of space, such that, when the number operator associated with region $\mathcal{R}$ acts on the state of the system, it tells you how many particles are present in that region. And similarly for the number operator associated with region $\mathcal{R}'$, and all other regions of space. Schematically,

$$N_{\mathcal{R}}|state\rangle = 3|\text{state}\rangle, \quad N_{\mathcal{R}'}|\text{state}\rangle = 5|\text{state}\rangle.$$

The second essential characteristic of a particle, according to the Received View, is what might be called *countability*. If a physical system consists of particles, it should be possible to begin with two *Here*, *Now*, add one *Here*, *Now*, and obtain three *Here*, *Now*. The Received View translates this intuition into mathematics by means of the requirement that any QFT that talks about particles must admit a *unique* Fock space formulation in which a *total number operator* occurs. Intuitively, a total number operator should be definable as the sum of all the local number operators associated with all regions of space. When you act with the total number operator on the state of a system of eight particles, it should tell you that there are eight, *and only* eight, particles present. Schematically,

$$N|\text{state}\rangle = \int N_{\mathcal{R}} d^3\mathbf{x}|\text{state}\rangle = 8|\text{state}\rangle.$$

The Received View then claims that RQFTs do not admit particle interpretations. It supports this claim by pointing to the following results:

(a) The Reeh-Schlieder theorem entails that there are no local number operators in RQFTs.
(b) The Unruh Effect indicates that there is no *unique* total number operator in non-interacting RQFTs.
(c) Haag's theorem entails that there are no total number operators in interacting RQFTs.

Thus, again, since the mathematical representations of particles are not supported in the formulations of RQFTs, these theories cannot be said to be about particles. My argument against the Received View will be based on the following claim:

> The existence of an absolute temporal metric is a necessary condition for the existence of local number operators and a unique total number operator.

The moral I will draw is that the Received View's concept of particle is informed by non-relativistic representations of localizability and countability associated with an absolute concept of time. To support this moral, I'll first consider the distinction between RQFTs and NQFTs in terms of spacetime structure.

3. Classical spacetimes and NQFTs

By an RQFT I will mean a quantum field theory invariant under the actions of the Poincaré group, the symmetry group of Minkowski spacetime. By an NQFT, I will mean a quantum field theory invariant under the actions of the symmetry group of a classical spacetime. A classical spacetime is a spacetime that minimally admits absolute spatial and temporal metrics that satisfy orthogonality and compatibility constraints. More precisely, a classical spacetime may be represented by a tuple $(M, h^{ab}, t_a, \nabla_a)$, where M is a differentiable manifold, h^{ab} is a (0, 1, 1, 1) symmetric tensor field on M identified as a spatial metric; t_a is a covariant vector field on M which induces a degenerate temporal metric $t_{ab}=t_a t_b$ with signature (1, 0, 0, 0); and ∇_a is a smooth derivative operator associated with a (non-unique) connection on M and compatible with the metrics in the sense $\nabla_c h^{ab}=\nabla_a t_b=0$. The spatial and temporal metrics are also required to be orthogonal in the sense $h^{ab}t_b=0$ (Bain, 2004, pp. 347–348). These conditions allow M to be decomposed into instantaneous three-dimensional spacelike hypersurfaces parameterized by a global time function t. In particular, they entail that the time interval between any two events is invariant, as well as the spatial distance between simultaneous events:

$$t_2 - t_1 = const., \tag{1}$$

$$|\mathbf{x}_2 - \mathbf{x}_1| = const., \quad \text{if } t_2 = t_1. \tag{2}$$

As Lévy-Leblond (1971, pp. 225) indicates, the most general *linear* transformations that preserve (1) and (2) are the symmetries of Neo-Newtonian spacetime, and these form the Galilei group. But if linearity is dropped, larger symmetry groups are allowed. The most general classical spacetime symmetry group is generated by vector fields x^a that Lie annihilate h^{ab} and t_a, subject to the orthogonality and compatibility conditions. Symbolically, we require $\pounds_x h^{ab}=\pounds_x t_a=0$, where $\pounds_x$ is the Lie derivative associated with x^a. Intuitively, this means that the transformations between reference frames defined by the integral curves of the vector fields x^a preserve the structure of the absolute spatial and temporal metrics. Additional constraints may be imposed on the curvature tensor $R^a{}_{bcd}$ associated with the derivative operator ∇_a, compatible with the relations (1) and (2). Two examples include Neo-Newtonian spacetime, characterized by $R^a{}_{bcd}=0$, encoding spatiotemporal flatness; and Maxwellian spacetime, characterized by $R^{ab}{}_{cd}=0$, encoding a rotation standard (Bain, 2004, pp. 348–352). The symmetries of Neo-Newtonian spacetime form the 10-parameter Galilei group (*Gal*) generated by vector fields x^a that Lie annihilate the spatial and temporal metrics, and the connection. Symbolically, $\pounds_x h^{ab}=\pounds_x t_a=\pounds_x \Gamma^a_{bc}=0$ (where Γ^a_{bc} is the connection defined by ∇_a). In coordinate form

$$\mathbf{x} \to \mathbf{x}' = R\mathbf{x} + \mathbf{v}t + \mathbf{a} \qquad (Gal)$$
$$t \to t' = t + b$$

where R is a constant orthogonal rotation matrix, $\mathbf{v}$, $\mathbf{a} \in \mathbb{R}^3$ are velocity boost and spatial translation vectors, and $b \in \mathbb{R}$ is a time translation. The symmetries of Maxwellian spacetime are given by the infinite dimensional Maxwell group (*Max*) generated by vector fields x^a that Lie annihilate the spatial and temporal metrics and the rotational part of the connection. Symbolically, $\pounds_x h^{ab}=\pounds_x t_a=\pounds_x \Gamma^a_{bc}=0$ (where $\Gamma^a_{bc}=h^{bd}\Gamma^a_{bc}$). In coordinate form

$$\mathbf{x} \to \mathbf{x}' = R\mathbf{x} + \mathbf{c}(t) \qquad (Max)$$
$$t \to t' = t + b$$

where R is a constant orthogonal rotation matrix, $\mathbf{c}(t) \in \mathbb{R}^3$ a time-dependent spatial boost vector, and $b \in \mathbb{R}$ a time translation.

Thus, just as there are different types of classical spacetimes, there are different types of NQFTs. A Galilei-invariant Quantum Field Theory (GQFT), for instance, is an NQFT invariant under (the central extension of) *Gal* (see, *e.g.*, Lévy-Leblond, 1967), and an Maxwell-invariant Quantum Field Theory (MQFT) is an NQFT invariant under *Max*. An example of the latter is Christian's (1997) Newtonian quantum gravity. This is an example of a QFT (technically invariant under an extension of *Max*; see, *e.g.*, Bain,

Table 1
Axioms for QFTs.

RQFT Axioms	GQFT Axioms
W1. ***Fields***. The fundamental dynamical variables of the theory are local field operators that act on a Hilbert space $\mathcal{H}$ of states.	L1. ***Fields***. The fundamental dynamical variables of the theory are local field operators that act on a Hilbert space $\mathcal{H}$ of states.
W2. ***Poincaré-Invariance***. $\mathcal{H}$ admits a unitary projective representation of the restricted Poincaré group, under which the fields transform appropriately.[a]	L2. ***Galilei-Invariance.*** $\mathcal{H}$ admits a unitary projective representation of the Galilei group, under which the fields transform appropriately.[a]
W3. ***Relativistic Local Commutativity***. The fields commute (or anti-commute) at spacelike separations.	L3. ***Non-relativistic Local Commutativity.*** At equal times, the fields commute/anti-commute for non-zero spatial separation.
W4. ***Vacuum State***. There exists a vector $\|0\rangle$ in $\mathcal{H}$ satisfying the following conditions:	L4. ***Vacuum State.*** There exists a vector $\|0\rangle$ in $\mathcal{H}$ satisfying the following conditions:
(i) $\|0\rangle$ is Poincaré-invariant. (*Invariance*)	(i) $\|0\rangle$ is Galilei-invariant. (*Invariance*)
(ii) $\|0\rangle$ is cyclic for $\mathcal{H}$. (*Cyclicity*)	(ii) $\|0\rangle$ is cyclic for $\mathcal{H}$ within a given mass sector. (*Cyclicity*)
(iii) The spectrum of the 4-momentum operator on the complement of $\|0\rangle$ is confined to the forward lightcone. (*Spectrum Condition*)	(iii) The spectrum of the internal energy operator on the complement of $\|0\rangle$ and within a given mass sector is bounded from below. (*Spectrum Condition*)

[a] For the relativistic case, see Araki (1999, pp. 103). For the Galilei case, see Lévy-Leblond (1967, pp. 163).

2004) in a curved classical spacetime; namely, Newton–Cartan spacetime.

In comparison, an RQFT is a QFT invariant under the actions of the Poincaré group, the isometry group of Minkowski spacetime. The Poincaré group is generated by vector fields that Lie annihilate the Minkowski metric; symbolically, $\pounds_x\eta^{ab}=0$, and in coordinate form

$$x^\mu \to x^{\mu'} = \Lambda^\mu_\nu x^\nu + d^\mu \qquad (Poincare')$$

where $\Lambda^\mu_\nu \in SL(2,\mathbb{C})$ is a pure Lorentz boost and $d^\mu \in \mathbb{R}^4$ a spacetime translation.

4. Axioms for QFTs

The distinctions between RQFTs and NQFTs can be made even more precise by means of axiomatic formulations. Table 1 compares the Wightman axioms for RQFTs with the Lévy-Leblond axioms for GQFTs.[1]

In general, the areas where the axioms differ are exactly those areas that involve spacetime structure. The most obvious difference in this regard is Axiom 2 of which more will be said in the following subsection. Axiom 3, Local Commutativity, encodes the requirement that fields (or field quantities) associated with causally separated regions of spacetime should be independent of each other (also referred to as "micro-causality"). The way "causal separation" gets fleshed out depends on the structure of the associated spacetime; so one would expect Axiom 3 to be different in the relativistic and non-relativistic contexts. Note that Non-Relativistic Local Commutativity only requires the existence of absolute spatial and temporal metrics (to guarantee the relations encoded in (1) and (2)), so it will be common to all formulations of NQFTs, and not just GQFTs in particular.

The 4th axiom describes the vacuum state: in the field-theoretic context, this is the state of zero energy. The last property of this state, the *Spectrum Condition* 4(iii), involves a restriction on the energies that other states of the theory can possess. In particular, W4(iii) guarantees positivity of energy in Lorentz frames, and its counterpart L4(iii) encodes the fact that, in non-relativistic mechanics, the potential energy of a single-particle state is a matter of convention. Technically, this is encoded in the fact that irreducible representations of the extended Galilei group (see later) that differ on their internal energies are projectively equivalent (Lévy-Leblond, 1971, pp. 277). Another feature of the extended Galilei group, to be explained below, requires the restriction to mass sectors in L4. Thus in the GQFT context, the constraints imposed by L4 can be explained by appeal to the structure of the spacetime symmetry group. While one would not expect this type of explanation to be available in the context of other classical spacetime symmetry groups, nevertheless it might be claimed that the constraints mandated by L4 should be imposed as a condition of physicality on all NQFTs, in so far as all such theories view mass as an absolute quantity distinct from energy.[2]

Thus to move from GQFTs to NQFTs, arguably, requires minimal modification of the Lévy-Leblond axioms. One simply replaces L2 and L4(i) with invariance under the appropriate classical spacetime symmetry group.

4.1. Irreducible representations of spacetime symmetry groups

Axiom 2 is perhaps the most significant from the point of view of spacetime structure. To unpack the significance of invariance under a spacetime symmetry group, first recall that a *representation* of a group $\mathcal{G}$ on a vector space V is a map U that takes elements of $\mathcal{G}$ to linear transformations on V and that preserves the group product. Elements of V are referred to as *carriers* of the representation of $\mathcal{G}$. An *irreducible representation* (IRREP) of $\mathcal{G}$ on V is one in there is no subspace of V invariant under the action of the image of U, other than the zero subspace or V itself. Intuitively, an IRREP cannot be "divided into parts". One can show that an IRREP is labeled uniquely by the eigenvalues of the Casimir invariants of $\mathcal{G}$'s Lie algebra, *i.e.*, those elements of the Lie algebra that commute with all other elements.

In the context of QFTs, since quantum states are physically distinct up to phase, one requires that the states be invariant under a projective representation of a spacetime symmetry group, *i.e.*, a representation that is unique up to a phase. In the relativistic case (see, *e.g.*, Weinberg, 1995, Chapter 2) one constructs projective IRREPs of the restricted Poincaré group, call it $\mathcal{P}$. These correspond to non-projective IRREPs of the universal covering of $\mathcal{P}$ (technically obtained by replacing the proper Lorentz subgroup $SO(3,1)$ of $\mathcal{P}$ with its universal covering group $SL(2,\mathbb{C})$). Such IRREPs are uniquely labeled by their mass and spin, these being

[1] I follow Araki's (1999, pp. 103–104) treatment of the Wightman axioms and adapt it, for the sake of comparison, to the treatment of the Lévy-Leblond axioms given in Lévy-Leblond (1967). In W1 and L1, the field operators should more precisely be defined as operator-valued distributions, and accommodations should be made for unbounded operators. These details will be glossed over in the following. Furthermore, in both cases, an additional axiom of asymptotic completeness, important for scattering theory, will not be needed in this essay. Finally, I consider only the $m \neq 0$ case for simplicity.

[2] *Cyclicity* of the vacuum, 4(ii), is the requirement that acting on the vacuum with operators defined on $\mathcal{H}$ (W4ii), or within a mass sector of $\mathcal{H}$ (L4ii) yields all states in $\mathcal{H}$ (resp. within the mass sector). In the axiomatic treatment, cyclicity guarantees that the fields form an irreducible representation of the equal time canonical commutation relations (Streater and Wightman, 1964/1989, pp. 101). This is a necessary condition for the construction of a Fock space representation, to be discussed later.

the eigenvalues of the Casimir invariants $P_\mu P^\mu$ and $S_\mu S^\mu$, $S_\mu = -1/2\varepsilon_{\mu\nu\rho\sigma}J^{\nu\rho}P^\sigma$, of the Poincaré Lie algebra (generated by infinitesimal spacetime-translations P^μ and Lorentz boosts $J^{\mu\nu}$). P^μ and S_μ admit representations in $\mathcal{H}$ as the 4-momentum and spin operators.

In the *Gal*-invariant case, projective IRREPs of *Gal* correspond to non-projective IRREPs of the extended Galilei group $\tilde{Gal}$.[3] These are uniquely labeled by their mass, internal energy, and spin, these being the eigenvalues of the Casimir invariants M, $U = H-(1/2m)\mathbf{P}^2$, and $\mathbf{S}^2 = (\mathbf{J}-(1/m)\mathbf{K}\times\mathbf{P})^2$ of the $\tilde{Gal}$ Lie algebra (generated by infinitesimal time-translations H, space-translations $\mathbf{P}$, rotations $\mathbf{J}$, Galilei boosts $\mathbf{K}$, and the one-parameter phase group M). M, U, and $\mathbf{S}$ admit representations in $\mathcal{H}$ as the mass, internal energy, and spin operators, respectively (Lévy-Leblond, 1967, pp. 161, 163).

Note that in the GQFT case, the symmetry group for the states ($\tilde{Gal}$) is distinct from the spacetime symmetry group (*Gal*), whereas this is not the case in the RQFT case. One notable consequence of this is that it gives rise to a superselection rule for GQFTs that prohibits superpositions of states with different masses (Lévy-Leblond, 1967, pp. 160). Briefly, $\tilde{Gal}$-invariant states are physically distinct up to phase, and this phase depends explicitly on the mass of the state (technically, to obtain $\tilde{Gal}$, one adds to *Gal* a central extension labeled by mass eigenvalues). Thus a transformation that only changes the phase of a state should not change the state's physical properties; and this will not be the case if one allows transformations between states with different masses.

Now so far in the axiomatic formulation, talk has only been about fields: Axiom 1, in particular, explicitly states that the fundamental variables of the theory represent fields. But there is a standard way of going from this "field-talk" to "particle-talk". This involves the construction of a Fock space.

4.2. From IRREPs to particles via Fock space

The carriers of projective IRREPs of a spacetime symmetry group are what Wigner identified as representing the states of "elementary systems". To identify these as single-particle states requires the construction of a Fock space.[4] The following briefly reviews four important steps in this process that are relevant to the Received View's concept of particle.

Step 1: *Hilbert space of states.* The first step involves the construction of a Hilbert space of states. Having obtained the IRREPs of a given spacetime symmetry group, one restricts attention to those whose carriers represent physically possible states. This involves restricting the ranges of the eigenvalues of the Casimir operators to physically possible values. In the relativistic case, for instance, one restricts attention to IRREPs labeled by $m=0$, for massless systems, and $m>0$, $P^\mu>0$, for massive systems with positive energies. In the $\tilde{Gal}$-invariant case, one restricts attention to IRREPS for which the internal energy U is bounded from below.[5] (These restrictions are enforced by the Spectrum Condition.) Under the assumption that the carriers of these "physical" IRREPs represent free particle states, one then forms a single-particle Hilbert space $\mathcal{H}$ as their span. (Here and below, for the $\tilde{Gal}$-invariant case, I will assume that the mass superselection rule on $\mathcal{H}$ and its constructs has been appropriately imposed.) This requires, in particular, the specification of a positive-definite inner product on the carrying space. This can be done without problem in the context of the Poincaré and extended Galilei groups, but may not be so easy in spacetimes that admit different symmetries (or none at all). More on this issue is given in Section 5.2 later.

Step 2: *Fock space.* One then identifies n-particle states as elements of the symmetrized or anti-symmetrized (depending on the spin) tensor product $\mathcal{H}^{(n)} \equiv \otimes^n\mathcal{H}$, where $\mathcal{H}^{(0)} \equiv \mathbb{C}$ for all spins. A Fock space $\mathcal{F}$ can now be defined as the direct sum of all such multiparticle state spaces: $\mathcal{F} = \oplus_{n=0}^{\infty}\mathcal{H}^{(n)}$,

Step 3: *Creation/annihilation operators*. Momentum space creation and annihilation operators $a^\dagger(q)$, $a(q)$ are now defined by their actions on n-particle states and on the vacuum (no-particle) state $|0\rangle$:

$$a^\dagger(q)|q_1\ldots q_n\rangle \equiv |qq_1\ldots q_n\rangle,\quad a^\dagger(q_1)\ldots a^\dagger(q_n)|0\rangle \equiv |q_1\ldots q_n\rangle,$$

$$a(q)|q_1\ldots q_n\rangle \equiv \sum\nolimits_{r=1}^{n}(\pm 1)^{r+1}\delta(q-q_r)|q_1\ldots q_{r-1}q_{r+1}\ldots q_n\rangle,$$
$$a(q)|0\rangle \equiv 0,$$

where the ± 1 sign depends on the spin, and q labels the appropriate Casimir invariants. These definitions guarantee that the creation and annihilation operators satisfy the canonical (anti-)commutation relations (CCRs)

$$[a^\dagger(q'),a^\dagger(q)]_\pm = 0,\ [a(q'),a(q)]_\pm = 0,\ [a(q'),a^\dagger(q)]_\pm = \delta(q'-q),$$

where $\pm$ indicates anti-commutator/commutator, depending on the spin. In non-axiomatic treatments, one now introduces configuration space field operators as Fourier transformations of $a^\dagger(q)$, $a(q)$. Schematically, $\phi^+_\ell(\mathbf{x},t) = \mathrm{F.T.}[a^\dagger(\mathbf{p},\sigma)]$, $\phi^-_\ell(\mathbf{x},t) = \mathrm{F.T.}[a^\dagger(\mathbf{p},\sigma)]$. The explicit form ultimately depends on the representation of the spacetime symmetry group that acts on the fields (here ℓ labels field components, $\mathbf{p}$ is the 3-momentum, and σ is the spin). These configuration space fields inherit the CCR structure of their momentum space counterparts. At this point, in non-axiomatic treatments, the micro-causality condition is imposed. One takes linear combinations of the configuration space fields $\phi_\ell(\mathbf{x},t) = \kappa\phi^+_\ell(\mathbf{x},t)+\lambda\phi^-_\ell(\mathbf{x},t)$ and requires that they satisfy *local commutativity*

$$[\phi_l{}^\dagger(\mathbf{x},t),\phi_l(x',t')]_\pm = 0,\quad \text{for } (\mathbf{x},t) \text{ and } (x',t') \text{ causally separated},$$

where, again, causal separation is cashed out in terms of the specific spacetime. This micro-causality constraint, together with the representation of the spacetime symmetry group under which the fields transform, uniquely determines the coefficients κ, λ, and the fields $\phi^+_\ell(\mathbf{x}, t)$, $\phi_\ell(\mathbf{x}, t)$ can now be said to be local in the sense of being independent of each other when they are causally separated.

Step 4: *Total Number Operator.* With this machinery in place, one can now define a number operator $N(q) = a^\dagger(q)a(q)$ such that $N(q)(a^\dagger(q)^n|0\rangle) = n(a^\dagger(q)^n|0\rangle)$. A total number operator that figures into the Received View's notion of particle may then be defined by $N = \int N(q)d^3q$. To fully justify identifying the states that it acts on as multiparticle states, one can observe that the eigenvectors of N are also eigenvectors of the Hamiltonian operator, which may be written as $H = \int E_q N(q)d^3q$ (where E_q is the energy of the qth state, the explicit form of which will depend on the spacetime symmetry group). This suggests that these eigenvectors represent states with a definite number of quanta with energies that are typical of that number of particles (Fraser, 2008, pp. 845–846).

Note that such a Fock space construction is only well-defined, in the relativistic context, for the case of a non-interacting quantum field theory. Within this context, it gives the Received View a rigorous way to define its notion of particle. And it

[3] More precisely, projective representations of *Gal* correspond to non-projective representations of the central extension of the universal covering of *Gal* (Lévy-Leblond, 1971, pp. 252). The universal covering of *Gal* is obtained by replacing its rotation subgroup $SO(3)$ with its universal covering group $SU(2)$.

[4] See, *e.g.*, Weinberg (1995, Chapters 2–5) for details for the relativistic case.

[5] In the Lévy-Leblond axioms for GQFTs there are no restrictions on the mass spectrum. In particular, states with opposite mass eigenvalues are interpreted as particle-antiparticle pairs to allow for particle production processes (Lévy-Leblond, 1967, pp. 162).

also gives it a rigorous way to implement its argument against particles.

5. Implications: no particles?

We now have the machinery in place to flesh out some of the details of the Received View's argument against particle interpretations of RQFTs. This argument splits into three parts, dealing the Reeh-Schlieder theorem, the Unruh Effect, and Haag's theorem.

5.1. The Reeh-Schlieder theorem and local number operators

The first part of the Received View's argument involves the Reeh-Schielder theorem and local number operators. One can show that the Spectrum Condition, in either its relativistic (W4iii) or non-relativistic (L4iii) form, entails that the vacuum state is *cyclic* for any local (von Neumann) algebra of operators $\mathfrak{R}(\mathcal{O})$ associated with a spatiotemporal region $\mathcal{O}$ of spacetime.[6] This means that the set of states $\{\phi|0\rangle;\ \phi\in\mathfrak{R}(\mathcal{O})\}$ generated by acting on the vacuum with any member of $\mathfrak{R}(\mathcal{O})$ is dense in $\mathcal{H}$.[7] The relativistic case was proven by Reeh and Schlieder in 1961 and is referred to as the Reeh-Schlieder theorem (see, *e.g.*, Streater and Wightman, 1964/1989, pp. 138), while Requardt (1982) demonstrated the non-relativistic case. To draw implications of this result for local number operators, one first notes the following general result in the theory of operator algebras:

> *General result* (see, *e.g.*, Bratelli and Robinson, 1987, pp. 85). Any cyclic vector for a von Neumann algebra $\mathfrak{R}$ is separating for its commutant $\mathfrak{R}'$.

The commutant $\mathfrak{R}'$ consists of operators that commute with all operators in $\mathfrak{R}$. Now Relativistic Local Commutativity (W3) entails that $\mathfrak{R}(\mathcal{O})'=\mathfrak{R}(\mathcal{O}')$, where $\mathcal{O}'$ is the *causal complement* of $\mathcal{O}$, defined as the set of all points *causally separated* (in this context, spacelike separated) from points in $\mathcal{O}$. In words: the commutant of a local algebra associated with a spacetime region $\mathcal{O}$ of Minkowski spacetime is the local algebra associated with the causal complement $\mathcal{O}'$ of $\mathcal{O}$. Thus, provided the causal complement of any region is non-empty, the general result entails that the vacuum is separating for any local algebra in Minkowski spacetime. We thus have the following *Separating Corollary*:

> *Separating Corollary* (Streater and Wightman, 1964/1989, pp. 139). Suppose (i) the vacuum is cyclic for $\mathfrak{R}(\mathcal{O})$; (ii) relativistic local commutativity (W3) holds; and (iii) the causal complement $\mathcal{O}'$ is non-empty. Then the vacuum is *separating* for $\mathfrak{R}(\mathcal{O})$.

Separability of the vacuum means that, given any bounded region $\mathcal{O}$ of Minkowski spacetime, and any operator ϕ associated with $\mathcal{O}$, if ϕ annihilates the vacuum, $\phi|0\rangle=0$, then it is identically zero, $\phi=0$. Now, for any bounded region $\mathcal{O}$ of Minkowski spacetime, $\mathcal{O}'$ is non-empty. Hence, the upshot of the Separating Corollary is that no bounded region of Minkowski spacetime can contain annihilation operators. These annihilate the vacuum, but cannot be identically zero, since they transform n-particle states into $(n-1)$-particle states. Thus no number operator can be associated with $\mathcal{O}$. Such a local number operator would act on states associated with $\mathcal{O}$ and return the number of particles present in $\mathcal{O}$. Thus local number operators do not exist for RQFTs.

Does this result hold in the non-relativistic context? In particular, does the Separating Corollary hold in the non-relativistic context? First note that Non-relativistic Local Commutativity (L3) only applies to *spatial* regions of classical spacetimes, *i.e.*, regions with zero temporal extent. Thus we should be a bit more explicit about the type of region we associate with a local algebra. In particular, one can consider two options:

(1) Associate local algebras with *spatiotemporal* regions, *i.e.*, regions with non-zero spatial and temporal extent.
(2) Associate local algebras with *spatial* regions, *i.e.*, regions with zero temporal extent.

In the relativistic context, this is a distinction that makes no difference. One can show that the local algebra associated with a spatial region $\mathcal{O}$ in Minkowski spacetime is the same as the local algebra associated with the domain of dependence $D(\mathcal{O})$ of $\mathcal{O}$ (Horuzhy, 1990, pp. 40–41, Theorem 1.3.14).[8] The latter, which consists of points p for which any (inextendible) causal worldline through p intersects $\mathcal{O}$, typically is a spatiotemporal region. On the other hand, in the non-relativistic context, the distinction between (1) and (2) is non-trivial. However, in both cases, one can show that the Separating Corollary cannot be derived.

To see this, first suppose that we adopt (1) for NQFTs. Then Requardt's (1982) proof of vacuum cyclicity goes through.[9] However, under reasonable assumptions, the causal complement of a spatiotemporal region of a classical spacetime is typically the empty set (barring topological mutants). In other words, all points outside a spatiotemporal region of a classical spacetime are typically causally connectible to some subset of points within that region. This assumes that the causal complement of a spatiotemporal region $\mathcal{O}$ of a classical spacetime consists of all points with zero temporal separation and non-zero spatial separation from all points in $\mathcal{O}$, which in turn assumes that infinite causal propagations are prohibited, but allows that finite causal propagations have no upper bound. These assumptions seem reasonable in so far as they follow from the requirement that simultaneous measurements be causally independent, which, arguably, motivates Local Commutativity in the non-relativistic context. The upshot is that Condition (iii) for the Separating Corollary is not met, and the vacuum is not separating.

Now suppose we adopt Option (2) and associate local algebras of operators with *spatial* regions of classical spacetimes. Then Requardt's (1982) proof of cyclicity fails, and Condition (i) of the Separating Corollary is not met.

Thus, the non-relativistic vacuum is not cyclic for *spatial* local algebras, and hence not separating; and while it is cyclic for *spatiotemporal* local algebras, it is not separating. Now one can argue that both of these results are due to the spatiotemporal structure of classical spacetimes; in particular, to the existence of an absolute temporal metric. First, it is the simultaneity structure

[6] In the axiomatic treatment of QFTs, one can associate to every bounded region $\mathcal{O}$ of spacetime, the polynomial algebra of operators, bounded and unbounded, smeared with test functions with support in $\mathcal{O}$. This algebra is supposed to provide candidates for local measurements confined to $\mathcal{O}$. In the following I will restrict attention to von Neumann algebras consisting of all such bounded operators.

[7] This is to be distinguished from the Cyclicity Axiom 4(ii), which is the requirement that the vacuum state be cyclic for the "global" algebra $\mathfrak{R}$; *i.e.*, the von Neumann algebra of all operators defined on $\mathcal{H}$, as opposed to a local algebra $\mathfrak{R}(\mathcal{O})$ of operators defined only on test functions with support in $\mathcal{O}$.

[8] Technically, this assumes a particular way of identifying spatial regions with subalgebras of operators on $\mathcal{H}$, namely, what Halvorson (2001, pg. 117) calls the *standard localization scheme*, which assigns to a spatial region $\mathcal{S}$, the relevant Cauchy data with support in $\mathcal{S}$ (intuitively, this Cauchy data is in 1-1 correspondence with those local field observables, viewed as solutions to the relevant field equations, with support in $\mathcal{S}$). Alternative localization schemes, such as Newton–Wigner, will not be a concern of this essay.

[9] In general, cyclicity for *spatiotemporal* local algebras, relativistic or non-relativistic, is a property of *any* state that is analytic in the energy.

associated with an absolute temporal metric that trivializes the causal complement of a spatiotemporal region of a classical spacetime; and this, in turn, prevents the non-relativistic vacuum from being separating for local algebras associated with such regions. Second, in brief, proofs of the cyclicity of the relativistic vacuum for spatial local algebras are based on the anti-local property, for spatial regions, of differential operators associated with relativistic field equations (Halvorson, 2001, pp. 118–119, reviews the Klein-Gordon case). This anti-local property for spatial regions is in part a consequence of the fact that relativistic field equations are *hyperbolic*, reflecting the Lorentzian metrics of relativistic spacetimes. In contrast, the differential operators associated with non-relativistic field equations are *parabolic*, reflecting the degenerate metrics in classical spacetimes; in particular, these spacetimes contain separate temporal metrics.[10] It is this feature that prevents parabolic differential operators from being anti-local for spatial regions, and thus is the cyclicity of the non-relativistic vacuum for spatial local algebras blocked. In the remaining subsection, I will attempt to substantiate this last claim.

Anti-locality, cyclicity, and classical spacetimes

An operator is said to be *anti-local* for a given region of spacetime just when a function and its transform under the operator can vanish in that region only if the function is identically zero (Segal and Goodman, 1965, pp. 630). The implication is that such an anti-local operator will transform a (non-zero) function with support entirely within a given region of spacetime into a function with "infinite tails". A series of results indicates that certain forms of elliptic differential operators are anti-local for Euclidean, or, in general, Riemannian spaces (Bär, 2000; Segal and Goodman, 1965; Strohmaier, 1999, 2000; Verch, 1993). These results are relevant to the study of RQFTs and NQFTs for two reasons. First, the elliptic operators in question are the "spatial" parts of the hyperbolic and parabolic differential operators that figure into relativistic and non-relativistic field equations. Hence anti-locality results derived for such elliptic operators may subsequently infect the full "spatiotemporal" versions of the hyperbolic and parabolic operators of which they are a part. This is, in fact, the case for the hyperbolic operators associated with relativistic field equations, and this has led to a literature on No-Go results for the existence of localized single-particle states and position operators in RQFTs (see, *e.g.*, Wallace, 2001, and references therein). This literature should be made distinct from the Received View's argument against particles. The latter argument, as I see it, is meant to be based on the stronger No-Go claim entailed by the Separating Corollary, namely that local number operators do not exist for RQFTs. The Received View is thus willing to allow that a coherent notion of particles may obtain even in situations that prohibit localized states, or position operators (see, *e.g.*, Halvorson and Clifton, 2002, pp. 17–18). In particular, the assumption is that local number operators provide a means to talk about the particle content in a region of spacetime in the absence of position operators and/or localized single-particle states.

This leads to the second reason anti-locality results are relevant to the study of RQFTs and NQFTs, namely one can demonstrate that anti-locality of the "spatial" part of certain relativistic differential operators entails the associated vacuum state has the Reeh-Schlieder property (*i.e.*, it is cyclic for any local algebra). Segal and Goodman (1965) initially demonstrated this for the vacuum state of the Klein-Gordon field in Minkowski spacetime, and their results have subsequently been extended to include the Klein-Gordon, Dirac, and Proca fields in ultrastatic and stationary Lorentzian spacetimes.[11] As Strohmaier indicates, these results are more general than they at first appear:

> As soon as a classical field satisfies a certain hyperbolic partial differential equation, a state over the field algebra of the quantized theory, which is a ground- or KMS-state with respect to the group of time translations, has the Reeh-Schlieder property (Strohmaier, 2000, pp. 106).

The hyperbolicity requirement is non-trivial. To see why, consider the following concrete examples (these expand on Saunders, 1992, pp. 372, and Streater, 1988, pp. 138). It turns out that a positive frequency solution ϕ to either the relativistic Klein-Gordon equation or the non-relativistic Schrödinger equation, as a real function of time, is a boundary value of a holomorphic function, call it **F**, defined on a complex extension of $\mathbb{R}^1$.[12] The Edge of the Wedge theorem then entails that if ϕ vanishes on some open set in time and then **F** vanishes everywhere (Streater and Wightman, 1964/1989, pp. 83, Theorem 2.17). Now if **F** vanishes everywhere, then so do all its boundary values. Hence ϕ cannot vanish on any open set in time unless it is identically zero. Thus the differential operators associated with the Klein-Gordon and Schrödinger equations are anti-local in *time*, but are they, in addition, anti-local in *space*? Suppose first that ϕ is a non-zero positive frequency solution to the Klein-Gordon equation and $\mathcal{S}$ is an open spatial region of Minkowski spacetime (*i.e.*, an open spacelike hypersurface). Then the *hyperbolicity* of the Klein-Gordon equation entails that if the relevant Cauchy data $(\phi, \partial\phi/dt)$ vanish on $\mathcal{S}$, then ϕ vanishes in the domain of dependence $D(\mathcal{S})$ of $\mathcal{S}$. In Minkowski spacetime, $D(\mathcal{S})$ is guaranteed to always have finite temporal extent; thus if ϕ vanishes in $D(\mathcal{S})$, it vanishes in some open set in time. The Edge of the Wedge theorem then entails that ϕ, as a non-zero boundary value of a holomorphic function in time, cannot vanish on open spatial sets. Hence the hyperbolicity of the Klein-Gordon equation entails that the Klein-Gordon operator is anti-local on spatial regions.

Now suppose ϕ is a non-zero positive frequency solution to the Schrödinger equation, and $\mathcal{S}$ is an open spatial region of a classical spacetime. Then the *parabolicity* of the Schrödinger equation entails that if the relevant Dirichlet (ϕ) or Neumann $(\partial\phi/dt)$ data vanish on $\mathcal{S}$, then ϕ vanishes in $D(\mathcal{S})$. However, in classical spacetimes $D(\mathcal{S}) = \mathcal{S}$, *i.e.*, any point p not in $\mathcal{S}$ is such that there will be a causal curve that intersects p and does not intersect $\mathcal{S}$. Hence $D(\mathcal{S})$ has *no* temporal extent. Thus if ϕ vanishes on $\mathcal{S}$, this does not entail that it vanishes on some open set in time. Thus the simultaneity structure of classical spacetimes, as encoded by the absolute temporal metric, entails that the Schrödinger operator is not anti-local on spatial regions. Note further that for a *spatiotemporal* region $\mathcal{O}$ of a classical spacetime, it is still the case that $D(\mathcal{O}) = \mathcal{O}$, but now if ϕ vanishes on $\mathcal{O}$, it also vanishes in some open set in time. Hence, the Edge of the Wedge theorem entails that the Schrödinger operator is anti-local for *spatiotemporal* regions, and this is consistent with Requardt's (1982)

[10] The partial differential equations (PDEs) of interest in non-relativistic and relativistic QFTs are of the parabolic form $u_t + Lu = 0$, and the hyperbolic form $u_{tt} + Lu = 0$, respectively, where L is a second-order elliptic operator dependent on the spatial coordinates. These PDEs are obtained as the configuration space representation of the Spectrum Condition (defined explicitly on momentum space variables) and inherit the signature of the spacetime through the (inverse) Fourier transformation of the momentum space variables. The result is an elliptic PDE in Riemannian spacetimes, a hyperbolic PDE in Lorentzian spacetimes, and a parabolic PDE in classical spacetimes (see, *e.g.*, McCabe, 2007, pp. 41–43).

[11] For the Klein-Gordon and Dirac fields in ultrastatic spacetimes, see Verch (1993) and Bär (2000), respectively. For the Dirac field in static globally hyperbolic spacetimes, see Strohmeier (1999). For the Klein-Gordon, Dirac, and Proca fields in stationary spacetimes, see Strohmeier (2000).

[12] This is the analyticity property referred to in footnote 9. It is enforced by the Spectrum Condition in the context of the Wightman or Lévy-Leblond axioms.

demonstration that the non-relativistic vacuum is cyclic for local algebras defined on such regions.

One might thus infer that anti-locality on open spatial regions of spacetime is a characteristic of the hyperbolic differential operators that appear in the field equations of RQFTs. This is because the Lorentzian spacetimes associated with such operators place bounds on the upper limit of propagations, and thus domains of dependence are guaranteed to have temporal extent. While parabolic equations do admit well-posed problems in the context of initial and/or boundary data on open spatial regions, the fact that the associated classical spacetimes place no bounds on the upper limit of propagations entails domains of dependence will always be trivial (barring topological mutants). And this ensures that the parabolic differential operators that appear in the field equations of NQFTs are not anti-local for spatial regions of spacetime.

Thus, to recap, it is the absolute temporal structure of classical spacetimes that prevents the vacuum in NQFTs from being separating, and this subsequently makes room for local number operators.

5.2. The Unruh Effect and unique total number operators

Another part of the Received View's argument against particles in RQFTs involves the Unruh Effect and unique total number operators for non-interacting RQFTs. Recall that the first step in constructing a total number operator in the Fock space formalism was the construction of a Hilbert space of single-particle states, and this required the definition of a positive-definite inner product on the carrying space of the irreducible representations of the relevant spacetime symmetry group. A sufficient condition for the existence of a positive-definite inner product on the state space of an RQFT in a Lorentzian spacetime is *global hyperbolicity* of the spacetime (Wald, 1994, pp. 64–65). Briefly, global hyperbolicity entails that the spacetime can be foliated by a family of smooth Cauchy surfaces $\sum_t$ parameterized by a global time function t.[13] This global time function then allows a decomposition of the solution space of hyperbolic PDEs into positive and negative frequency subspaces. One can then uniquely define a positive-definite inner-product on the space of positive frequency solutions, and a Fock space with its attendant total number operator can then be constructed.

Now while global hyperbolicity guarantees the *existence* of a total number operator, it does not guarantee *uniqueness*. Globally hyperbolic spacetimes may admit more than one foliation $\sum_t$, and hence more than one global time function. Thus there may be more than one way to "split the frequencies" of the solutions to hyperbolic PDEs. Most authors take Minkowski spacetime as an example: the Unruh Effect occurs when an observer, accelerating relative to the Minkowski vacuum, experiences a thermal bath of quanta. This is due to the presence (in the right Rindler wedge) of a timelike Killing vector field (*i.e.*, a timelike isometry) distinct from the one associated with Poincaré symmetry. These distinct timelike isometries induce distinct global time functions, and this allows for distinct unitarily inequivalent Fock representations of the canonical commutation relations (the Minkowski and Rindler representations). So accelerating and inertial observers in Minkowski spacetime make use of *different* total number operators, and hence disagree on the total number of particles present.

Now what would guarantee *uniqueness* of a total number operator would be the presence of a unique global time function. And this is only guaranteed in those (topologically well-behaved) spacetimes that admit an absolute temporal metric.[14] The moral then is that the existence of a unique total number operator *reflects* the non-relativistic structure of classical spacetimes.

Note, finally, that some authors (Arageorgis et al. 2003, pp. 180–181) have claimed that the Rindler representation in the Unruh Effect is unphysical; thus there is no problem of uniqueness in Minkowski spacetime.[15] Instead of entering into this debate, for those convinced that the uniqueness of a "physical" total number operator is not an issue for non-interacting RQFTs, I caution, "Beware of Haag". Whether or not classical spacetimes are necessary for the existence of a unique physical total number operator in non-interacting QFTs, they are arguably so for interacting QFTs. And surely interacting QFTs are the more empirically successful theories, and hence should warrant our attention more. Let's thus move on to Haag.

5.3. Haag's theorem and total number operators

The last part of the Received View's argument against particles in RQFTs involves Haag's theorem and total number operators for interacting QFTs. In one form, Haag's theorem implies that, under fairly reasonable assumptions, one cannot construct unitarily equivalent representations of the CCRs that describe both free and interacting fields (see, *e.g.*, Earman and Fraser, 2006, pp. 316). The immediate upshot of this is that in an interacting RQFT, the Fock space representation of free particles cannot be used to describe interacting particles. Moreover, Fraser (2008) has argued that other attempts at constructing Fock space representations for interacting particles in RQFTs fail. In particular, attempts to construct an appropriate Fock space representation of interacting particles by second-quantizing classical interacting relativistic fields, or by defining interacting creation and annihilation operators directly with respect to classical interacting relativistic fields, fail (Fraser, 2008 pp. 849–855). Taken together, these results indicate that total number operators that can be interpreted as counting particles do not exist for interacting RQFTs. My response is going to be that the reason why total number operators do not exist in interacting RQFTs is that their existence requires absolute spacetime structures; in this case, an absolute temporal metric.

To see why, consider what Earman and Fraser (2006, pp. 313–314) term the Haag–Hall–Wightman (HHW) version of Haag's theorem. For two local fields ϕ_1, ϕ_2, the first part of the HHW theorem demonstrates that, under the assumptions,

(a) the fields belong to irreducible representations of the equal-time canonical commutation relations;
(b) there are unique Euclidean-invariant vacuum states $|0_1\rangle$, $|0_2\rangle$;
(c) there is a unitary transformation $V(t)$ that relates the fields at a given time;

then the vacuum states are constant multiples of each other: $c|0_1\rangle = V(t)|0_2\rangle$, $|c|=1$. The second part of the HHW theorem demonstrates that, under the assumptions,

[13] Wald (1994, pp. 56). A spacetime is globally hyperbolic just when it admits a Cauchy surface, *i.e.*, a spacelike surface $\sum$ that intersects every causal curve.

[14] The compatibility condition, $\nabla_a t_b = 0$, on the temporal metric of a classical spacetime entails t_a is closed, and thus locally exact. If M is topologically well-behaved (if, for instance, it is simply connected), then t_a is globally exact, and there exists a unique globally defined time function $t: M \to \mathbb{R}$ satisfying $t_a = \nabla_a t$. On the other hand, suppose there exists a global time function $t: M \to \mathbb{R}$. Then a temporal metric t_{ab} compatible with a connection ∇_a can be defined by $t_{ab} = (\nabla_a t)(\nabla_b t)$.

[15] Briefly, these authors first note that the right Rindler wedge is *extendible*; *i.e.*, it can be isometrically embedded as a proper subset of another spacetime, namely, full Minkowski spacetime. Intuitively, this suggests that the time function associated with the right Rindler wedge should not count as a global way to "split the frequencies" of solutions to hyperbolic PDEs in Minkowski spacetime. To make this more precise, Arageorgis et al. point out that the Rindler vacuum cannot be extended to a physically realizable state (in the sense of satisfying the Hadamard condition) on the global algebra of Minkowski spacetime.

(d) (a)–(c) above;
(e) the fields and vacuum states are Poincaré-invariant;

then the first four vacuum expectation values of the two fields are equal. One can then demonstrate that if one of the fields is free, both are free (Earman and Fraser, 2006, pp. 314). Thus if we insist that there are such things as *interacting relativistic* quantum fields, then in the first instance, we have to deny that the vacuum state of any such interacting field is a constant multiple of the corresponding free vacuum state. And, in the second instance, this entails that we must give up one or more of Assumptions (a), (b), or (c). Now if one is wedded to the notion that number operators (for whatever reason) are essential aspects of the formulation of a QFT, one may be loath to give up assumptions (a) and (b), in so far as both appear to be necessary for Fock space representations of free fields. One is thus compelled to reject (c).[16]

Now the condition that the vacuum states of a free and an interacting field are multiples of each other is equivalent to the condition that the interaction does not polarize the vacuum. Vacuum polarization occurs when the Hamiltonian operator that describes the interacting field fails to annihilate the vacuum of the free field. (Let $H=H_0+H_I$ be an interacting Hamiltonian operator with free part H_0 and interaction part H_I. Suppose further that there is a free vacuum $|0_F\rangle$ such that $H_0|0_F\rangle=0$. Then the interaction is said to polarize the vacuum just when $H|0_F\rangle\neq 0$.) Vacuum polarization is conceptually problematic: the Hamiltonian operator encodes the energy of the system, so it should act on the state with zero energy (the vacuum state) and produce zero. In the HHW theorem, Part I, let H_1 and H_2 be Hamiltonian operators associated with the fields ϕ_1, ϕ_2, such that $H_1|0_1\rangle=0=H_2|0_2\rangle$. Then, since $c|0_1\rangle=V(t)|0_2\rangle$ implies that $H_2|0_1\rangle=0$, we can replace the condition that the vacuum states are constant multiples of each other with the condition that vacuum polarization does not occur. (And, obviously, if vacuum polarization does not occur, then the free and interacting vacuum states must be constant multiples of each other.)

This gives us two necessary conditions for the existence of interacting fields that are unitarily equivalent to free fields, namely

(i) The interaction polarizes the vacuum or
(ii) Poincaré-invaraince does not hold.

One can now argue that it's the absolute temporal structure of classical spacetimes that allows NQFTs to satisfy (ii) while denying the conceptually problematic (i).

This argument was suggested by Lévy-Leblond (1967, pp. 160–161) to explain how interacting GQFTs avoid Haag's theorem. He asks us to consider the structure of the (extended) Galilei group, as encoded in its Lie algebra. This structure is encoded in the algebraic commutation relations between 5 generators, which are responsible for time-translations (H), space-translations ($\mathbf{P}$), velocity boosts ($\mathbf{K}$), rotations ($\mathbf{J}$), and mass scalings (M).[17] Lévy-Leblond points out that the generator of time-translations nowhere occurs on the right hand side of these relations. Intuitively, time-translations are "independent" of the other generators, and this encodes the fact that the temporal metric is absolute in Neo-Newtonian spacetime. One can now consider a representation of the generators on a state space that encodes the time-translation generator as the Hamiltonian operator H_0 of a free field. If we then construct a Hamiltonian operator $H=H_0+H_I$ that consists of this free part and a part H_I that describes an interaction, then the "free" representation (H_0, $\mathbf{P}$, $\mathbf{K}$, $\mathbf{J}$, M) will be unitarily equivalent to the "interacting" representation (H, $\mathbf{P}$, $\mathbf{K}$, $\mathbf{J}$, M) in the sense of satisfying the same commutation relations. The only constraint is that the interaction term H_I be Galilei-invariant. And one can then show that if the free Hamiltonian annihilates a vacuum state, then so does the interacting Hamiltonian (Fraser, 2006, pp. 40, footnote 23). Hence no vacuum polarization occurs.

Lévy-Leblond now asks us to contrast this with the situation arising in the case of the Poincaré group. The commutation relations that define its generators include one in which the generator of time-translations is "mixed up" with the generators of pure Lorentz boosts and space-translations: $[\mathbf{K},\ \mathbf{P}]=iH$.[18] This encodes the fact that there is no independently occurring absolute temporal metric in Minkowski spacetime. Hence requiring that H_I be Poincaré-invariant will not guarantee that H_0+H_I will preserve the Lie bracket structure of the representation in which H_0 appears (Fraser, 2006, pp. 41). In general, another structurally distinct representation of the Poincaré generators will have to be constructed for the interacting Hamiltonian H_0+H_I. Thus if a free Hamiltonian annihilates a Poincaré-invariant vacuum state, this does not guarantee that an interacting Hamiltonian will do so, too.

The moral then is that the existence of interacting GQFTs that do not polarize the vacuum reflects the absolute temporal structure of Neo-Newtonian spacetime. Now one would expect that this way of avoiding Haag's theorem extends to all NQFTs, given that all have in common the classical spacetime structure associated with an absolute temporal metric. The key condition is that the generator of time translations be independent, as it were, of the other generators, and this will be the case for the symmetry group of any spacetime with an absolute temporal metric. This inference is given support by explicit examples of Fock space representations of interacting GQFTs, as mentioned above, as well as Christian's (1997) Maxwell-invariant QFT of Newtonian gravity. The latter is an interacting NQFT that admits a Fock space representation and a total number operator (Christian, 1997, pp. 4872). Thus one might infer that any NQFT will not run afoul of Haag's theorem.[19]

6. Conclusion

Schematically, the Received View's argument against particle interpretations of RQFTs can be reconstructed in the following way:

(RV1) (Particle concept) ⇒ (localizability/countability)
(RV2) (Localizability/countability) ⇒ (local/unique total number operators)

[16] If one is not wedded to number operator chauvinism, one may give up (b) and avoid Haag's Theorem by inserting a cut-off into one's interacting theory and renormalize the fields. Alternatively, some authors have proposed giving up (a) (Streater and Wightman, 1964/1989, pp. 101).

[17] The generators of the extended Galilei Lie algebra (H, $\mathbf{P}$, $\mathbf{K}$, $\mathbf{J}$, M) satisfy the following commutation relations:
$[J_i,J_j]=i\varepsilon_{ijk}J_k \quad [J_i,K_j]=i\varepsilon_{ijk}K_k \quad [J_i,P_j]=i\varepsilon_{ijk}P_k$
$[K_i,P_j]=iM\delta_{ij} \quad [K_i,\ H]=iP_i$
$[J_i,H]=[K_i,K_j]=[P_i,P_i]=[P_i,H]=0$
$[H,M]=[J_i,M]=[P_i,M]=[K_i,M]=0$

[18] In a 3-dimensional notation, the Poincaré Lie algebra is generated by time translations H, space translations P_i, rotations J_i, and pure Lorentz boosts K_i that satisfy the following commutation relations: $[J_i,J_j]=i\varepsilon_{ijk}J_k \quad [J_i,K_j]=i\varepsilon_{ijk}K_k \quad [J_i,P_j]=i\varepsilon_{ijk}P_k \quad [K_i,P_j]=iH\delta_{ij} \quad [K_i,K_j]=-i\varepsilon_{ijk}J_k \quad [K_i,H]=iP_i \quad [J_i,H]=[P_i,H]=[H,H]=0$.

[19] Note that Fraser (2006, pp. 46) indicates that non-relativistic local commutativity L3 blocks the Streit-Emch version of Haag's theorem (which does not explicitly require Poincaré invariance). Lévy-Leblond (1967, pp. 166) suggests the Spectrum Condition L4(iii) also plays a role. Since these two axioms are (arguably) shared by any NQFT, this further suggests that nothing unique to the Galilei group beyond what it shares in common with other non-relativistic spacetime symmetry groups does the work in avoiding the consequences of Haag's theorem.

(RV3) (RQFT) $\Rightarrow \neg$(local/unique total number operators)

$\therefore$ (RQFT) $\Rightarrow \neg$(particle concept)

(RV1) encodes the Received View's pre-theoretic particle concept, embodied in the necessary properties of localizability and countability. (RV2) encodes the manner in which the Received View chooses to mathematically represent this concept in the forms of local and unique total number operators.

I have argued that a necessary condition for the existence of local and unique total number operators is the existence of an absolute temporal metric:

(Local/unique total number operators)
$\Rightarrow$ (absolute temporal metric) (∗)

First, the existence of local number operators requires the absolute temporal metric of a classical spacetime. This structure allows NQFTs to avoid the consequences of the Reeh-Schlieder theorem. In particular, it prevents the non-relativistic vacuum state from being separating for any local algebra of operators, and this allows for the possibility of local number operators. Second, the existence of a unique total number operator requires the absolute temporal metric of a classical spacetime. An absolute temporal metric guarantees the existence of a unique global time function for non-interacting NQFTs, and hence a unique way to define an inner-product (or its equivalent) on the space of single-particle states. This ultimately leads to a uniquely defined total number operator *via* a Fock space construction, thus avoiding the implications of the Unruh Effect. Finally, an absolute temporal metric allows interacting NQFTs to avoid polarizing the vacuum, and this immunizes such theories against the consequences of Haag's theorem. In particular, interacting NQFTs exist that are unitarily equivalent to non-interacting NQFTs, and hence the former can appropriate the Fock space structure of the latter, and, in particular, the total number operators defined in the latter.

The claim (∗) suggests that the Received View is (implicitly) appealing to the existence of an absolute temporal metric in its representations of the pre-theoretic particle concepts of localizability and countability. In particular, it suggests that (RV2) is informed (implicitly) by non-relativistic intuitions, and thus should be rejected in the context of interpretations of relativistic QFTs.

What are our options? We might reject (RV2) while upholding (RV1). This would require identifying mathematical objects, other than Fock space number operators, that encode the notions of localizability and countability and that are supported by the formalisms in which RQFTs, both free and interacting, are presented. On the other hand, one might reject (RV1) and attempt to identify alternative conditions of adequacy for a particle concept that are compatible with the relativistic context. The concept of localizability, for instance, might be weakened to allow for asymptotically localized states (Bain, 2000), or "effectively localized" states (Wallace, 2001).[20] What this essay warns against, however, is the implicit adoption of mathematical objects that require structures associated with classical spacetimes, either in the representations of localizability and/or countability, or in the representations of any particle concepts that are meant to replace them. Given this warning, it may turn out that the identification of appropriate mathematical representations goes hand in hand with alternative conditions of adequacy for the particle concept.

Regardless of the strategy one adopts, what I hope to have made plausible is that the debate over whether or not RQFTs admit particle interpretations has yet to be settled.

[20] Halvorson and Clifton (2002, pp. 16) prove a No-Go theorem for the existence of "unsharply localizable" particles. But as they state, their theorem". . . only show[s] that it is impossible to define position operators that obey appropriate relativistic constraints. But it does not immediately follow from this that we lack any notion of localization in relativistic quantum theories" (pp. 18). This motivates them to replace position operators with local number operators as the way to mathematically represent the concept of particle localizability. They go on to prove a Reeh-Schlieder-like theorem from which they conclude: "This serves as a *reducio ad absurdum* for a notion of localizable particles in any relativistic quantum theory" (pp. 20). But again, the qualification should be that Reeh-Schlieder-type No-Go theorems militate against attempts to represent localizability by local number operators.

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