Studies in History and Philosophy of Modern Physics 56 (2016) 27–38

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Emergence and mechanism in the fractional quantum Hall effect

Jonathan Bain

Tandon School of Engineering, New York University, Department of Technology, Culture and Society, 6 Metrotech Center, Brooklyn, NY 11201, United States

ARTICLE INFO

Article history:
Received 27 February 2016
Received in revised form
24 August 2016
Accepted 29 September 2016
Available online 2 November 2016

Keywords:
Emergence
Mechanism
Quantum Hall effect
Condensed matter physics

ABSTRACT

For some authors, an adequate notion of emergence must include an account of a mechanism by means of which emergent behavior is realized. This appeal to mechanism is problematic in the case of the fractional quantum Hall effect (FQHE). There is a consensus among physicists that the FQHE exhibits emergent phenomena, but there are at least four alternative explanations of the latter that, arguably, appeal to ontologically distinct mechanisms, both at the microphysics level and at the level of general organizing principles. In light of this underdetermination of mechanism, one is faced with the following options: (I) deny that emergence is present in the FQHE; (II) argue for the priority of one mechanistic explanation over the others; or (III) temper the desire for a mechanism-centric account of emergence. I will argue that there are good reasons to reject (I) and (II) and accept (III). In particular, I will suggest that a law-centric account of emergence does just fine in explaining the emergent phenomena associated with the FQHE.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

1. Introduction

For some authors, an adequate notion of emergence must include an account of a mechanism by means of which emergent behavior is realized. These authors maintain that without such an account, emergence risks becoming a trivial concept that is appealed to whenever we lack epistemic access to a physical phenomenon, or the technical skill required to provide a complete description of it. According to Mainwood (2006, 284), for instance, "…emergent properties are not a panacea, to be appealed to whenever we are puzzled by the properties of large systems. In each case, we must produce a detailed physical mechanism for emergence, which rigorously explains the qualitative difference that we see with the microphysical". The mechanism of most interest to Mainwood in the context of condensed matter physics is spontaneous symmetry breaking (SSB). Morrison (2012, 160) similarly claims that emergence in condensed matter systems must be underwritten by a physical mechanism, and in particular SSB: "The important issue here is not just the elimination of irrelevant degrees of freedom; rather it is the existence or emergence of cooperative behavior and the nature of the order parameter (associated with symmetry breaking) that characterizes the different kinds of systems." Finally, Lancaster and Pexton (2015) note that while the fractional quantum Hall effect (FQHE) cannot be explained in terms of SSB, nevertheless a physical mechanism can be associated with it; namely, what Wen (2013) refers to as "long-range entanglement", and it is in terms of this mechanism that emergence in the FQHE should be understood.

The aim of this essay is to question this mechanism-centric view of emergence by considering Lancaster and Pexton's example of the FQHE in a bit more detail.[1] The consensus among physicists is that this effect exhibits emergence, but there are at least four alternative explanations of it that, arguably, appeal to distinct ontological mechanisms, at both the microphysical level and the level of what have been called higher organizing principles. These explanations include (1) the Laughlin ground state account, (2) the composite fermion account, (3) the composite boson account, and (4) the topological order account. The FQHE is described by these accounts as (i) a many-body Coulomb effect of electrons, (ii) a one-body effect of composite fermions, (iii) a many-body effect of composite bosons, and (iv) a many-body entangled effect of electrons, respectively. These ontologically distinct microphysical

[1] In addition to Lancaster and Pexton (2015), recent philosophical discussions of the FQHE include Shech (2015), and Lederer (2015).

http://dx.doi.org/10.1016/j.shpsb.2016.09.002
1355-2198/

mechanistic accounts are underwritten by the following ontologically distinct high-level mechanistic accounts: (a) localization (accounts 1 and 2); (b) spontaneous symmetry breaking (account 3), and (c) long-range entanglement (account 4).

In light of this underdetermination of mechanism, one is faced with the following options: (I) deny that emergence is present in the FQHE; (II) argue for the priority of one mechanistic explanation over the others; or (III) temper the desire for a mechanism-centric account of emergence. I will argue that there are good reasons to reject (I) and (II) and accept (III). In particular, I will suggest that emergence in the FQHE is best described in terms of a law-centric view of emergence. According to this view, emergence is characterized, in part, by novelty, and novelty is underwritten by an appeal to distinct laws, cashed out as the equations of motion associated with formally distinct Lagrangian densities.

Section 2 contrasts mechanism-centric and law-centric views by means of a particular notion of emergence relevant to the FQHE. Sections 3 and 4 describe the quantum Hall effect and alternative mechanistic accounts of the FQHE. Section 5 makes the case for a law-centric view of emergence in the FQHE.

2. Two versions of emergence

I will make the distinction between the mechanism-centric and law-centric views of emergence in terms of a particular ontological account of emergence. The intent is to capture a sense of emergence that is relevant to the FQHE, on the one hand, and yet general enough to underwrite the mechanism-centric/law-centric distinction, on the other. The account I will consider is based on two conditions, inspired by Mainwood (2006, 20):

(a) *Microphysicalism*: An emergent system is composed of microphysical systems that comprise the fundamental system and that obey the fundamental system's laws.
(b) *Novelty*: The properties of the emergent system are *dynamically independent* of, and *dynamically robust* with respect to, the properties of the fundamental system.

Microphysicalism is intended to capture the intuition that an emergent system does not “float free” of the fundamental system from which it emerges; rather, there must be a sense in which the fundamental system ontologically determines the properties of the emergent system. This sense cannot be too strong, however, and this is the motivation for *novelty*. To say an emergent property is *dynamically independent* of a fundamental property is to say the former is independent of the dynamics that governs the latter. To say an emergent property is *dynamically robust* with respect to a fundamental property is to say the former is dynamically independent of the latter, and remains so, despite changes in the dynamics of the latter.

Dynamical independence is supposed to guarantee that, while the emergent system is ontologically determined in a minimal sense by the fundamental system, insofar as it is ultimately composed of microphysical systems that comprise the fundamental system and that obey the fundamental system's laws, it is not completely determined by the fundamental system, insofar as, even though its *microphysical constituents* obey the fundamental system's laws, it does not; hence the dynamics of the fundamental system fails to specify how the emergent system behaves. One way (but perhaps not the only way) to cash out the notion of dynamical independence is in terms of a mathematical distinction between equations of motion. Thus if the equations of motion that govern the properties of a given system are distinct from those that govern the properties of another system, the former properties can be said to be dynamically independent of the latter properties.

Dynamical robustness is supposed to guarantee that this independence is persistent; it is not just due to a particular realization of the fundamental system's dynamics, but rather persists under slight perturbations of the latter. Suppose, for instance that the dynamics of systems S and S' are encoded in equations of motion that differ only in an interaction term (suppose S' is a relativistic scalar field with an interaction described by a potential $V'(\varphi)$, and S is a relativistic scalar field with an interaction described by a potential $V(\varphi) \neq V'(\varphi)$. Then S' is dynamically independent of S, insofar as the behavior of (the properties of) S' will not be determined by the dynamics of (the properties of) S. But S' is not dynamically robust with respect to S, insofar as a change in the dynamics of S that maps $V(\varphi)$ onto $V'(\varphi)$ will result in a dynamics that completely determines the behavior of S'. The failure of dynamical robustness in this example suggests that S' is not dynamically independent of S in an essential way. Rather, $S'$and S seem better understood as the same system undergoing different interactions.

Note that when the dynamics of S' and S are sufficiently distinct, dynamical robustness is somewhat trivial. For instance, if S' is a scalar field and S is a Maxwell field, then S' is dynamically independent of, and dynamically robust with respect to, S. Changes to the dynamics of the Maxwell field obviously will not map its dynamics onto the dynamics of the scalar field, simply because the dynamics of a Maxwell field is unrelated in any way to the dynamics of a scalar field. Dynamical robustness becomes more interesting when the dynamics of S' and S are related in a way that does not affect their independence. On the surface, this may seem strange: how can two types of dynamics be *independent* of each other yet still be *related*. Arguably, this is the case when the dynamics of S' encodes the low-energy dynamics of S; i.e., when the theory T' that describes S' is a low-energy effective theory of a high-energy theory T that describes S. In this case, S' is dynamically independent of S, insofar as T' and T are formally distinct (at the level of equations of motion, say). Moreover, more than one high-energy theory can be associated with the same low-energy effective theory T': any theory T^* that differs from T only in its high-energy degrees of freedom will have the same low-energy effective theory T' as T. In other words, changes to the high-energy degrees of freedom of T will not affect its relation to T'. This suggests that, in such cases, S' is “non-trivially” dynamically robust with respect to S.[2]

Dynamical independence and dynamical robustness are intended to be instances of Butterfield (2011, 921) more general concepts of “novelty” and “robustness”, which are defined relative to a comparison class as “not definable from the comparison class”, and “the same for various choices of, or assumptions about, the comparison class”, respectively. Note that the above account emphasizes the role that dynamics plays in underwriting these concepts, but remains agnostic about how dynamics is to be understood (i.e., whether in terms of causes, mechanisms, dynamical

[2] To make this a bit more precise would require fleshing out some of the details involved in the construction of an effective field theory (EFT) (see, e.g., Bain, 2013, 258–61). For EFT aficionados, dynamical independence of S' from S holds insofar as the effective Lagrangian density $\mathcal{L}_{T'}[\theta]$ that encodes T' is formally distinct from the high-energy Lagrangian density $\mathcal{L}_T[\phi]$ that encodes T, where ϕ are the degrees of freedom of S and θ are the degrees of freedom of S'. The construction of $\mathcal{L}_{T'}$ assumes that there is a characteristic energy Λ with respect to which ϕ can be split into a high-energy regime and a low energy regime θ. Dynamical robustness of S' with respect to S holds insofar as, (a) we assume that T is “realistic” in the sense of being 4-dim, and this entails that T' is characterized by a finite number of “marginal” and “relevant” couplings (i.e., couplings that are significant for energies $E << \Lambda$), which encode the contributions from T; and (b) this finite number only depends on the dimension of spacetime and the symmetries at low-energies; in particular, the effects of any other high-energy theory T^* that differs from T only in its high-energy degrees of freedom with respect to Λ can be encoded in the same finite number of relevant and marginal couplings in T'.

laws, or something else). Note, too, that whereas these concepts are the sole conditions for emergence according to Butterfield, the above account adds microphysicalism. It is thus similar to Crowther's (2015, 431–3) account of emergence, which organizes Butterfield's criteria under the general heading of "Independence" and adds an additional condition called "Dependence". But whereas Crowther allows "Dependence" to be cashed out in terms of supervenience, I will allow that microphysicalism is compatible with the particular type of failure of supervenience associated with a system in an entangled state.[3] The reason for this is to be able to account for explanations of emergence in the FQHE that attribute it to a holism associated with a particular type of entanglement. One such example will be discussed in Section 4.4.

The task of further fleshing out the above notion of emergence is to resolve the tension between the "dependence" criterion of microphysicalism and the "independence" criterion of novelty (to use Crowther's terminology). A *mechanism-centric* view of emergence resolves this tension by positing a mechanism that is responsible for dynamical independence and robustness in the light of microphysicalism. In this context, there are two ways of understanding the notion of a mechanism. The first is at the level of "microphysics", as a particular collection of entities and activities that are organized in such a way that they realize a regularity, law, principle, etc. (Weber, van Bouwel, & de Vreese, 2013, 59). Alternatively, a high-level mechanism can be understood as a general physical process that can be instantiated by any of a number of distinct microphysical processes (mechanisms in the first sense). This second sense of mechanism is advocated by Morrison (2012, 149), and Laughlin and Pines (2000, 28), who refer to it as a "structural/dynamical feature of physical systems" and a "higher organizing principle", respectively. According to these views, when a general physical process is instantiated by a variety of ontologically distinct microphysical systems that all exhibit the same set of universal properties, the latter are novel and autonomous in senses relevant to their being taken to be emergent.

An alternative view of emergence underwrites novelty by an appeal to distinct laws. I will call this a *law-centric* view. By a law I will mean, very generally, a way to specify or constrain the *dynamics* of a *particular* physical system: a law is supposed to describe how a state of a given system changes as a function of time. A law differs from a high-level mechanism in two ways: First, whereas the essential characteristic of a high-level mechanism is multiple realizability, a law (as here understood) need not be multiply realizable; indeed the dynamics of a given physical system is, in some sense, unique to that system. Moreover, and perhaps more importantly, a high-level mechanism, as understood by its advocates, is not a constraint on the time-evolution of a system (i.e., on a system's dynamics); rather, it is a kinematical constraint that delimits the possible states a system may possess (a dynamical constraint, on the other hand, delimits the possible *dynamical* states a system may possess). A kinematical constraint is specified independently of the specification of dynamics. For instance, according to Morrison (2012, 149), the high-level mechanism of spontaneous symmetry breaking (SSB) "…functions as a structural constraint on many different kinds of systems in both high-energy physics and condensed matter physics". SSB, as a structural constraint, is purely kinematical: it is an indication that the kinematical structure of the algebra of observables associated with a physical system admits unitarily inequivalent representations of the canonical commutation relations (equivalently, SSB is an indication that a symmetry possessed by the system cannot be unitarily implemented).[4]

Two other examples of high-level mechanisms that are relevant to this essay are localization and long-range entanglement. Localization, as a microphysical mechanism, involves interactions in a conductor between electrons and impurities which result in the electrons becoming bound to the impurity sites. Localization, as a high-level mechanism, involves a scaling theory, according to which the binding of conduction electrons by impurities is a universal property that depends only on the global symmetry of the system, and not on its microphysical makeup (see, e.g., Yoshioka, 2002, 31–2). As a high-level mechanism, localization is arguably a kinematical constraint on the type of allowable states that a physical system admits, insofar as it is independent of the dynamics of the system (i.e., the particular ways in which conduction electrons interact with impurities). Finally, to say a system admits a description in terms of a (long-range) entangled state is to say something about its kinematics, not its dynamics; it is to say something about the possible states the system can be in, independently of a specification of the system's dynamics.

Bain (2013) claims that the law-centric view is exemplified by effective field theories (EFTs). In such theories, one can distinguish between a high-energy regime, characterized schematically[5] by a Lagrangian density $\mathcal{L}[\phi]$ with equations of motion

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial(\partial \phi)} = 0, \tag{1}$$

and a low-energy regime, characterized by an effective Lagrangian density $\mathcal{L}_{eff}[\theta] \neq \mathcal{L}[\phi]$, with equations of motion

$$\frac{\partial \mathcal{L}_{eff}}{\partial \theta} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}_{eff}}{\partial(\partial \theta)} = 0. \tag{2}$$

To the extent that the behavior of the low-energy regime is governed by laws (2) and dynamical variables θ that are formally distinct from the laws (1) and dynamical variables ϕ that govern the high-energy regime, the behavior of the former is dynamically independent of, and dynamically robust with respect to, the behavior of the latter. Hence the low-energy regime is characterized by novelty. The low-energy regime is also characterized by microphysicalism insofar as the low-energy variables, while distinct from the high-energy variables, are still, in an appropriate sense, the low-energy degrees of freedom of the high-energy theory (Bain, 2013, 262–3).

This law-centric view is intended to be neutral with respect to any particular account of laws. The role of laws is to underwrite novelty; i.e., a distinction in laws is supposed to be evidence for dynamical independence and robustness. It should not matter whether this is based on a distinction between different types of regularities, say, or between different types of nomic structure. In the relevant cases in physics, this distinction can be made in terms of (non-trivially) formally distinct Lagrangian densities.

[3] A set of (top) properties T supervenes on a set of (base) properties B just when any two objects that agree on their B-properties also agree on their T-properties; conversely, just when any two objects that disagree on their T-properties also disagree on their B-properties. Whether or not the properties of a system in an entangled state fail to supervene on base properties will depend on how the base properties are identified. For instance, a failure of supervenience will occur when the base properties are identified as properties of the subsystems of a decomposition of the system with respect to which a Bell inequality can be constructed.

[4] See, e.g., Earman (2003). If the unitarily inequivalent representations take the form of Fock space representations, then there is a degeneracy of the vacuum state, one per representation. If these representations are taken to act on a single Hilbert space, then the latter is divided into dynamically isolated superselection sectors. Again, the point is that SSB is a kinematical constraint that is independent of the specification of a system's dynamics.

[5] In general a Lagrangian density $\mathcal{L}\left[\phi_i, \phi_i^n\right]$, $i = 1 \dots N$, is a functional of N field variables ϕ_i and their first and possibly higher-order derivatives $\phi_i^n \equiv \partial^n \phi_i / \partial x^n$. The corresponding Euler–Lagrange equations of motion are given by $\partial \mathcal{L}/\partial \phi_i + (-1)^n (\partial^n/\partial x^n)(\partial \mathcal{L}/\partial \phi_i^n) = 0$.

What this view does assume, however, is that a non-trivial formal distinction in Lagrangian densities entails an ontological distinction in the physical systems they represent. This is at odds with authors who claim that the construction of an EFT typically involves eliminating predictively and/or explanatorily irrelevant degrees of freedom, and the reasons for doing this need not reflect ontological commitments; rather, they may reflect epistemic (lack of knowledge) or pragmatic (ease of calculations) concerns. Thus a notion of emergence that takes its inspiration from EFTs risks conflating ontological emergence (a relation that holds between physical systems) with epistemological emergence (in this context, perhaps, a relation that holds between theories, instrumentally construed). As Lancaster and Pexton (2015, 10) note, "…throwing away explanatorily irrelevant details is not enough for ontological emergence". (This concern assumedly is absent from a mechanism-centric view in which the novelty of an emergent phenomenon is tied explicitly to an ontological mechanism.) Again, this concern is over how EFTs should be interpreted; in particular, whether we are justified in literally interpreting an EFT as describing a physical system that is ontologically distinct from the physical system described by the corresponding high-energy theory (when it exists). The law-centric view assumes we are so-justified. In particular, when the formal distinction between an effective Lagrangian density and that for an associated high-energy theory is such that (a) it supports the ascription of dynamical independence and dynamical robustness to the phenomena of the EFT, with respect to the phenomena of the high-energy theory, and (b) the dynamical variables of the effective Lagrangian density are formally distinct from those of the high-energy Lagrangian density, then we are justified in interpreting the phenomena described by the EFT as ontologically distinct from those described by the high-energy theory, *regardless* of what the motivations were for the construction of the EFT in the first place.[6]

In the rest of this essay, I will argue that, whereas the mechanism-centric view of emergence fails to adequately account for the emergent phenomena associated with the fractional quantum Hall effect, the law-centric view does just fine.

3. The quantum Hall effect

The setup for the quantum Hall effect consists of a current in a 2-dim conductor in the presence of a magnetic field perpendicular to the conductor's surface.[7] The magnetic field deflects electrons towards the conductor's edge, resulting in a build-up of charge and a transverse electric field. The classical Hall effect occurs when the force due to this electric field balances the force due to the magnetic field, with the result that the electrons are no longer deflected. When this occurs, the transverse, or Hall, resistance R_H is linearly related to the magnetic field B by

$$R_H = B/eN \tag{3}$$

where N is the number of electrons per unit area. The quantum Hall effect occurs at low temperature ($\sim 0.02\ K$) and strong magnetic field ($\sim 30\ T$), for which the linear plot of Hall resistance to magnetic field displays plateaus at values of R_H given by

$$R_H = (h/e^2)(1/n) \tag{4}$$

where h is Planck's constant and n is either an integer or a fraction. Each plateau is characterized by a constant value of R_H over a finite range of values of B. Moreover, at these plateaus, the longitudinal resistance R (in the direction of the current) is observed to vanish. Thus the quantum Hall effect consists of two observations:

(I) The Hall resistance R_H exhibits plateaus at values given by Eq. (4).
(II) The longitudinal resistance R vanishes for values of R_H given by Eq. (4).

The explanation of (II) for the integer quantum Hall effect (IQHE), for which n is an integer, involves the fact that a 2-dim system of an electron coupled to a magnetic field is characterized by discretely spaced energy levels (called "Landau levels"), each with a degeneracy of $D = B(e/h)$ states per unit area.[8] From the degeneracy D, one can define the "filling factor" $\nu \equiv N/D = Nh/eB$, which gives the number of completely filled Landau levels.[9] One then argues that, when $\nu = n =$ integer, the lowest n Landau levels are completely filled, and hence, due to the gaps between Landau levels, the system becomes *incompressible*; i.e., there are no accessible energy states for electrons to scatter into. Thus conduction electrons cannot dissipate energy, and the longitudinal resistance subsequently vanishes. Moreover, at these integer values of ν, the magnetic field takes special values $B_n = Nh/en$, at which R_H takes the values in Eq. (4).

The explanation of observation (I) requires an additional hypothesis; namely, the existence of impurities in the conductor. One first notes that slight changes in B will result in slight changes in D, and this should result in changes in R_H and R due to electrons scattering into now available energy states. However, the existence of impurities serves to trap such electrons in localized states, thus preventing them from contributing to the current. Incompressibility thus persists until the number of available energy states exceeds the number of impurity sites.

These explanations provide answers to two distinct why questions:

A. Why is the system incompressible at integer values of ν?
B. Why does incompressibility persist in the system for small changes in the special values B_n of the magnetic field?

The answer to question A involves an appeal to the Hamiltonian that describes an electron coupled to a magnetic field. As Ezawa (2008, 169) notes, this explains incompressibility in the IQHE by describing it as a "one-body" effect of electrons coupled to a magnetic field ("one-body" as opposed to "many-body" insofar as it ignores electron–electron interactions). This suggests that incompressibility is not a novel property of an IQHE state, insofar as it can be derived from the Hamiltonian that describes the fundamental system; in the jargon of Section 2, it fails to satisfy *dynamical independence*. Thus, under Section 2's concept of emergence, incompressibility in the IQHE is not an emergent property. On the other hand, some authors have suggested that the novelty associated with the quantum Hall effect in general is to be found in the impurity hypothesis that explains observation (I). For instance,

[6] According to Bain (2013, 262), an example of ontologically distinct phenomena that differ only in their dynamics is a physical system described by a non-relativistic scalar field and a physical system described by a relativistic scalar field of the same rest mass: whereas the former satisfies the Schrödinger equation, the latter satisfies the Klein–Gordon equation. An example of ontologically distinct phenomena that differ both in their dynamics and the variables that appear in the Lagrangian densities that describe them is a physical system described by a scalar field $\phi(x)$ solution to the Klein–Gordon equation, and a physical system described by a tensor field $F_{\mu\nu}(x)$ solution to Maxwell's equations.

[7] The following is based on the discussion in Eisenstein and Stormer (1990).

[8] See, e.g., Yoshioka (2002, 22–5), Ezawa (2008, 167). In the following I will assume the electrons are spinless to simply the discussion. Spin degrees of freedom result in additional splitting of the Landau levels.

[9] The degeneracy of a Landau level can be written as $D = B/\phi_0$, where $\phi_0 \equiv h/e$ is a quantum of magnetic flux. Hence D is also a measure of the density of flux quanta with respect to the magnetic field B, thus the filling factor also gives the ratio of electrons to flux quanta.

Laughlin and Pines (2000, 28) argue that "[t]he quantum Hall effect is exact because of localization" (i.e., the effect of impurities) and this cannot be "deduced from microscopics". This suggests to Morrison (2012, 149) and Mainwood (2006 93) that localization provides the mechanism (general physical process) that underwrites emergence in the quantum Hall effect.[10]

In the FQHE, we shall find the same answer to question B as the one given for the IQHE; namely, particle–impurity interactions account for the persistence of incompressibility for slight changes in the magnetic field (the only difference will that, in the FQHE explanation, the "particles" are not electrons). On the other hand, the answer to question A for the FQHE (with "integer" replaced by "fractional") will be fundamentally different from the one given for the IQHE. In fact, as we will see below, most authors view the answer to question A for the FQHE as providing the justification for describing the FQHE as an example of emergence.

4. The underdetermination of mechanism in the FQHE

The FQHE occurs at values of n in Eq. (4) given by fractions. If, as in the IQHE, we identify the filling factor ν with such values, this entails that the highest occupied Landau level is only partially filled. Hence, if, as in the IQHE, we are going to appeal to incompressibility in the explanations of observations (I) and (II) in Section 3, we will need a new account of how it arises. I will initially restrict attention to odd denominator filling factors of the form $\nu=1/(2p+1)$, where p is an integer (other fractional values will be considered in Section 4.5). In these cases, $1/(2p+1)$ of the states in the highest occupied Landau level are occupied, while $2p/(2p+1)$ are unoccupied. This should allow conduction electrons to dissipate energy by moving to unoccupied states: What prevents this from happening? There are four alternative explanations: the Laughlin ground state account, the composite fermion account, the composite boson account, and the topological order account. The goal of this section is to expound enough of these accounts to be able to answer the following questions:

(i) Do these accounts exhibit ontologically distinct mechanisms?
(ii) Do these accounts exhibit emergence?
Section 5 takes on the related question,
(iii) Is the type of emergence exhibited by these accounts best described as mechanism-centric or law-centric?

In anticipation of the subsequent discussion, I will argue that the answer to questions (i) and (ii) is "yes", and the answer to question (iii) is "law-centric".

4.1. The Laughlin ground state

The Laughlin ground state explanation of observations (I) and (II) of Section 3 for $\nu=1/(2p+1)$ rests on the claim that the ground state of a 2-dimensional *many-electron* system in which the electrons couple to a magnetic field and, in addition, interact with each other via a Coulomb potential is a highly-correlated liquid state at $\nu=1/(2p+1)$ that exhibits gapped excitations (and hence is incompressible).[11] Laughlin (1983) derived these results from an ansatz for the ground state given by

$$\psi_m(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \prod_{i>j} (z_i - z_j)^m e^{-\sum_i |z_i|^2/4} \quad (5)$$

where m is an odd integer which can be shown to be equal to $1/\nu$ in the limit of large electron density (and z_i is the complex coordinate of the ith electron in the plane).[12] For the sake of comparison with other accounts, (5) may be considered an informed guess for the ground state of a system described by the Lagrangian density[13]:

$$\mathcal{L} = \psi^\dagger iD_0\psi + \frac{1}{2m}\psi^\dagger D_i^2\psi + V(\psi^\dagger, \psi) \quad (6)$$

where ψ is a second-quantized electron field, the covariant derivative $D_\mu=\partial_\mu+ieA_\mu$ couples the electrons to the external magnetic potential A_μ, and the last term encodes the Coulomb interaction between electrons.

Laughlin's assumption of electron–electron interactions is significant: It suggests that, for values of the filling factor given by $\nu=1/(2p+1)$, the highest occupied Landau level is partially filled, but electron–electron interactions prevent electrons from moving to unoccupied states, thus establishing incompressibility. This suffices to explain observation (II).

To explain observation (I) the hypothesis of impurity interactions is again appealed to, but in this case the role of the impurities is slightly different. The electrons in the highest occupied Landau level are now trapped by electron–electron Coulomb interactions, which prevents them from scattering into unoccupied states, but for slight changes in ν (and hence in B), gapped excitations are produced. These take the form of fractionally charged quasiparticles, which can potentially contribute to the current. Impurities in the conductor now serve to trap these quasiparticles in localized states. Thus incompressibility is maintained, until the number of available energy states for quasiparticles exceeds the number of impurity sites.

Recall that incompressibility in the IQHE is a one-body effect, insofar as it is derivable from a Hamiltonian for a single electron coupled to a magnetic field. In the Laughlin account of the FQHE, incompressibility is a *many-body* effect, insofar as it follows from an analysis of a many-body Hamiltonian with an electron–electron Coulomb interaction term. Moreover, to the extent that the Laughlin ground state describes a highly-correlated liquid state (a superfluid) distinct from the Fermi liquid state of electrons in a normal conductor, the transition to the FQHE can be characterized as a phase transition. Many authors take this as providing justification for viewing the FQHE as an emergent phenomenon. The high-level mechanism responsible for this phase transition might be identified with the general physical process of localization, which instantiates itself in this particular context in the form of quasiparticle–impurity interactions.

4.2. Composite fermions

The composite fermion explanation of observations (I) and (II) for $\nu=1/(2p+1)$ rests on the following result[14]:

> The $\nu = 1/(2p+1)$ FQHE for electrons is equivalent to an IQHE for composite fermions with effective filling factor $\nu_{\text{eff}}=1$, where a composite fermion is an electron with an even number, $2p$, of magnetic fluxes attached to it.

Flux attachment is formally established by coupling the electrons to a Chern–Simons (CS) gauge field. An appropriate Lagrangian density is given by[15]:

[10] None of these authors distinguishes the IQHE from the FQHE.

[11] This account originated in Laughlin (1983). For discussion see Yoshioka (2002, 65–70), Fradkin (2013, 480–91).

[12] These results can also be derived by numerical approximation techniques (Yoshioka, Halperin, & Lee, 1983).

[13] See, e.g., Zee (2010, 324; 1995, 113), Zhang (1992, 31), Fradkin (2013, 502).

[14] This account originated in Jain (1989). For discussions see Yoshioka (2002, 112–5), Ezawa (2008,227–8), Fradkin (2013, 512–35).

[15] Zee (1995, 113), Fradkin (2013, 513). This is the Chern–Simons–Landau–

$$\mathcal{L}_{CF} = \psi'^{\dagger} iD_0 \psi' + \frac{1}{2m} \psi'^{\dagger} D_i^2 \psi' + \frac{1}{4\pi(2p)} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + V(\psi'^{\dagger}, \psi') \tag{7}$$

which is identical to the Lagrangian density (6), except for the third term involving the CS potential a_μ (which encodes the attachment of an even number $2p$ of CS fluxes to each fermion field ψ'), and the definition of the covariant derivative, which now includes couplings to both the external magnetic potential and the CS potential: $D_\mu = \partial_\mu + ie(A_\mu + a_\mu)$. One can show that a composite particle (i.e., an electron with fluxes attached to it) experiences an effective magnetic field given by

$$B^{eff} \equiv B - (\textit{CS flux contribution}) \tag{8}$$

where $B = N\phi_0(2p+1)$, in which $\phi_0 \equiv h/e$ is the flux quantum,[16] and the CS flux contribution is given by $N\phi_0$ times the number of fluxes per particle. When the particles are composite fermions, this becomes

$$B^{eff} = N\phi_0(2p+1) - N\phi_0(2p) = B/(2p+1). \tag{9}$$

Thus $\nu_{eff} \equiv N\phi_0/B^{eff} = (2p+1)\,\nu = 1$. In words: a system of electrons coupled to a magnetic field for which the highest occupied Landau level is $1/(2p+1)$ filled, is equivalent to a system of composite fermions for which the highest occupied Landau level is completely filled. This serves to explain observation (II). Observation (I) is explained by appeal to particle–impurity interactions, but this time the particles are composite fermions.

In this account, incompressibility is due to a *one-body* effect of composite fermions, in analogy with the IQHE. If this analogy is taken seriously, it suggests that whether or not we ascribe emergence to the composite fermion account of the FQHE will depend on whether or not we ascribe emergence to the IQHE. If we are willing to do the latter, then we might appeal to the high-level mechanism of localization to underwrite emergence in both the IQHE and the composite fermion account of the FQHE. However, if we have reason to believe that emergence is present in the FQHE but not in the IQHE (since the former, but not the latter, exhibits a phase change, say), then there should be an associated mechanism that is present in the former but not in the latter. The high-level mechanism of localization will not do, since it is present in both cases. On the other hand, flux attachment is present in the composite fermion account, but not in the IQHE. Thus, one option for a mechanism-centric advocate is to identify the low-level mechanism of flux attachment as underwriting emergence in the composite fermion account.[17]

4.3. Composite bosons

The composite boson explanation of observations (I) and (II) for $\nu = 1/(2p+1)$ rests on the following result[18]:

> The $\nu = 1/(2p+1)$ FQHE for electrons is equivalent to a Bose–Einstein condensate of composite bosons, where a composite boson is an electron with an odd number $(2p+1)$ of magnetic fluxes attached to it.

Flux attachment is again formally established by coupling the electrons to a Chern–Simons (CS) gauge field. An appropriate Lagrangian density is given by[19]:

$$\mathcal{L}_{CB} = \phi^{\dagger} iD_0 \phi + \frac{1}{2m} \phi^{\dagger} D_i^2 \phi + \frac{1}{4\pi(2p+1)} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + V(\phi^{\dagger}, \phi) \tag{10}$$

which is identical to the composite fermion Lagrangian density (7) except for the appearance of a second-quantized boson field ϕ and the strength of the CS coupling (which, in this case, can be interpreted as attaching an odd number $(2p+1)$ of CS fluxes to the bosons). Relation (8) then entails

$$B^{eff} = N\phi_0 2(2p+1) - N\phi_0 2(2p+1) = 0. \tag{11}$$

Thus the composite bosons experience no effective magnetic field, and at low temperatures, they will condense to form a Bose–Einstein condensate (BEC). The constituents of this BEC are charged bosons, hence it exhibits the properties of a charged superfluid; namely, dissipationless transport and the Meissner effect. The latter is the expulsion of a magnetic field from the bulk of a superconductor. If we identify the so-expelled magnetic field with the CS field, this entails that the composite boson density is constant (being determined by the CS field), and hence the bulk superfluid is incompressible.

Another property of a superconductor is the appearance of pinned charged vortices near its surface in the presence of a magnetic field. Being charged, these vortices should contribute to the current flow and hence allow energy dissipation. Thus, in order for a persistent current to be maintained, impurities must be present that trap vortices. This pinning of vortices then corresponds to the localization of quasiparticles that explains the plateaus in the FQHE.

In this account, incompressibility is due to a many-body effect of composite bosons whereby the latter condense to form a BEC. Typical accounts of BEC formation appeal to the high-level mechanism of spontaneous symmetry breaking (SSB). This suggests that a high-level mechanism-centric view of emergence could identify SSB as underwriting emergence in the composite boson account.

Note that in the Laughlin ground state account, it is assumed that the initial conductor is comprised of strongly interacting electrons, and that this initial system has the same symmetry as the FQHE system; and moreover, technically, the transition between the two systems is characterized as a quantum phase transition (a phase transition in which quantum fluctuations dominate thermal fluctuations). In the composite boson account, on the other hand, the initial system is taken to be a weakly-interacting boson gas (in which the particles are composite bosons), and the transition to the FQHE system is represented by Bose–Einstein condensation, which is characterized by a classical (i.e., thermal) phase transition. On the surface, these are ontologically distinct accounts: they are distinct in terms of the entities they posit, and in terms of the way these entities interact, both among themselves and with the external magnetic field. On the other hand, they make exactly the same predictions. As Zhang (1992, 35) notes, the composite boson Lagrangian density (10) is "an exact representation" of the original electron problem, and "…while it is logically independent of Laughlin's wave function approach, it leads to the same phenomenological consequences" (pg. 56).

(*footnote continued*)
Ginsburg formulation of the composite fermion approach developed by Lopez and Fradkin (1991).

[16] This follows from the definition of the filling factor $1/(2p+1) = \nu \equiv Nh/eB$.

[17] If one adopts this point of view, then one might be less willing to view the FQHE as a one-body effect analogous to the IQHE, insofar as flux attachment is absent in the latter (thanks to a reviewer for raising this concern). On the other hand, to the extent that flux attachment can be encoded in a one-body Hamiltonian that describes an electron coupled to a Chern–Simons field in the absence of electron–electron interactions, one might still claim this account is a one-body account.

[18] This account reached fruition in Zhang, Hansson, and Kivelson (1989). For discussions see Zhang (1992), Yoshioka (2002, 102–12), Ezawa (2008,227–8), Fradkin (2013, 503–12).

[19] Zee (1995, 113), Fradkin (2013, 503), Zhang (1992, 35). This is the Chern–Simons–Landau–Ginsburg formulation of the composite boson approach developed by Zhang et al. (1989).

4.4. Topological order and long-range entanglement

The topological order explanation of observations (I) and (II) is motivated by the following claims[20]:

(a) The FQHE involves a phase transition to a gapped quantum liquid state.
(b) Different FQHE ground states have the same symmetry, but may differ on their degeneracy and geometric phase.
(c) FQHE ground states do not exhibit exact off-diagonal long-range order (ODLRO).

Claim (a) is based on the results of Laughlin (1983) and Yoshioka et al. (1983), as discussed in Section 4.1. Claim (b) is based on the work of Wen (1990), which may be consulted for an account of how FQHE states may be characterized in terms of their geometric phases. In claim (c), ODLRO is a measure of the order present in a system according to the Landau–Ginsburg theory of phase transitions. It requires that a system's single-particle density matrix $\rho(\mathbf{r}-\mathbf{r}') \equiv \langle 0|\Psi(\mathbf{r})\Psi^{\dagger}(\mathbf{r}')|0\rangle$, where $|0\rangle$ is the ground state, and $\Psi(\mathbf{r})$ is an order parameter, satisfies

$$\rho(\mathbf{r}-\mathbf{r}') \neq 0, \quad \text{as } |\mathbf{r}-\mathbf{r}'| \to \infty. \tag{12}$$

This is interpreted as signifying the presence of long-range order in the form of correlations between the system's constituents (Yoshioka, 2002, 90). It then transpires that the electron single-particle density matrix for an FQHE ground state does not satisfy (12), but rather exponentially decays to zero at large separation distances.[21]

Wen (2013, 2004) takes claims (a), (b), and (c) to demonstrate that the order exhibited by FQHE states cannot be fully described by the Landau–Ginsburg theory of phase transitions. Wen (1990) adopted the term "topological order" to refer to this non-Landau–Ginsburg type of order. It is topological insofar as its defining characteristics – gapped ground state degeneracy and geometric phase – are topological properties, in the sense of being robust under local permutations of the Hamiltonian. Moreover, the effective field theory that describes the FQHE can be put into the form of a topological quantum field theory (as we shall see in Section 5).

The mechanism associated with transitions between Landau–Ginsburg orders is spontaneous symmetry breaking. Chen, Gu, & Wen, (2010, 4) suggest that the mechanism associated with transitions between topological orders be identified with what they call *long-range entanglement*. They define a long-range entangled (LRE) state to be a state that cannot be transformed into an unentangled (i.e. direct-product) state by a "local unitary evolution", where the latter is a unitary operation generated by the time evolution associated with a local Hamiltonian over a finite time.

Note that any state can be transformed into a direct-product state by means of a local unitary transformation. An LRE state cannot be so-transformed by a particular type of unitary transformation; namely, one that is generated by a Hamiltonian. The motivation for identifying long-range entanglement as the mechanism for transitions between topological orders is based on the claim that two gapped states belong to the same phase if and only if they are related by a local unitary evolution.[22] This entails that *short-range entangled* gapped states (i.e., gapped states that can be disentangled by a local unitary evolution) belong to the same phase, which subsequently suggests that LRE gapped states do not.

In this account, incompressibility is due to a *many-body* long-range entangled effect of electrons, which serves to explain observation (II), and observation (I) is explained by the hypothesis of quasiparticle–impurity interactions. In what sense can emergence be associated with this account? According to Lancaster and Pexton (2015, 1), "…the presence of topological order in the FQHE is indicative of an intrinsic holism to the FQH system", and thus "…the FQHE bears serious consideration as an example of a metaphysically significant, 'strongly' emergent phenomena". The intrinsic holism is underwritten by the mechanism of long-range entanglement, and the type of emergence that Lancaster and Pexton associate with this holism is captured by the failure of mereological supervenience (Lancaster and Pexton 2015, 2, 11). This type of emergence is supposed to characterize the properties of a composite system that are novel with respect to the properties of its parts, when those parts are considered as subsystems of the whole (as opposed to considered in isolation from the whole). Thus incompressibility, presumably, is an emergent property in the FQHE insofar as it is a novel property of an FQHE ground state taken as a whole, with respect to its constituent electrons.

In this account of emergence in the FQHE, long-range entanglement is the high-level mechanism that produces holism and thus emergence, appropriately construed. Here are a few concerns with this account. First, one might initially be concerned with associating entanglement with holism. Earman (2015) argues that the notion of an entangled state is prone to an ambiguity that makes it problematic to interpret entangled states as exhibiting holism. In particular, according to Earman (2015, 305), it is a truism that "…entanglement means entanglement of a state on a system algebra with respect to a decomposition of the system algebra into subsystem algebras", and thus "…the very same system state may be unentangled with respect to one decomposition but entangled with respect to another". Assumedly, a mechanism-centric notion of emergence requires an absolute judgment on whether or not a composite system exhibits emergence: it should not be the case that a composite system exhibits emergence with respect to one decomposition of it into subsystems, but not with respect to another. For Earman, whether a physical system exhibits holism is better addressed by focusing on its algebra of observables than on its state; in particular, holism is better associated with a failure of additivity of a system's algebra of observables.[23] Thus for authors who wish to identify emergence in the FQHE with holism, Earman's advice presumably would be to demonstrate that the associated algebra of observables is not additive.

On the other hand, Lancaster and Pexton (2015, 11–12) are careful to say that the emergence exhibited by the FQHE is not underwritten by *generic* entanglement, but rather *long-range* entanglement (LRE): "We are not claiming that the FQHE is emergent in an interesting sense merely because there is *some*

[20] This account originated in Wen (1990). For discussion, see Wen (2013; 2004, 338–41; 1995), Lancaster and Pexton (2015).

[21] Girvin and MacDonald (1987) identified the FQHE order parameter with a composite boson (an electron with an odd number of attached fluxes) and showed that the corresponding density matrix exhibits *quasi*-ODLRO, in the sense that it algebraically decays for large separation distances. Read (1989) identified the FQHE order parameter with an electron-quasihole composite and showed that the corresponding density matrix exhibits *exact* ODLRO. Some authors take these results to appropriately characterize order in the FQHE (Girvin, 1990, 406; Yoshioka 2002, 91; Shi, 2004, 6816).

[22] Chen et al. (2010, 26–8). A gapped state is a ground state of a local Hamiltonian that exhibits an energy gap. A phase transition in this context involves the following considerations (Chen et al. 2010, 3): Let $H(g)$ be a Hamiltonian with a smooth dependence on some parameter g. This induces a dependence $\langle O\rangle(g)$ on the ground state expectation value of a local operator O. The system described by $H(g)$ is said to undergo a phase transition at $g=g_c$ just when the function $\langle O\rangle(g)$ has a singularity at g_c in the infinite volume limit.

[23] Additivity requires that any local algebra $\mathfrak{R}(O)$ associated with a region O of spacetime be generated by the local algebras associated with any of its open coverings: $\mathfrak{R}(O)=\vee_i\mathfrak{R}(O_i)$, for $O=\bigcup_i O_i$. According to Earman (2015, 335), this is "…a precise way of capturing in algebraic terms the idea that the whole is not greater than the sum of its parts".

entanglement. Rather, it is the *specific type of entanglement* that makes the… emergence of the FQHE non-trivial." Note that there seems to be more to LRE than just a failure of mereological supervenience. An LRE state cannot be "dynamically" disentangled in the sense that it cannot be transformed into a product state by means of a Hamiltonian-induced transformation. If it exhibits a failure of supervenience, this failure is thus dynamically robust: it is preserved under local perturbations of the Hamiltonian. (This is not in general the case for a generically entangled state).

The type of emergence associated with the topological order account of the FQHE might thus be identified with a *dynamically robust* failure of supervenience, as exhibited by states that are long-range entangled with respect to a decomposition into electron single-particle states. Moreover, one might excise reference to supervenience altogether from this account. A many-body system in a long-range entangled state might be thought of as satisfying both the *microphysicalism* condition and the *novelty* condition of the concept of emergence in Section 2. On the one hand, it satisfies *microphysicalism* if we allow that its single-particle subsystems obey a fundamental dynamics (expressed by the Schrödinger equation, say). On the other hand, it is *dynamically independent* of its single-particle subsystems to the extent that its state cannot be transformed into a product state comprised of single-particle states of the latter by means of a transformation induced by their dynamics. And it is dynamically robust with respect to its subsystems insofar as this failure to disentangle is robust under small changes to this dynamics.

But even this modified account needs a bit more defense against Earman's criticism, insofar as it may still be the case that whether a many-body system exhibits long-range entanglement depends on how it is decomposed into subsystems. Shi (2004, 6814–16) for instance shows that a notion of "interaction-induced entanglement", based on the decomposition of a many-particle state in the basis of eigenstates of a corresponding single-particle Hamiltonian yields different results for the FQHE, depending on what single-particle Hamiltonian one chooses: An FQHE state is interaction-induced entangled with respect to the electron single-particle basis, but not with respect to the composite boson single-particle basis. Moreover, Shi agrees with authors who allow that the order exhibited by FQHE states is encoded in (quasi-) ODLRO (see footnote 21). In Shi's analysis, then, the electron basis exhibits entanglement but not ODLRO, whereas the composite boson basis exhibits (quasi-) ODLRO but not entanglement.[24]

Earman (2015, 305) views this problem of alternative decompositions in terms of a debate between realists and pragmatists. In the context of the FQHE, to paraphrase Earman, a realist might argue that *real* long-range entanglement is LRE over subalgebras corresponding to real subsystems, and the latter should be identified as electrons. A pragmatist, on the other hand, might argue that the observables associated with the FQHE, in particular those responsible for the plateaus in the Hall resistance and the vanishing of the longitudinal resistance, can equally well be constructed out of composite particles as out of electrons. Thus, under pragmatist scruples, to the extent to which composite particle decompositions of an FQHE state do not yield long-range entanglement, the role of the latter in articulating an appropriate notion of emergence associated with the FQHE may be questioned.

4.5. Comparison

The four accounts of the FQHE for filling factor $\nu = 1/(2p+1)$ reviewed above make appeals to ontologically distinct mechanisms, both at the microphysical level and at the level of general physical processes (high-level mechanism). To see this, first note that the classical regime (high temperature, weak magnetic field) is ontologically characterized by a one-body system of non-interacting electrons coupled to an external magnetic field. To explain the transition to the FQHE regime (low temperature, strong magnetic field, fractional filling factor), each of the four accounts *redescribes* the ontology of the system in the following ways:

(a) *Laughlin ground state account*. The system is taken to be a *many-body* system of electrons coupled to an external magnetic field and interacting with each other *via* a Coulomb potential.
(b) *Composite boson account*. The system is taken to be a weakly-interacting boson gas comprised of electrons with an odd number of attached Chern–Simons fluxes.
(c) *Composite fermion account*. The system is taken to be a one-body system of non-interacting composite fermions that are coupled to an external magnetic field (weaker than the field in (a)), and that are comprised of electrons with an even number of attached Chern–Simons fluxes.
(d) *Topological order account*. The system is taken to be a many-body system of electrons coupled to an external magnetic field and interacting with each other *via* a Coulomb potential.

The transition to the FQHE regime is then given by four ontologically distinct mechanisms, as summarized in Table 1.

Table 1 describes four mechanistic accounts of the two essential observations associated with the FQHE; namely, the vanishing of the longitudinal resistance ($R=0$), and the plateaus in the Hall resistance R_H. These accounts are *ontologically distinct* at the level of microphysics: In the Laughlin account, the vanishing of R is explained in terms of an ontology of strongly interacting electrons, and the Hall plateaus are explained in terms of quasiparticles interacting with impurities. In the composite fermion account, the vanishing of R is explained in terms of an ontology of non-interacting composite fermions, and the Hall plateaus are explained in terms of composite fermions interacting with impurities. In the composite boson account, the vanishing of R is explained in terms of the formation of a Bose–Einstein condensate of composite bosons *via* spontaneous symmetry breaking (SSB), and the Hall plateaus are explained in terms of vortices interacting with impurities; and in the topological order account, the vanishing of R is explained in terms of an ontology of long-range entangled (LRE) electrons, and the Hall plateaus are explained in terms of quasiparticle excitations interacting with impurities.

These accounts differ in the entities they posit (electrons, composite bosons, composite fermions, quasiparticle excitations, vortices), and in the properties and processes they ascribe to these entities (strong many-body Coulomb interaction, non-interacting one-body IQHE, weakly interacting SSB, LRE). Thus they can be said to posit ontologically distinct microphysical mechanisms. These accounts also differ on high-level mechanisms. Arguably, three high-level mechanisms can be identified: SSB, LRE, and localization. The latter appears in all four explanations of the Hall plateaus (although instantiated in different ways), whereas SSB and LRE occur in the explanations of the vanishing of the longitudinal resistance in the composite boson and topological order accounts, respectively.

An advocate of a low-level mechanism-centric view of emergence in the FQHE is in the position of arguing for one of the four accounts in Table 1. An advocate of a high-level mechanism-centric view of emergence can readily account for the Hall plateaus (*via* the high-level mechanism of localization), but when it comes to the vanishing of the longitudinal resistance, seems to be in the position

[24] Shi's analysis (see also Shi, 2003 and Jaeger and Sarkar, 2003) indicates that the type of long-range order exhibited by highly-correlated many-body condensed matter systems should not necessarily be identified with entanglement, as some philosophers have suggested (e.g., Howard, 2007, 153).

Table 1
Alternative mechanistic accounts of the FQHE.

	Mechanism	
	$R=0$	*Plateaus in R_H*
Laughlin ground state	A many-body Coulomb effect of strongly interacting electrons.	Localization: quasiparticle–impurity interactions.
Composite fermion	A one-body IQHE effect of non-interacting composite fermions.	Localization: composite fermion–impurity interactions.
Composite boson	A many-body effect in which weakly-interacting composite bosons form a Bose–Einstein condensate *via* spontaneous symmetry breaking.	Localization: vortex–impurity interactions.
Topological order	A many-body, long-range entangled effect of electrons.	Localization: quasiparticle–impurity interactions.

of aruging against the first two accounts (which don't seem to provide high-level mechanisms in this context), and in favor of one or the other of the remaining two accounts. Such arguments are made problematic by the fact that, at a purely formal level, the Lagrangian densities (6), (7), and (10) that encode the Laughlin, composite fermion, and composite boson accounts can be transformed into each other (Fradkin, 2013, 502–3, 513). Moreover, as we'll see in Section 5, the topological order account is associated with a low-energy effective Langrangian from which (7) and (10) can be derived (Zee, 1995, 122). This suggests that, formally, these accounts are notational variants of each other.

On the other hand, these accounts provide explanations of observations (I) and (II) of Section 3 for simple odd-denominator fractional filling factors $\nu=1/(2p+1)$ and, experimentally, observations (I) and (II) also hold for more complicated fractional values of ν. Some authors have argued that a choice can be made between the accounts in Table 1 based on how well they treat these more complex values. The rest of this subsection argues that currently there is no consensus on which of these extensions is best suited to this task.

To account for more complicated observed fractions, hierarchy schemes have been developed that complement the composite boson account. In the Haldane–Halperin scheme, deviations from $\nu=1/(2p+1)$ are associated with the creation of quasiparticles, and when the density of these becomes great enough, they form quasiparticle-flux composite bosons (i.e., quasiparticles with attached fluxes that obey Bose–Einstein statistics) and BEC-condense to form a new FQHE state. These daughter states then produce quasiparticles that subsequently BEC-condense at relevant densities to form granddaughter FQHE states, and so forth. One can show that the electron filling factor of this bosonic quasiparticle hierarchy is given by (Yoshioka, 2002, 84),

$$\nu = \cfrac{1}{q + \cfrac{\alpha_1}{2p_1 + \cfrac{\alpha_2}{2p_2 + \dots}}} \tag{13}$$

where each α_i is $+1$ or -1 (depending on whether the quasiparticles are quasiholes or quasielectrons, respectively) or 0, and p_i is an arbitrary integer, where i labels the quasiparticle "generation". Daughter states occur for $\alpha_i=0$, $i \geq 2$, granddaughter states occur for $\alpha_i=0$, $i \geq 3$, etc. On the other hand, this scheme requires large densities of quasiparticles to create daughter states for any given generation, and this suggests that at some point, the stability of daughter states becomes questionable, as well as an essential assumption that quasiparticle states are non-overlapping. This is problematic since stable FQHE states have been observed along the "principle sequences" $\nu=n/(2pn \pm 1)$, for n an arbitrary integer. These electron FQHE states are naturally explained in the composite fermion account insofar as one can show that they correspond to composite fermion filling factor $\nu_{eff}=n$ IQHE states. This might suggest that the composite fermion account is to be preferred. Additional evidence comes in the form of observations of oscillations in the longitudinal resistance in the vicinity of $\nu=1/2$ which are similar to those associated with free electrons (what are called Shubnikov-de Haas oscillations; Ezawa, 2008, 293). These observations can be explained in the composite fermion account by noting that for filling factors $\nu=1/2p$, the effective magnetic field for composite fermions vanishes (substitute $\nu=1/2p$ for the value of B in (8)), and the composite fermions can then be thought of as behaving like free electrons.[25]

These considerations suggest to some authors that the composite fermion account is to be preferred (Jain, 2007). On the other hand, as Ezawa (2008, 296) notes, there are observed FQHE states at fractions $\nu=n/m$ other than the principle sequences. These observations cannot be accounted for in terms of IQHE states of composite fermions, hence the composite fermion account loses its initial appeal. Ezawa has proposed a modified composite fermion account for these observations that includes both composite fermions and composite bosons. Similarly, Fradkin (2013, 520) notes that

> All the [experimentally observed] states can also be described by the 'bosonic' Haldane–Halperin hierarchy. However, empirically the stronger fractional quantum Hall states, defined by the width of the observed plateau in the Hall conductance, are naturally described by the Jain [i.e., principle] sequences. On the other hand, there are several observed fractional quantum Hall states that do not fit in the Jain sequences, such as the state at filling fraction 4/11. Such a state can be described as a fractional quantum Hall state in the bosonic hierarchy or as a 'next generation' Jain state, a fractional quantum Hall state of the quasiparticles (vortices) of the primary Jain sequence. More interesting are the states with even denominators, such as at $\nu=5/2$, which cannot be described by either hierarchy.

Yoshioka (2002) suggests that, even though the composite fermion account is "quite effective" at $\nu=1/2$ (139), "…the two theories for the hierarchy [i.e., the composite boson approach and the composite fermion approach]… are not different theories. They describe the same state from different directions" (115). These considerations suggest that the choice between the composite boson and composite fermion accounts based on extensions to fractions other than $\nu=1/(2p+1)$ is still a matter of debate.

Are there reasons to favor the topological order account, underwritten by long-range entanglement, over the others? Granted, this account is associated with a growing body of research on orders in condensed matter systems that cannot be described by the standard Landau–Ginsburg theory (in addition to intrinsic

[25] Ezawa (2008, 294) also reports that there is evidence for the effective cyclotron motion of composite fermions near $\nu=1/2$, and estimates of their effective mass can be calculated (see, also, Yoshioka, 2002, Chapter 7).

topological orders characterized by long-range entanglement, there are also symmetry-protected topological orders characterized by short-range entanglement; see, e.g., Gu and Wen, 2014). On the other hand, the long-range entangled account assumes a decomposition of the FQHE many-body system into electron single-particle subsystems (i.e., that the constituents of the FQHE system are electrons in long-range entangled states), whereas the observable facts (i.e., observations (I) and (II) of Section 3) underdetermine such a decomposition (i.e., the facts can be accounted for by decompositions into composite fermion or composite boson single-particle subsystems).

It should be pointed out that this is a concern with a mechanism-centric view of topological order, according to which a state exhibits topological order just when it is characterized by long-range entanglement. Under a law-centric view, one might claim that a state exhibits topological order just when it is characterized by a topological quantum field theory. Moreover, as will be noted in Section 5, the topological quantum field theory associated with $\nu = 1/(2p+1)$ FQHE states has a natural extension to other fractional states. Thus, at the end of the day, if one is drawn towards the topological order account of the FQHE, one need not adopt a mechanism-centric interpretation of it, and hence a mechanism-centric view of emergence.

5. The law-centric account of emergence in the FQHE

Section 4 suggests that there is an underdetermination of mechanism in causal-mechanical explanations of the FQHE. This underdetermination holds for notions of mechanism both at the level of microphysics and at the level of general physical processes (i.e., the "higher organizing principles" of Laughlin and Pines, 2000). Examples like this should be of concern to advocates of the mechanism-centric view of emergence described in Section 2. According to that view, an emergent system is characterized, in part, by novelty, and the latter is underwritten by an appeal to a mechanism. This view seems to require realism with respect to mechanisms. The appeal to an underlying mechanism, whether it be a particular microphysical mechanism or a general physical process, is supposed to guarantee that the description of the phenomenon as emergent is ontological and doesn't simply reflect epistemic or methodological aims (i.e., it shouldn't reflect a lack of knowledge, or a pragmatic concern with simplifying calculations). But if we agree that there is a real emergent phenomenon associated with the FQHE, it would then seem odd that it can be understood equally well in terms of ontologically distinct mechanisms.[26]

The appeal to an underlying mechanism is also supposed to make the ascription of emergence to the corresponding phenomena nontrivial. Emergence is not, as Mainwood (2006, 284) declares, "a panacea, to be appealed to whenever we are puzzled by the properties of large systems". To avoid triviality, mechanism-centric advocates "…must produce a detailed physical mechanism for emergence, which rigorously explains the qualitative difference that we see with the microphysical"; otherwise, "…if such a mechanism is missing, an appeal that a suggested property is 'emergent' does nothing more than give it a comforting name". Assumedly, then, a mechanism-centric advocate cannot simply claim that a detailed physical mechanism exists in the context of the FQHE, but plead ignorance as to what it is, since this would risk trivializing the ascription of emergence to the latter. The concern would be that if the mechanism-centric advocate is allowed to adopt an agnostic view of mechanism, nothing would prevent her from claiming that emergence is rampant: in any given circumstance, an agnostic could claim that a mechanism that underwrites emergence is present, but we have yet to correctly identify it.[27]

Note in particular that this underdetermination problem applies to mechanism-centric views of emergence that identify general physical processes as the mechanisms that underwrite novelty (e.g., Morrison, 2012, Laughlin and Pines, 2000). Recall from Section 2 that such views gain their purchase from the multiple realizability of a general physical process in ontologically distinct microphysical systems (in cases where the multiple realizability is underwritten by the presence of universal properties that the microphysical systems have in common). This mechanism-centric view of emergence would presumably have no problem allowing for the alternative microphysical accounts of the FQHE. On the other hand, to the extent that the FQHE admits descriptions in terms of ontologically distinct general physical processes, and does not admit a description in terms of universal properties of the relevant sort, this "high-level" mechanism-centric view fails to account for emergence in the FQHE.

The law-centric view of emergence described in Section 2 does not face this underdetermination problem. According to this view, the novelty that characterizes an emergent phenomenon is underwritten by the distinct laws that govern the phenomenon, compared to those that govern the fundamental system from which it emerges. To the extent that laws are independent of causal-mechanical processes, law-centrism can remain agnostic about any particular causal-mechanical account of how emergence is supposed to occur.

In Section 2, it was argued that law-centric emergence characterizes the low-energy phenomena described by a theory that admits the construction of an EFT. For the FQHE, the relevant theory is that for a many-body system of electrons coupled to a magnetic field and interacting via a Coulomb potential. This theory is encoded in the Lagrangian density (6) of Section 4.1:

$$\mathcal{L}[\psi, A_\mu] = \psi^\dagger iD_0\psi + \frac{1}{2m}\psi^\dagger D_i^2\psi + V(\psi^\dagger, \psi) \tag{14}$$

where the dynamical variables ψ, A_μ encode the degrees of freedom of electrons and magnetic field, respectively. A low-energy EFT for the FQHE is encoded in the effective Lagrangian density

$$\mathcal{L}_{eff}[a_\mu, A_\mu, j^\mu] = -\frac{m}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + j^\mu a_\mu \tag{15}$$

where m is an odd integer, the first term is the CS flux attachment term, the second term encodes the coupling of an electromagnetic potential A_μ to the CS potential, and the third term encodes the coupling of a source j^μ of quasiparticles to the CS potential.[28] One

[26] In other words, a mechanism-centric advocate views the alternative mechanistic accounts of the FQHE as alternative theories (of emergence, say) and adopts a scientific realist attitude towards them: we should take the claims they make about theoretical entities (in this case, theoretical entities involved in mechanisms) literally, and we are warranted in believing these claims. This raises the specter of underdetermination: to the extent that the claims of these theories are incompatible (i.e., to the extent that they tell incompatible causal/mechanical stories about the FQHE), the mechanism-centric advocate cannot both uphold the semantic component of realism (the desire to literally interpret epistemically warranted theories) and the epistemic component (the desire to believe the claims made by epistemically warranted theories).

[27] Pleading ignorance amounts to upholding the semantic component of realism with respect to the competing mechanistic accounts, and relinquishing the epistemic component; i.e., claiming that we should take the mechanistic accounts at their face value, but that we are not warranted in believing any given mechanistic account; rather, we should remain agnostic as to which account is correct. In the scientific realism debate, this is the *anti-realist* position of a constructive empiricist. Granted this is one possible stance to adopt towards the notion of emergence, but it is not the stance of a mechanism-centric advocate described in the text above.

[28] The effective Lagrangian density (14) can be derived from a handful of

can show that (15) reproduces observations (I) and (II) of Section 3 for filling factors $\nu = 1/m$. Moreover, a hierarchical extension of (15) can be constructed to describe FQHE states for more complex fractional filling factors (Wen, 2004, 301). Finally, Zee (1995, 122) indicates that a slight generalization of (15) (under a duality transformation) reproduces the composite boson Lagrangian density (10).

Note that (14) and (15) differ in a substantial way: while (14) describes a non-relativistic quantum field theory, (15) describes a topological quantum field theory. The latter uses the totally anti-symmetric tensor $\epsilon^{\mu\nu\lambda}$ to contract tensorial indices, as opposed to a spacetime metric, which appears implicitly in (14). This entails that (14) is invariant under spacetime isometries (in this case Galilean symmetries), whereas (15) is not. Thus if the laws of a theory are encoded in its dynamical equations of motion, the laws that govern the phenomena described by (14) and (15) differ, and they differ substantially. Moreover, the dynamical variables (a_μ, A_μ, j^μ) that encode the behavior of the low-energy system are formally distinct from the dynamical variables ($\in_\Psi$, A_μ) that encode the behavior of the high-energy system. These considerations suggest that the properties associated with (15) are both *dynamically independent* of, and *dynamically robust* with respect to, those associated with (14).[29]

Thus, under a law-centric view of the account of emergence given in Section 2, the properties of the FQHE system described by the Lagrangian density (15) can be said to emerge from the properties of the electron system described by the Lagrangian density (14). The FQHE system exhibits microphysicalism with respect to the electron system, and the novelty of the former with respect to the latter is underwritten by the distinct laws and dynamical variables that characterize both.

6. Conclusion

This essay has argued that there is an underdetermination of mechanistic accounts of the fractional quantum Hall effect (FQHE), at both the microphysical level and the level of general physical processes. This underdetermination is pernicious for a mechanism-centric view of emergence in the FQHE, since alternative mechanistic accounts of the FQHE appeal to ontologically distinct mechanisms, and there are currently no good reasons to prefer one of these accounts over the others. A law-centric view of emergence avoids this underdetermination by avoiding reference to mechanisms. Under a law-centric view, the novelty exhibited by a fractional quantum Hall liquid, with respect to the fundamental electron system out of which it emerges, is not explained by appealing to a mechanism; but rather by appealing to the distinct laws that govern both systems. This distinction manifests itself in the formally distinct Lagrangian densities that describe these systems. This formal distinction has two aspects. First, it entails a distinction in the equations of motion that the two systems obey. Second, it is a distinction not only in the form of these equations of motion, but also in the dynamical variables that appear in them. These differences suggest that the fractional quantum Hall liquid is both dynamically independent of, and dynamically robust with respect to the fundamental electron system.

(*footnote continued*)
general principles in a "bottom-up" approach (Zee, 2010, 326; Lancaster and Blundell, 2014, 419), or by starting with the "microscopic" theory described by the Lagrangian density (14) and eliminating high-energy degrees of freedom in a "top-down" approach (Zee, 1995, 110; Wen, 2004, 298).

[29] Granted, (15), as an EFT, is obtainable from (14) by identifying and integrating out high-energy degrees of freedom from the latter. Hence there is a sense in which (15) is not entirely independent of (14). One might claim that, formally, (15) is related to (14) *via* a renormalization group flow: (15) is a description of the physical system in the vicinity of a fixed point of this flow, whereas (14) is a description of the system away from this fixed point. On the other hand, if all we were handed were (14) and (15), without prior knowledge that the latter was an EFT of the former, we would think we were dealing with two dynamically distinct physical systems.

References

Bain, J. (2013). Emergence in effective field theories. *European Journal for Philosophy of Science*, *3*, 257–273.
Butterfield, J. (2011). Emergence, reduction and supervenience: A varied landscape. *Foundations of Physics*, *41*, 920–959.
Chen, X., Gu, Z.-C., & Wen, X.-G. (2010). Local unitary transformation, long-range quantum entanglement, wave function renormalization, and topological order. *Physical Review B*, *82*, 155138.
Crowther, K. (2015). Decoupling emergence and reduction in physics. *European Journal for Philosophy of Science*, *5*, 419–445.
Earman, J. (2003). Rough guide to spontaneous symmetry breaking In: K. Brading, & E. Castellani (Eds.), *Symmetries in physics* (pp. 335–346). Cambridge: Cambridge University Press.
Earman, J. (2015). Some puzzles and unresolved issues about quantum entanglement. *Erkenntnis*, *80*, 303–337.
Eisenstein, J., & Stormer, H. (1990). The fractional quantum hall effect. *Science*, *248*, 1510–1516.
Ezawa, Z. (2008). *Quantum hall effects* (2nd Ed.). Singapore: World Scientific.
Fradkin, E. (2013). *Field Theories of Condensed Matter Physics*. Cambridge: Cambridge University Press.
Girvin, S. (1990). Recent developments In: R. Prange, & S. Girvin (Eds.), *The quantum hall effect* (2nd Ed.). Berlin: Springer.
Girvin, S., & MacDonald, A. (1987). Off-diagonal long-range order, oblique confinement, and the fractional quantum hall effect. *Physical Review Letters*, *58*, 1252–1255.
Gu, Z.-C., & Wen, X.-G. (2014). Symmetry-protected topological orders for interacting fermions: fermionic topological nonlinear σ models and a special group supercohomology theory. *Physical Review B*, *90*, 115141.
Howard, D. (2007). Reduction and emergence in the physical sciences: some lessons from the particle physics and condensed matter debate In: N. Murphy, & W. Stoeger (Eds.), *Evolution and emergence* (pp. 141–157). Oxford: Oxford University Press.
Jaeger, G., & Sarkar, S. (2003). Coherence, entanglement, and reductionist explanation in quantum physics In: A. Ashtekar, R. Cohen, D. Howard, J. Renn, S. Sarkar, & A. Shimony (Eds.), *Revisiting the foundations of relativistic physics* (pp. 523–542). Berlin: Kluwer Academic Publishers.
Jain, J. (1989). Composite-fermion approach for the fractional quantum hall effect. *Physical Review Letters*, *63*, 199–202.
Jain, J. (2007). *Composite fermions*. Cambridge: Cambridge University Press.
Lancaster, T., & Blundell, S. (2014). *Quantum field theory for the gifted amateur*. Oxford: Oxford University Press.
Lancaster, T., & Pexton, M. (2015). Reduction and emergence in the fractional quantum hall effect. *Studies in History and Philosophy of Modern Physics*. http://dx.doi.org/10.1016/j.shpsb.2015.10.004.
Laughlin, R. (1983). Anomalous quantum hall effect: An incompressible quantum fluid with fractionally charged excitations. *Physical Review Letters*, *50*, 1395–1398.
Laughlin, R., & Pines, D. (2000). The theory of everything. *Proceedings of the National Academy of Science*, *97*, 28–31.
Lederer, P. (2015). The quantum hall effects: Philosophical approach. *Studies in History and Philosophy of Modern Physics*, *50*, 25–42.
Lopez, A., & Fradkin, E. (1991). Fractional quantum hall effect and Chern–Simons gauge theories. *Physical Review B*, *44*, 5246–5262.
Mainwood, P. (2006). Is more different? Emergent properties in physics, (Ph.D. Dissertation), University of Oxford, Online at PhilSci Archive: ⟨http://philsci-archive.pitt.edu/8339⟩.
Morrison,, M. (2012). Emergent physics and micro-ontology. *Philosophy of Science*, *79*, 141–166.
Read, N. (1989). Order parameter and Ginzburg–Landau theory for the fractional quantum Hall effect. *Physical Review Letters*, *62*, 86–89.
Shech, E. (2015). Two approaches to fractional statistics in the quantum Hall effect: idealizations and the curious case of the anyon. *Foundations of Physics*, *45*, 1063–1100.
Shi, Y. (2003). Quantum disentanglement in long-range orders and spontaneous symmetry breaking. *Physics Letters*, *A309*, 254–261.
Shi, Y. (2004). Quantum entanglement in second-quantized condensed matter systems. *Journal of Physics*, *A37*, 6807–6822.
Weber, E., van Bouwel, J., & de Vreese, L. (2013). *Scientific explanation*. Dordrecht: Springer.
Wen, X.-G. (1990). Topological orders in rigid states. *International Journal of Modern Physics B4* (pp. 239–271), 239–271.
Wen, X.-G. (1995). Topological orders and edge excitations in fractional quantum Hall states. *Advances in Physics*, *44*, 405–473.
Wen, X.-G. (2004). *Quantum field theory of many-body systems*. Oxford: Oxford University Press.

Wen, X.-G. (2013). Topological order: from long-range entangled quantum matter to a unified origin of light and electrons. *ISRN Condensed Matter Physics*, *2013*, 1–20.
Yoshioka, D. (2002). *The quantum hall effect*. Berlin: Springer.
Yoshioka, D., Halperin, B., & Lee, P. (1983). Ground state of two-dimensional electrons in strong magnetic fields and 1/3 quantized Hall effect. *Physical Review Letters*, *50*, 1219–1222.
Zee, A. (1995). Quantum hall fluids In: H. Geyer (Ed.), *Field theory, topology, and condensed matter physics* (pp. 99–153). Berlin: Springer Verlag.
Zee, A. (2010). *Quantum Field Theory in a Nutshell*. Princeton: Princeton University Press.
Zhang, S.-C. (1992). The Chern–Simons–Landau–Ginzburg theory of the fractional quantum hall effect. *International Journal of Modern Physics B* (pp. 25–58), 25–58.
Zhang, S.-C., Hansson, T., & Kivelson, S. (1989). Effective-field-theory model for the fractional quantum hall effect. *Physical Review Letters*, *62*, 82–85.