

# Interpreting Effective Field Theories

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*Abstract:* An effective field theory (EFT) is a theory of the dynamics of a physical system at energies small compared to a given cut-off. Low-energy states with respect to this cut-off are effectively independent of states at high energies; hence one may study the low-energy dynamics without the need for a detailed description of the high-energy dynamics. Many authors have suggested that, because of the essential role the cut-off plays in the standard (Wilsonian) method of constructing an EFT, an appropriate interpretation of an EFT requires a realistic interpretation of the cut-off. For some, this suggests an ontology of "quasi-autonomous domains" (Cao and Schweber 1993); for others, it suggests an ontology in which space is discrete and finite (Fraser 2009); and for yet others, it suggests that EFTs engage in idealizations and are inherently approximate (Fraser 2009, Castellani 2002). I argue that these interpretations are not forced upon us, in so far as there is an alternative to the Wilsonian method for constructing an EFT that does not explicitly employ a cut-off.

*Keywords:* effective field theory, quantum field theory, ontology, empirical equivalence.

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## 1. Introduction

An effective field theory (EFT) of a physical system is a theory of the dynamics of the system at energies small compared to a given cut-off. For some systems, low-energy states with respect to this cut-off are effectively independent of ("decoupled from") states at high energies. Hence one may study the low-energy sector of the theory without the need for a detailed description of the high-energy sector, in so far as one can demonstrate that the EFT is empirically equivalent to the underlying high-energy theory, restricted to the appropriate energy scale and to any given range of accuracy. The standard (Wilsonian) approach to the construction of an EFT puts emphasis on the cut-off: under this approach, an EFT is constructed from an underlying high-energy theory by identifying fields with energies above the cut-off and then integrating them out of the high-energy action. Many authors have claimed that this process requires a

realistic interpretation of the cut-off, but just what this entails has been a source of debate. Cao and Schweber (1993), for instance, make use of a formal result (the "Decoupling Theorem") that the Wilsonian approach guarantees, given certain assumptions about the underlying high-energy theory, in order to justify an ontology of "quasi-autonomous domains". But this ontology has been disputed by authors who observe that the formal result does not hold in the general case (Hartmann 2001; Grinbaum 2008). For other authors, a realistic interpretation of the cut-off suggests an ontology in which space is discrete and finite (Fraser 2009). Under this view, the standard (Wilsonian) construction of an EFT is characterized by an idealization under which a continuum high-energy theory is placed on a lattice. In general the claim that EFTs involve this or similar types of idealization has motivated the view that EFTs are intrinsically approximate and context-dependent (Castellani 2002).

I will argue that these interpretations are not forced upon us, in so far as there is an alternative to the standard (Wilsonian) method for constructing an EFT that does not formally employ a cut-off. This alternative produces what Georgi (1992, 1993) refers to as a "continuum EFT". I will argue that a realistic interpretation of a continuum EFT (a) provides better support for Cao and Schweber's (1993) ontology of quasi-autonomous domains; (b) supports an interpretation of EFTs in which space is continuous and not discrete; and (c) supports an interpretation of EFTs in which they need not be considered intrinsically approximate and context-dependent.

Section 2 briefly reviews the general steps in the construction of a Wilsonian EFT. Section 3 indicates the extent to which the explicit form of an EFT depends on the type of renormalization scheme one employs to handle divergent integrals that can arise when one uses the EFT to calculate the values of observable quantities. Briefly, under Georgi's (1992, 1993) classification, a "Wilsonian EFT" employs momentum cut-off regularization to tame divergent integrals, and a *mass-dependent* subtraction scheme to subtract the resulting infinities from physical quantities. In contrast, a "continuum EFT" employs dimensional regularization and a *mass-independent* subtraction scheme. Ultimately, the choice of renormalization scheme has no effect on the values of observable quantities; thus the Wilsonian EFT of a given high-energy theory and its continuum EFT are empirically equivalent. However, to the extent that this choice determines the explicit form of the EFT (*i.e.*, to the extent that a Wilsonian EFT is formally distinct from a continuum EFT), arguably, it has non-trivial consequences when it comes to the question of how the EFT can be interpreted. These consequences are investigated in Section 4.

## 2. How to Construct an EFT

The construction of an EFT follows one of two general procedures, top-down and bottom-up, depending on whether the underlying high-energy theory is known. Both procedures are based on an expansion of the effective action (which formally describes the EFT) in terms of an infinite sum of local operators, constrained by symmetry and "naturalness" considerations. They differ on how this expansion is derived: the top-down approach derives it by eliminating degrees of freedom from the high-energy theory, whereas the bottom-up approach derives it from scratch.

The top-down approach starts with a known theory and then systematically eliminates degrees of freedom associated with energies above some characteristic energy scale  $E_0$ . The practical goal is to obtain a low-energy theory that allows one to more easily calculate the values of observable quantities associated with energies below  $E_0$  than in the original theory. One is typically instructed to proceed by means of a two step process: (I) One first integrates out the high momenta (*i.e.*, "heavy") fields, which results in non-local interactions among the low momenta ("light") fields; and (II) one then performs an operator product expansion on these non-local interactions to produce the local interactions of the effective theory. The following describes these steps in slightly more detail.<sup>1</sup>

- (I) Given a field theory described by an action  $S$  and possessing a characteristic energy scale  $E_0$ , suppose we are interested in the physics at a lower scale  $E \ll E_0$ . First choose a cut-off  $\Lambda$  at or slightly below  $E_0$  and divide the fields  $\phi$  into high and low momenta parts with respect to  $\Lambda$ :  $\phi = \phi_H + \phi_L$ , where  $\phi_H$  have momenta  $k > \Lambda$  and  $\phi_L$  have momenta  $k < \Lambda$ . Now integrate out the high momenta fields. In the path integral formalism, do the integral over the  $\phi_H$ . Schematically

$$\int \mathcal{D}\phi_L \int \mathcal{D}\phi_H e^{iS[\phi_H, \phi_L]} = \int \mathcal{D}\phi_L e^{iS_\Lambda[\phi_L]}, \quad (1)$$

where the Wilsonian effective action  $S_\Lambda$  is given by  $e^{iS_\Lambda[\phi_L]} = \int \mathcal{D}\phi_H e^{iS[\phi_L, \phi_H]}$ . The effective Lagrangian density  $\mathcal{L}_{\text{eff}}$  is thus given by  $S_\Lambda[\phi_L] = \int d^D x \mathcal{L}_{\text{eff}}[\phi_L]$ , where  $D$  is the dimension of the spacetime.

- (II) Typically, the integration over the heavy fields will result in a non-local effective action (*i.e.*, one in which terms occur that consist of operators and derivatives that are not all evaluated at the same spacetime point). In practice, this is addressed by expanding the effective action in a set of local operators:

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<sup>1</sup> This exposition is based on Polchinski (1993), and Campbell-Smith and Mavromatos (1998).

$$S_\Lambda = S_0(\Lambda, g^*) + \sum_i \int d^D x g^i \mathcal{O}_i, \quad (2)$$

where the sum runs over all local operators  $\mathcal{O}_i$  allowed by the symmetries of the initial theory, and the  $g_i$  are coupling constants. Assuming weak coupling, the expansion point  $S_0$  may be taken to be the free action of the initial theory, so that  $g^* = 0$ .

To see how the effective action relates to the initial high-energy action, one can perform a dimensional analysis on the operators that appear in (2). This allows one to obtain information about their behavior as the cutoff  $\Lambda$  is lowered. This analysis involves three steps:

- (i) Choose units in which the action is dimensionless ( $\hbar = 1, c = 1$ ). In such units, energy has dimension  $+1$  while length has dimension  $-1$ . We now use the free action to determine units for the field operators.<sup>2</sup> This then determines units for the coupling constants, and subsequently, terms in the expansion (2). For instance, if an operator  $\mathcal{O}_i$  has been determined to have units  $E^{\delta_i}$  (thus dimension  $\delta_i$ ), then its coupling constant  $g_i$  has units  $E^{D-\delta_i}$ , and the magnitude of the  $i$ th term is  $\int d^D x \mathcal{O}_i \sim E^{\delta_i-D}$ .
- (ii) To make the cutoff dependence of the terms explicit, one can define dimensionless coupling constants by  $\lambda_i = \Lambda^{\delta_i-D} g_i$ . The order of the  $i$ th term in (2) is then

$$\lambda_i (E/\Lambda)^{\delta_i-D}. \quad (3)$$

- (iii) We can now classify terms in the expansion (2) into three types:

- *Irrelevant:*  $\delta_i > D$ . This type of term falls at low energies as  $E \rightarrow 0$ . Such terms are suppressed by powers of  $E/\Lambda$ .
- *Relevant:*  $\delta_i < D$ . This type of term grows at low energies as  $E \rightarrow 0$ .
- *Marginal:*  $\delta_i = D$ . This type of term is constant and equally important at low and high energies (in so far as quantum effects can modify its scaling behavior towards either relevancy or irrelevancy).

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<sup>2</sup> Consider, for instance, a scalar field theory with free action  $S = (1/2) \int d^D x (\partial_\mu \phi)^2$ . This action contains  $D$  powers of the spacetime coordinate from  $d^D x$  (with total energy units  $E^{-D}$ ), and  $-2$  powers from the two occurrences of the spacetime derivative  $\partial_\mu \equiv \partial/\partial x^\mu$  (with total units  $E^2$ ). Thus, in order for the action to be dimensionless (with units  $E^0$ ), the field  $\phi$  must have units  $E^y$  satisfying  $E^{-D} E^2 E^y E^y = E^0$ , and thus dimension  $y = D/2 - 1$ .

This dimensional analysis indicates that, in most cases, there will only be a finite number of relevant and marginal terms in (2).<sup>3</sup> Thus, in such cases, the low-energy EFT will only depend on the underlying high-energy theory through a finite number of parameters.<sup>4</sup> It is typical in the literature on EFTs to elevate these considerations to a principle. Polchinski (1993, pg. 6) articulates such a principle in the following way:

*The low energy physics depends on the short distance theory only through the relevant and marginal couplings, and possibly through some leading irrelevant couplings if one measures small enough effects.* (Polchinski 1993, pg. 6.)

Note that arbitrarily many irrelevant terms can occur in (2), but they are suppressed by powers of  $E/\Lambda$ . Moreover, the cut-off can be used as a regulator for any divergences associated with these terms in calculations of the values of observable quantities. Thus, "... even though the [effective] Lagrangian may contain arbitrarily many terms, and so potentially arbitrarily many coupling constants, it is nonetheless predictive so long as its predictions are only made for low-energy processes, for which  $E/\Lambda \ll 1$ " (Burgess 2004, pg. 17).

Finally, in addition to symmetry considerations, a further constraint is typically applied to the expansion in (2). Dimensional analysis entails that, without unnatural fine-tuning of the original parameters, the coefficients of the operators in the expansion in (2) are dimensionless of order 1. This entails that mass terms, which have coefficients proportional to powers of the cut-off, cannot appear in this expansion (such mass terms are relevant). This motivates the following naturalness condition:<sup>5</sup>

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<sup>3</sup> Consider, again, scalar field theory. From footnote 2, the dimension of a scalar field  $\phi$  is given by  $D/2 - 1$ ; hence, in general, an operator  $\mathcal{O}_i$  constructed from  $M$   $\phi$ 's and  $N$  derivatives will have dimension  $\delta_i = M(D/2 - 1) + N$ . For  $D \geq 3$ , there are only a finite number of ways in which  $\delta_i < D$  and  $\delta_i = D$ .

<sup>4</sup> Two caveats should be mentioned for this dimensional method of justifying the finite dependence of an EFT on the high-energy theory. First, it is based on using the free theory to determine units, and this assumes the high-energy theory is weakly coupled. Second, dimensional arguments work when using the EFT to make simple "tree-level" calculations of the values of observed quantities, but for higher-level "loop" corrections to such calculations, scaling based on dimensional arguments may break down, and one may have to appeal to a particular renormalization scheme in order to justify the explicit reliance of an EFT on a finite number of parameters (see Section 3 below).

<sup>5</sup> Polchinski (1993, pg. 9). Another way to motivate this restriction is by noting that mass terms correspond to gaps in the energy spectrum in so far as such terms describe excitations with finite rest energies that cannot be made arbitrarily small. These gaps create problems when taking a smooth low-energy limit (in the sense of a smooth renormalization group evolution of parameters). Thus for Weinberg (1996, pg. 145), renormalization group theory can only be applied to EFTs that are massless or nearly massless.

*EFTs must be natural, meaning that all possible masses must be forbidden by symmetries.*

The procedure outlined above for constructing an EFT requires having in one's possession the action (or Lagrangian density) of a high-energy theory. In some cases of interest, the fundamental high-energy theory is not known, but an EFT is, nonetheless, still constructible. One simply begins with the operator expansion (2) and includes all terms consistent with the naturalness constraint and with the symmetries and interactions assumed to be relevant at the given energy scale. One can then determine how these terms scale when a given cut-off is raised (as opposed to lowered). Examples of such "bottom-up" EFTs include the Four-Fermi theory (as it was originally constructed) that describes low-energy weak interactions; and, in the view of many physicists, the Standard Model itself (see, *e.g.*, Hartmann 2001 for discussion). Another example is effective field theoretic formulations of general relativity in which the Hilbert action is identified as the first term of the expansion (2) (Burgess 2004).

### **3. On Renormalization Schemes and Interpretations of EFTs**

As Manohar (1997, pg. 326) observes, knowing the Lagrangian density of a quantum field theory is not enough to calculate the values of observable quantities. To accomplish the latter (using perturbative techniques) requires expanding the Greens function that represents a particular observable quantity in an infinite series in which, typically, divergent integrals appear. This is the problem of renormalization in quantum field theory. Thus in addition to knowing the Lagrangian density of a quantum field theory, one needs to specify a renormalization scheme. This is a method that specifies (i) a means of regulating divergent integrals, and (ii) a means of subtracting the associated infinities in a systematic way. There are a number of different methods that accomplish this, two of which are important in the context of interpreting EFTs. The first adopts momentum cut-off regularization and a mass-dependent method of subtraction, and is used (at least implicitly) in the Wilsonian approach to constructing EFTs (outlined above in Section 2). The second adopts dimensional regularization and a mass-dependent method of subtraction, and is associated with what Georgi (1992, pg. 1; 1993, pg. 215) has called "continuum EFTs".

#### *3.1. Mass-Dependent Schemes and Wilsonian EFTs*

In a Wilsonian EFT, the explicit appearance of the cut-off  $\Lambda$  that defines the border between the low-energy physics and the high-energy physics suggests employing it as a means to regulate the particular type of divergent integrals that appear in calculations of the values of observable quantities. Given such a divergent integral of the schematic

form  $\int_0^\infty d^D p \kappa(p)$ , where  $D$  is the dimension of spacetime and  $\kappa(p)$  is a particular function of momentum  $p$ , one can insert the cut-off  $\Lambda$  and rewrite the integral as the sum of a finite piece and an infinite piece:

$$\int_0^\Lambda d^D p \kappa(p) + \int_\Lambda^\infty d^D p \kappa(p) . \quad (4)$$

For the types of divergent integrals under consideration, the infinite piece can be absorbed into a redefinition of the parameters of the theory through the introduction of renormalization constants. It turns out that, in this method of regularization, these constants are dependent on the heavy masses that appear in the high-energy theory; hence, the manner in which they are defined is referred to as a *mass-dependent* subtraction scheme.<sup>6</sup>

There are two main advantages of employing this type of renormalization scheme in the context of EFTs. First, it is conceptually consistent with the image of an EFT as a low-energy approximation to a high-energy theory based on a restriction of the latter to a particular energy scale. This scale is explicitly represented by the cut-off  $\Lambda$  which thus plays a *double role* in designating the appropriate energy scale, and in cutting-off divergent integrals. The second advantage of using this renormalization scheme is that it guarantees that the Decoupling Theorem holds, given a few other assumptions.

The Decoupling Theorem is due to Appelquist and Carazzone (1975). Hartmann (2001) describes it thusly:

For two coupled systems with different energy scales  $m_1$  and  $m_2$  ( $m_2 > m_1$ ) and described by a renormalizable theory, there is always a renormalization condition according to which the effects of the physics at scale  $m_2$  can be effectively included in the theory with the smaller scale  $m_1$  by changing the parameters of the corresponding theory. (Hartman 2001, pg. 283.)

This theorem is a formal guarantee of the informal EFT "ideology" of Polchinski (1993, pg. 6), stated above in Section 2. Hartmann (2001, pg. 284) is careful to note that it requires that there is an underlying high-energy theory that is renormalizable, and that different mass scales exist in this theory. Moreover, as Georgi (1992, pg. 3) indicates, the renormalization condition that the theorem refers to is, in fact, a mass-dependent subtraction scheme.

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<sup>6</sup> More precisely, a mass-dependent subtraction scheme is one in which anomalous dimensions and renormalization group  $\beta$  functions explicitly depend on  $\mu/M$ , where  $\mu$  is the renormalization scale and  $M$  is the heavy mass (Georgi 1993, pg. 221).

The above advantages of cut-off regulated, mass-dependent renormalization schemes are balanced by the following disadvantages:

- (1) A momentum cut-off regularization method violates Poincaré invariance of the underlying high-energy theory, as well as any gauge invariance it may possess.
- (2) Mass-dependent subtraction schemes typically prevent the justification, based on dimensional analysis, that allows one to ignore the potentially infinite number of irrelevant terms in the effective action (2) from being extended from tree-level calculations to higher-order loop corrections. The reason is that in mass-dependent schemes, the simple tree-level dependence of irrelevant terms on orders of  $1/\Lambda$  can break down when doing higher-order loop corrections. In particular, in these higher-order corrections, the dependence of irrelevant terms on the cut-off may be of order 1 (in general, such terms have a power law dependence on the cut-off), and thus such terms cannot be ignored (Manohar 1997, pp. 327-228; Pich 1998, pg. 14). Note that this does not prevent loop calculations from proceeding in mass-dependent schemes; rather it makes them more difficult (Manohar 1997, pg. 329).

### 3.2. Mass-independent Schemes and Continuum EFTs

To address Problem (2), many authors suggest adopting a mass-independent renormalization scheme. In this type of scheme, the dimensional parameter  $\mu$  (analogous to the momentum cut-off  $\Lambda$  in the cut-off approach) only appears in loop corrections in logarithms, and not powers, thus the relevant integrals are small at scales much smaller than the heavy fields (Manohar 1997, pg. 238; Pich 1998, pg. 15). This allows one to effectively ignore the contributions of irrelevant terms, not only at tree-level as naive dimensional analysis allows, but also for higher-order loop corrections as well.

Mass-independent renormalization schemes are typically associated with dimensional regularization. This takes advantage of the mathematical fact that the particular types of divergent integrals that arise in quantum field theoretic calculations, again represented schematically by  $\int_0^\infty d^D p \kappa(p)$ , will converge for sufficiently small values of  $D$ . Formally, one lets  $D = 4 - \epsilon$  in the integral (where  $\epsilon$  is a very small constant), and then analytically continues  $D$  to 4. This process picks up poles in  $D$ -dimensional momentum space, and these can be absorbed into a redefinition of the parameters of the theory. In this case, such redefinitions are independent of the masses, hence the term *mass-independent subtraction scheme*.



In the context of EFTs, there are two main advantages of employing a mass-independent renormalization scheme based on dimensional regularization. First, dimensional regularization respects Poincaré and gauge invariance. Second, as indicated above, mass-independent renormalization schemes allow one to truncate the effective action (2) to a finite list of terms, not only for tree-level calculations, but also for higher-order loop calculations. However, it turns out that this simplification comes at the cost of having terms that explicitly include the heavy fields appear in this finite list (see, *e.g.*, Burgess 2004, pg. 19). This has the following consequences:

- (1') Many authors consider mass-independent schemes to violate the "spirit" of an EFT, to the extent that the latter is based on the notion of a cut-off, below which the physics is explicitly described by only the light fields (Burgess 2004, pg. 19; Burgess 2007, pg. 343; Polchinski 1993, pg. 5).
- (2') Perhaps more importantly, the presence of heavy field terms in an effective action employing a mass-independent renormalization scheme prevents the application of the Decoupling Theorem (Georgi 1993, pg. 225; Manohar 1997, pg. 329). As mentioned above in Section 3.1, the latter holds only for mass-dependent schemes.

It turns out that Problem (2') can be addressed by inserting decoupling by hand into an EFT that employs a mass-independent scheme, but this requires a slight reconceptualization of the nature of an EFT. This results in what Georgi (1992, pg. 1; 1993, pg. 215) refers to as a "continuum EFT".

How can an EFT be constructed without initial appeal to a cutoff? Briefly, for top-down constructions, the initial momentum splitting of the fields in a Wilsonian EFT and the integration over the heavy modes, is replaced in a continuum EFT by the following steps (after Georgi 1993, pg. 228; see, also, Burgess 2004, pp. 19-20; Burgess 2007, pg. 344):

- (I') Start with a dimensionally-regularized theory with Lagrangian density  $\mathcal{L}_H(\chi, \phi) + \mathcal{L}(\phi)$  at a large scale  $\mu$ , where  $\mathcal{L}(\phi)$  describes the light fields and  $\mathcal{L}_H(\chi, \phi)$  describes everything else (where  $\chi$  are the heavy fields of mass  $M$ ). Now evolve the theory to lower scales using the renormalization group: This allows you to go from scale  $\mu$  to scale  $\mu - d\mu$  without changing the content of the theory.
- (II') When  $\mu$  gets below  $M$ , the effective theory is changed to a new one without the heavy fields:  $\mathcal{L}(\phi) + \delta\mathcal{L}(\phi)$ , where  $\delta\mathcal{L}(\phi)$  encodes a "matching correction" that includes any new nonrenormalizable interactions that may be required. The matching correction is made so that the physics of the light fields is the same in the two theories at the boundary  $\mu = M$ . To explicitly calculate  $\delta\mathcal{L}(\phi)$ , one

expands it in a complete set of local operators in the same manner that the expansion (2) for Wilsonian EFTs is performed:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}(\phi) + \sum_n \delta\mathcal{L}^n(\phi) . \quad (5)$$

Dimensional analysis can now be applied to determine the scaling behavior of the terms in (5), in the same way it is applied in Wilsonian EFTs. Again, in the case of a continuum EFT, this analysis is valid not just for tree-level calculations, but also for higher-order loop calculations as well.

To summarize and compare, in the construction of a Wilsonian EFT, the heavy fields are first integrated out of the underlying high-energy theory and the resulting Wilsonian effective action is then expanded in a series of local operator terms. The cutoff  $\Lambda$  in a Wilsonian EFT plays a double role: first, through the definition of the heavy and light fields, in explicitly demarcating the low-energy physics from the high-energy physics; and second, in regulating divergent integrals in the calculation of observable quantities. In the construction of a continuum EFT, the heavy fields are initially left alone in the underlying high-energy theory, which is first evolved down to the appropriate energy scale. The continuum EFT is then constructed by completely removing the heavy fields from the high-energy theory, as opposed to integrating them out; and this removal is compensated for by an appropriate matching calculation (this latter is ultimately responsible for the appearance of heavy modes into the operator expansion (5)). In a continuum EFT, the first role of the Wilsonian cutoff  $\Lambda$  is played by the renormalization scale  $\mu$  that demarcates the low-energy physics from the high-energy physics. The second role of the Wilsonian cut-off is dropped, the procedure of dimensional regularization taking its place.

This last observation suggests the motivation for Georgi's phrase "continuum EFT". In a Wilsonian EFT, the regularization of divergent integrals is performed by restricting the range of momentum variables in integrals over momentum space. The Fourier transform equivalent of this procedure is a restriction of the range of coordinate variables in integrals over coordinate space. Hence regularization in a Wilsonian EFT is analogous to placing a high-energy continuum theory on a discrete lattice, and this is the reason why momentum cut-off regularization violates Poincaré invariance. In contrast, in a "continuum EFT", the regularization of divergent integrals is performed by calculating them in a continuous spacetime of dimension  $D$  (the Fourier transform equivalent of  $D$ -dimensional momentum space). This is the reason why dimensional regularization does not violate Poincaré invariance.

## 4. Implications

Different renormalization schemes ultimately all agree on the values of physical quantities. In particular, both mass-dependent and mass-independent schemes will, at the end of the day, agree on all empirically measured quantities.<sup>7</sup> Thus a Wilsonian EFT for a given physical system is empirically equivalent to that system's continuum EFT. On the other hand, the fact that these types of EFT place different emphasis on the nature of the cut-off suggests they can be interpreted as telling us different things about the world. I now consider some of the implications this has for debates over the ontological status of EFTs.

#### 4.1. *Decoupling and Quasi-Autonomous Domains*

Cao & Schweber (1993) emphasize the role of the Decoupling Theorem in understanding the ontological significance of EFTs:

Thus, with the decoupling theorem and the concept of EFT emerges a hierarchical picture of nature offered by QFT [quantum field theory], one that explains why the description at any one level is so stable and is not disturbed by whatever happens at higher energies, and thus justifies the use of such descriptions. (Cao and Schweber 1993, pg. 64.)

In this picture, the [physical world] can be considered as layered into quasi-autonomous domains, each layer having its own ontology and associated 'fundamental law'. (Cao and Schweber 1993, pg. 72.)

They further suggest EFTs entail "... an antifoundationalism in epistemology and an antireductionism in methodology" (Cao and Schweber 1993, pg. 69). Huggett and Weingard (1995, pg. 187) interpret this as a view that "... holds that nature is described by a genuinely never-ending tower of theories, and that the competing possibilities of unification and new physics should be abandoned".

Hartmann (2001, pg. 298) claims that "Cao and Schweber's talk of quasi-autonomous domains rests on the validity of the decoupling theorem", and then rightly points out that the latter is not necessarily valid in all cases in which EFTs exist. In particular, it requires the existence of an underlying high-energy renormalizable theory with different mass scales. This vitiates Cao and Schweber's antifoundationalism and/or antireductionism, if they are taken to entail that there is no underlying theory.

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<sup>7</sup> Burgess (2004, pg. 20) explains this in the following way: "... the difference between the cutoff- and dimensionally regularized low-energy theory can itself be parameterized by appropriate local effective couplings within the low-energy theory. Consequently, any regularization-dependent properties will necessarily drop out of final physical results, once the (renormalized) effective couplings are traded for physical observables."

On the other hand, Castellani (2002) appears to endorse an aspect of the "quasi-autonomous domain" interpretation:

The EFT approach in its extreme version provides a level structure ("tower") of EFTs, each theory connected with the preceding one (going "up" in the tower) by means of the [renormalization group] equations and the matching conditions at the boundary... (Castellani 2002, pg. 263.)

It appears that the participants in this debate are talking past each other, having implicitly adopted different notions of an EFT. Cao and Schweber, in particular, are sometimes ambiguous on what concept of EFT they are employing. On the one hand, in their emphasis on the Decoupling Theorem and momentum cut-off regularization, they appear to adopt a Wilsonian notion of an EFT. Moreover, in the context of discussing the ontological significance of momentum cut-off regularization, they observe that

The following can be stated for other regularization schemes... but not for dimensional regularization, which is more formalistic and irrelevant to the point discussed here. (Cao and Schweber 1993, pg. 92, Footnote 17.)

On the other hand, immediately before introducing the EFT-inspired hierarchical picture of nature, they describe an EFT in the following terms:

The EFT can be obtained by deleting all heavy fields from the complete renormalizable theory and suitably redefining the coupling constants, masses, and the scale of the Green's functions, using the renormalization group equations. (Cao and Schweber 1993, pg. 64.)

This appears to be a description of the construction of a continuum EFT as outlined in Section 3.2. Hartmann (2001) makes it evident in his critique of Cao and Schweber that he implicitly has adopted the Wilsonian notion of an EFT. Finally, Castellani implicitly adopts a continuum notion of an EFT, describing its construction as one based on matching conditions (see, *e.g.*, Castellani 2002, pg. 262).

Note that the notion of EFT one adopts should be important in this debate. The Decoupling Theorem was proven in the context of Wilsonian EFTs of a particular kind; namely, those for which there exists an underlying renormalizable high-energy theory with different mass scales. Thus, if by "EFT" Cao and Schweber mean "Wilsonian EFT", then Hartmann's critique goes through: Wilsonian EFTs do not, by themselves, support an ontology of quasi-autonomous domains. On the other hand, the Decoupling Theorem fails for continuum EFTs, but, arguably this does not prevent them from

supporting a well-defined notion of quasi-autonomous domains. This is because in continuum EFTs, decoupling is inserted "by hand" in the form of matching calculations. Thus, if by "EFT" Cao and Schweber mean "continuum EFT", then, arguably, Hartmann's critique does not go through: continuum EFTs do, by themselves, support an ontology of quasi-autonomous domains. Hence, provided by "EFT", Castellani means "continuum EFT", her endorsement of such an ontology is justified.<sup>8</sup>

#### 4.2. *Realistic Interpretations of the Cut-off*

In typical expositions of EFTs, emphasis is placed on a realistic interpretation of the cut-off. Many authors claim that such a realistic interpretation is what separates the contemporary concept of an EFT from older views of renormalization (Hartmann 2001, pg. 282; Castellani 2002, pg. 261; Grinbaum 2008, pg. 37). Under these older views, a cut-off, when it occurred in accounts of QFTs, only played a role as a regulator of integrals, and was taken to infinity at the end of the day (if this resulted in a finite theory, then the theory was deemed to be renormalizable). The contemporary concept of an EFT, so the story goes, is based on viewing the cut-off realistically in a different role; namely, as a means of demarcating low-energy physics from high-energy physics. By now it should be obvious from the discussion in Section 3 that this standard account is only half the story; it, perhaps unfairly, privileges Wilsonian EFTs and the double role the cut-off plays in their construction, over continuum EFTs.

This is not to say that a realistic interpretation of the cut-off cannot be made in the context of continuum EFTs. Recall that in continuum EFTs, the role that the Wilsonian cut-off  $\Lambda$  plays in demarcating low-energy physics from high-energy physics is played by a scaling constant  $\mu$  (*i.e.*, the renormalization scale that appears in the renormalization group equations). Certainly this scaling constant can be realistically interpreted, perhaps as a basis for an ontology of quasi-autonomous domains; and it might even be referred to as a cut-off, in this context. The important point is that, in a continuum EFT, this scaling constant does not play the second role that the Wilsonian cut-off  $\Lambda$  plays; namely, as a regulator of divergent integrals.

Thus to realistically interpret the cut-off in an EFT could mean one of two things:

- (a) The Wilsonian regulator  $\Lambda$  should be realistically interpreted.

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<sup>8</sup> Grinbaum (2008, pg. 40), notably, makes a distinction between the "strict" form of decoupling associated with the Appelquist-Carazzone theorem, and a "milder" empirical decoupling thesis, which evidently is to be associated with the matching calculations involved in the construction of continuum EFTs.

- (b) The constant that demarcates low-energy physics from high-energy physics (given by  $\Lambda$  in Wilsonian EFTs and by  $\mu$  in continuum EFTs) should be realistically interpreted.

As the discussion at the end of Section 3.2 suggests, adopting (a) might motivate an ontology in which space is discrete: Momentum cut-off regularization is analogous to placing a continuum theory on a discrete lattice. But such an ontology is *not* forced upon us by a realistic interpretation of the cut-off. Simply put, (b) does not entail (a). One can adopt a realistic interpretation of the cut-off in a continuum EFT and, at the same time, an ontology in which spacetime is continuous.

This conclusion affects part of a recent debate over interpretations of quantum field theory (QFT). Wallace (2006) adopts an interpretation of QFT in which a cut-off is inserted and realistically interpreted and justifies it by appealing to features of EFTs (see, *e.g.*, pg. 43). Fraser (2009) criticizes this "cut-off variant" of QFT in the following way:

If the cutoffs are taken seriously, then they must be interpreted realistically; that is, space is really discrete and of finite extent according to the cutoff variant of QFT. (Fraser 2009, pg. 552.)

Fraser suggests this makes cut-off QFT unsatisfactory: If the cut-off is not taken seriously, then cut-off QFT reduces to "infinitely renormalized QFT", which is the standard textbook account in which cut-offs, when they appear, are taken to infinity in renormalization schemes. An appeal to Haag's theorem then indicates this textbook account is inconsistent. On the other hand, if the cut-off is taken seriously, then cut-off QFTs avoid Haag's theorem (which, among other things, requires Poincaré invariance); but they entail space is discrete and finite, and "... nobody defends the position that QFT provides evidence that space is discrete and the universe is finite" (Fraser 2009, pg. 552).

Once again, being clear on the type of EFT one adopts on which to base the cut-off variant of QFT will make a difference in this debate. If by "cut-off QFT" one means "Wilsonian EFT", then Fraser's critique, arguably, goes through. However, if by "cut-off QFT" one means "continuum EFT", then the above argument does not go through: continuum EFTs support an ontology in which spacetime is continuous.<sup>9</sup> Thus, provided one can demonstrate that continuum EFTs avoid Haag's theorem, a cut-off

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<sup>9</sup> This suggests that the categories of empirically equivalent variants of QFT that Fraser (2009, pg. 538) identifies should be expanded. Her "cut-off QFT" category should be split into "cut-off regularized QFT" and "dimensionally regularized QFT".

version of QFT based on continuum EFTs is a viable alternative to the "formal variant" (*i.e.*, axiomatic QFT) that Fraser (2009, pg. 538) advocates.

### 4.3. EFTs and Idealizations

Finally, two further examples of how the distinction between types of EFTs is important for issues of interpretation involve the notions of idealization and approximation. The fact that Wilsonian EFTs support an ontology in which space is discrete suggests to Fraser (2009, pp. 564) that the cutoff variant of QFT is an indispensable idealization (in so far as QFTs in general should be interpreted as referring to continuous spacetime). Again, continuum EFTs do not make such an idealization, hence a version of cut-off QFT based on the latter is not an idealization of this type.

Relatedly, Castellani (2002, pp. 260, 263) suggests that EFTs are "intrinsically approximate and context-dependent". An EFT, under this view, is an approximation of an underlying high-energy theory and is valid only within a specified energy range. Now, arguably, Cao and Schweber's ontology of quasi-autonomous domains is intended in part to address claims of this type. Under Cao and Schweber's view, an EFT describes a quasi-autonomous domain by means of a complete description of phenomena within a given energy range, independent for the most part of descriptions at higher or lower energies (Cao and Schweber 1993, pg. 64, are careful to explain how high-energy effects do make themselves present in relevant and marginal terms of an effective Lagrangian density). Thus, the discussion in Section 4.1 entails that EFTs need not be interpreted as intrinsically approximate, provided one adopts continuum EFTs as the object of one's interpretation.

## 4. Conclusion

One implication of the above discussion is that, in order to understand how EFTs can be interpreted, one needs to understand the methods that physicists use in applying them. By focusing attention on different renormalization schemes that practicing physicists actually employ, one can discern two types of empirically equivalent EFTs -- Wilsonian EFTs and continuum EFTs. These are non-trivial examples of empirically equivalent theories in so far as they suggest distinct ontologies. Continuum EFTs support an ontology of quasi-autonomous domains, whereas Wilsonian EFTs do not. Continuum EFTs support an ontology with a continuous spacetime, whereas Wilsonian EFTs require space to be discrete and finite. These features of Wilsonian EFTs have contributed to the view that EFTs in general engage in idealization and are inherently approximate. The fact that continuum EFTs do not engage in such idealizations

suggests that EFTs do admit interpretations in which they are not considered inherently approximate.

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