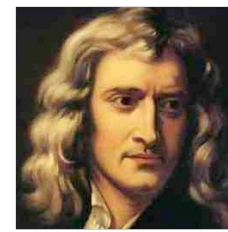


19. Newton

1. Newton and Ancient Wisdom

- (1687) *Mathematical Principles of Natural Philosophy* (*Philosophia Naturalis Principia Mathematica*)

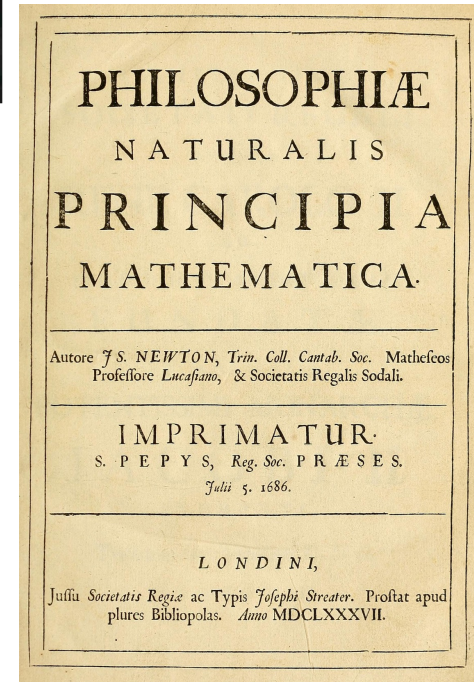


Isaac Newton
(1643-1727)

1. Ancient Wisdom
2. Critique of Descartes
3. Newton's Cosmos

"Classical Scholia"

- Commentary on Propositions IV to IX of Book 3.
 - General claim: *The views expressed in Props IV-IX were anticipated by ancient doctrines.*
- Written in 1690s and intended for 2nd edition of *Principia* (but never included in full).
- Cited by Newton's associates (Fatio de Duillier, David Gregory, Colin MacLaurin) and used in their publications.
- Used by Newton in parts of 2nd edition of *Principia* (*General Scholium*) and in the *Opticks* (1704).



- Contents of *Principia*:
 - Definitions.
 - Axioms, or Laws of Motion.
 - Book 1: The Motion of Bodies.
 - Book 2: The Motion of Bodies (In Resisting Mediums).
 - Book 3. The System of the World (In Mathematical Treatment).

Book 3:

- Prop. I. *That the forces by which the circumjovial planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to Jupiter's center; and are inversely as the squares of the distances of the places of those planets from that center.*
- Prop. II. *That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun; and are inversely as the squares of the distances of the places of those planets from the sun's center.*
- Prop. III. *That the force by which the moon is retained in its orbit tends to the earth; and is inversely as the square of the distance of its place from the earth's center.*

Significance of I-III

- Circumjovial planets, primary planets, and the moon are all retained in their orbits by an inverse-square law force.
 - *Purely mathematical description of phenomena.*

- Prop. IV. *That the moon gravitates towards the earth, and by the force of gravity is continually drawn off from a rectilinear motion, and retained in its orbit.*
- Prop. V. *That the circumjovial planets gravitate towards Jupiter; the circumsaturnal towards Saturn; the circumsolar towards the sun; and by the forces of their gravity are drawn off from rectilinear motions, and retained in curvilinear orbits.*
- Prop. VI. *That all bodies gravitate towards every planet; and that the weights of bodies towards any one planet, at equal distances from the center of the planet, are proportional to the quantities of matter which they severally contain.*
- Prop. VII. *That there is a power of gravity pertaining to all bodies, proportional to the several quantities of matter which they contain.*
- Prop. VIII. *In two spheres gravitating each towards the other, if the matter in places on all sides round about and equidistant from the centres is similar, the weight of either sphere towards the other will be inversely as the square of the distance between their centers.*
- Prop. IX. *That the force of gravity, considered downwards from the surface of the planets, decreases nearly in the proportion of the distances from the center of the planets.*

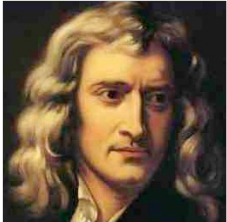
Significance of IV-IX

- Postulate *real* force in nature (gravity).
- Proportional to the quantity of matter in *any* given body (sensible, component, and/or celestial).
- Upshot: Gravity is a *real universal* force.

Claim (McGuire & Ratansi 1966)

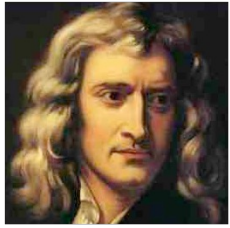
The Classical Scholia to Props. IV-IX were intended by Newton as evidence to establish that there was ancient knowledge of the following theses:

1. Matter is atomic in structure and moves by gravity through a vacuum.
2. Gravity is a universal force.
3. Gravity diminishes in the ratio of the inverse square of the distance between bodies.
4. The cause of gravity is the direct action of God.



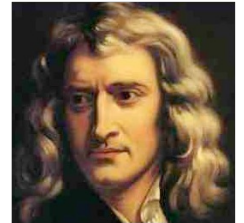
"Even the ancients were aware that all bodies which are round about the Earth, air and fire as well as the rest, have gravity towards the Earth, and that their gravity is proportional to the quantity of matter of which they consist. Lucretius thus argues in proof of the void."

On the atomic theory of matter



"Among the philosophers therefore who have held that bodies are composed of atoms, it was a received opinion that gravity accrues both to atoms and to composite bodies, and that in individual bodies it is proportional to the quantity of matter. That bodies are compound of atoms was the view of both Ionic and Italic philosophers. *The followers of Thales and Pythagoras*, Plutarch observes, *deny that the section of those bodies which are subject to movement proceeds to infinity but ceases at those things which are individual and are called atoms.*"

"That all matter consists of atoms was a very ancient opinion. This was the teaching of the multitude of philosophers who preceded Aristotle, namely Epicurus, Democritus, Ecphantus, Empedocles, Zenocrates, Heraclides, Asclepiades, Diodorus, Metrodorus of Chios, Pythagoras, and previous to these Moschus the Phoenician whom Strabo declares older than the Trojan war. For I think that same opinion obtained in that mystic philosophy which flowed down to the Greeks from Egypt and Phoenicia, since atoms are sometimes found to be designated by the mystics as monads. For the mysteries of numbers equally with the rest of hieroglyphics had regard to the mystical philosophy."

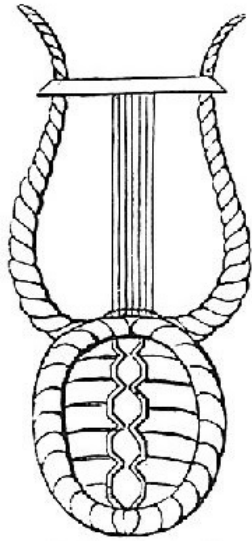


On the inverse square law (Prop. VIII)

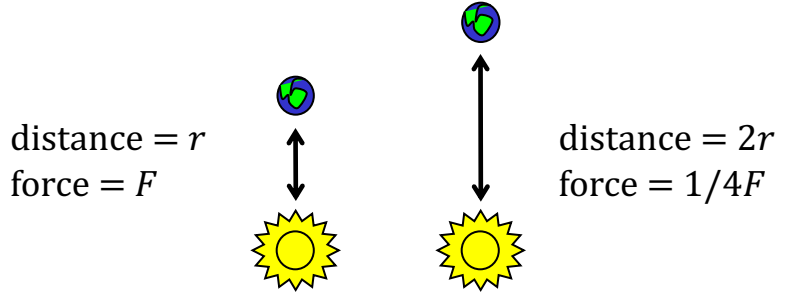
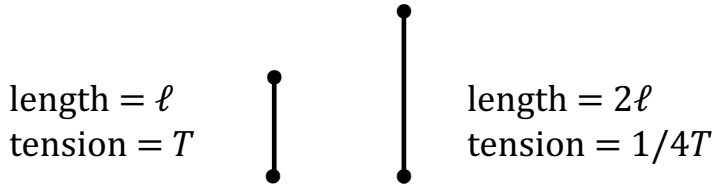
"By what proportion gravity decreases by receding from the Planets the ancients have not sufficiently explained. Yet they appear to have adumbrated it by the harmony of the celestial spheres, designating the Sun and the remaining six planets, Mercury, Venus, Earth, Mars, Jupiter, Saturn, by means of Apollo with the Lyre of seven strings, and measuring the intervals of the spheres by the intervals of the tones."



"But by this symbol they indicated that the Sun by his own force acts upon the planets in that harmonic ratio of distances by which the force of tension acts upon strings of different lengths, that is reciprocally in the duplicate ratio of the distances. For the force by which the same tension acts on the same string of different lengths is reciprocally as the square of the length of the string."



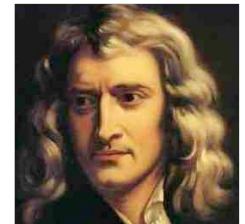
- Apollo's 7-stringed lyre encodes 7 planets.
- Harmonic chords on lyre correspond to inverse square law for planets.



Same note when plucked!
Inverse square law: $T \propto 1/\ell^2$

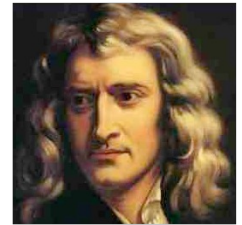
Inverse square law for gravity: $F \propto 1/r^2$

"But the Philosophers loved so to mitigate their mystical discourses that in the presence of the vulgar they foolishly propounded vulgar matters for the sake of ridicule, and hid the truth beneath discourses of this kind. In this sense Pythagoras numbered his musical tones from the Earth, as though from here to the Moon were a tone, and thence to Mercury a semitone, and from thence to the rest of the Planets other musical intervals. But he taught that the sounds were emitted by the motion and attrition of the solid spheres, as though a greater sphere emitted a heavier tone as happens when iron hammers are smitten. And from this, it seems, was born the Ptolemaic system of solid orbs, when meanwhile Pythagoras beneath parables of this sort was hiding his own system and the true harmony of the heavens."

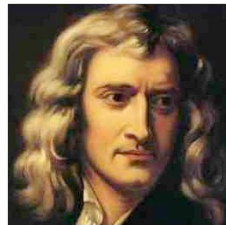


On the cause of gravity

"To some such laws the ancient Philosophers seem to have alluded when they called God Harmony and signified his actuating matter harmonically by the God Pan's playing upon a Pipe and attributing musick to the spheres made the distances and motions of the heavenly bodies to be harmonical, and represented the Planets by the seven strings of Apollo's Harp."



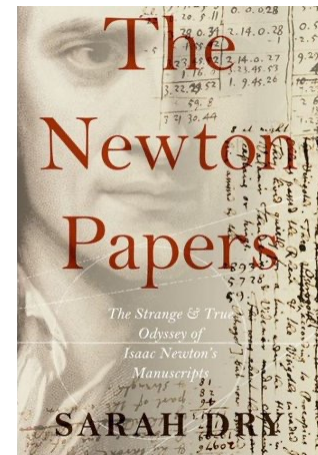
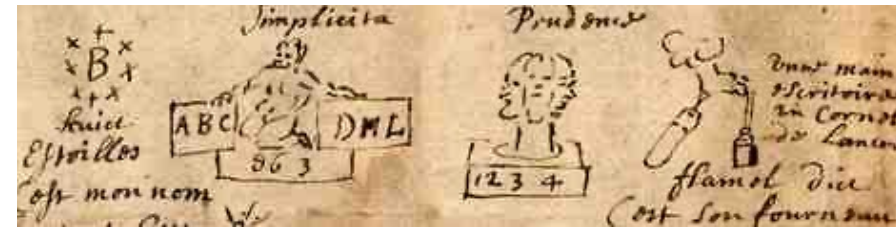
"...those ancients who more rightly held unimpaired the mystical philosophy as Thales and the Stoics, taught that a certain infinite spirit pervades all space into infinity, and contains and vivifies the entire world. And this spirit was their supreme divinity, according to the Poet cited by the Apostle. In him we live and move and have our being."



Three interrelated pursuits of Newton

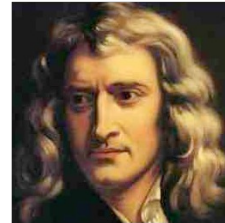
- (1) *Prisca sapientia*: Interpretation of texts of ancient natural philosophy.
- (2) Alchemy.

- Large collection of unpublished notes and manuscripts on alchemy ("chymistry"). (Portsmouth papers.)
- Descriptions of alchemical experiments performed at Cambridge.



Dry, S. (2014)
The Newton Papers

"I made Jupiter fly on his eagle."

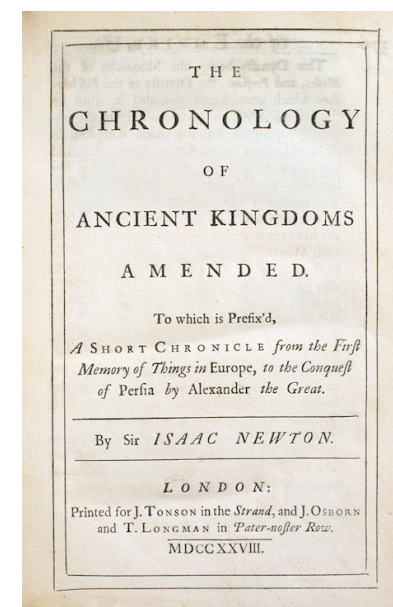
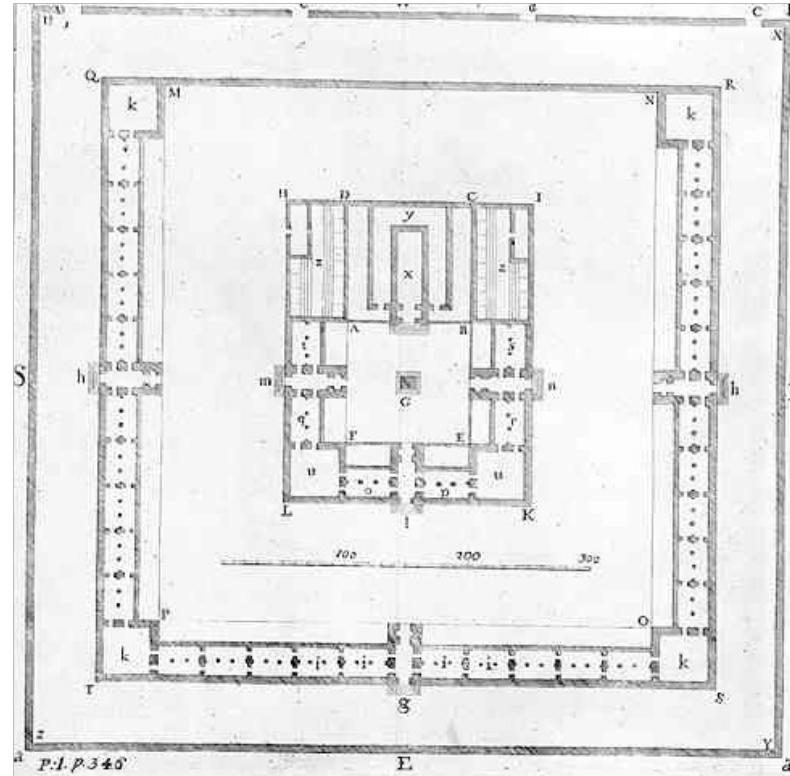


- Westfall (1984): Newton drew the concept of attraction out of the alchemical tradition.
- Newman (preprint): Newton's experiments on the spectral decomposition of sunlight were heavily influenced by his "chymical" pursuits.

(3) Biblical studies: Interpretation of prophetic books of Old Testament.

- (1728) *The Chronology of Ancient Kingdoms*

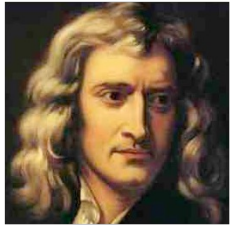
- Chapter V: A study of King Solomon's temple.
- Reflects heliocentric structure of universe (temple flame in center corresponding to sun).



All three pursuits "...rested on the assumption that a true body of knowledge had been available to wise men in the remotest antiquity, and that the knowledge was couched in an enigmatical, symbolical form to conceal it from the vulgar" (McGuire & Rattansi, pg. 136)

2. Newton's Critique of Descartes

- (~1670) *De Gravitatione et Aequipondio Fluidorum*
(*On the Gravity and Equilibrium of Fluids*)



"...[Descartes] says that speaking properly and according to philosophical sense the Earth and the other Planets do not move, and that he who declares it to be moved because of its translation with respect to the fixed stars speaks without reason and only in the vulgar fashion. Yet later he attributes to the Earth and Planets a tendency to recede from the Sun as from a centre about which they are revolved, by which they are balanced at their [due] distances from the Sun by a similar tendency of the gyrating vortex. What then? Is this tendency to be derived from the (according to Descartes) true and philosophical rest of the planets, or rather from [their] common and non-philosophical motion?"

Descartes is committed to three claims:

- The Earth is at rest in its immediate surroundings (it is at "true" rest).
- The Earth exhibits a tendency to recede from the sun (rotational motion).
- The Earth is accelerating (Laws of Nature 1 & 2).

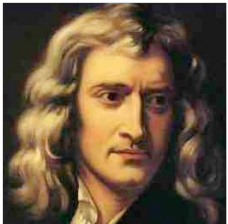
Newton asks: What is the *cause* of the Earth's recession?

(a) "True" (*philosophical*) motion of the Earth?

- *But then Earth is both at rest and accelerating.*

(b) Apparent (*non-philosophical*) motion with respect to some other body?

- *But which one: fixed stars? Saturn? Jupiter? etc.*



"Unless it is conceded that there can be a single physical motion of any body, and that the rest of its changes of relation and position with respect to other bodies are so many external designations, it follows that the Earth (for example) endeavours to recede from the centre of the Sun on account of a motion relative to the fixed stars, and endeavours the less to recede on account of a lesser motion relative to Saturn..., and still less relative to Jupiter..., and also less relative to Mars... Since all of these endeavours and non-endeavours cannot absolutely agree, it is rather to be said that only the motion which causes the Earth to endeavour to recede from the Sun is to be declared the Earth's natural and absolute motion."

- Newton's claim: The cause must be a *single* motion of the Earth; namely, its "absolute" motion. That is, its motion with respect to *absolute space*.

Two Characteristics of Absolute Space

1. *Substantialism*:

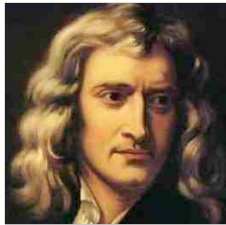
(a) Space is geometrical: consists of Euclidean points, lines, surfaces, *etc.*

(b) Spatial points exist independently of matter:

- matter = substance + properties

- space = something else... (*not* a property, not entirely substantial either)

... a precondition for the existence of matter and God.

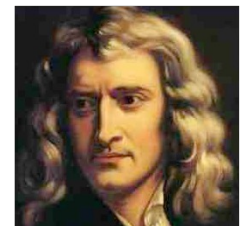


"It is not substance; on the one hand, because it is not absolute in itself, but is as it were an emanent effect of God, or a disposition of all being..."

2. *Immutability*: Space is unaffected by matter.

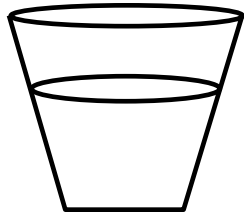
- Provides the single absolute, immovable reference frame with respect to which absolute motion can be defined.

"The positions, distances, and local motions of bodies are to be referred to the parts of space... space is eternal in duration and immutable in nature, and this because it is the emanent effect of an eternal and immutable being.."

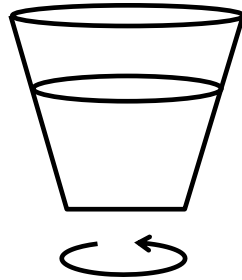


The Bucket Thought Experiment (Scholium to Principia)

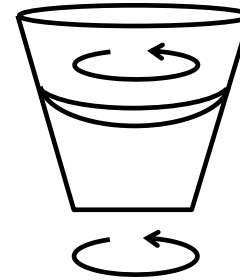
- Claim: Rotational motion is best explained as motion with respect to absolute space, and not as motion with respect to other physical objects.
- Set Up: Consider a water-filled bucket suspended from a rope. Twist up the rope and release the bucket. Observe its motion at various stages:



Stage 1
water at rest
bucket at rest



Stage 2
water at rest
bucket rotating

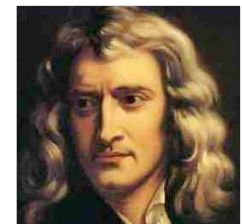


Stage 3
water rotating
bucket rotating at same rate.

*surface of water
is concave*

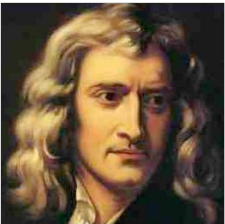
- Question: At what stage is the water in rotation?
- Newton: Stage 3: Presence of force of recession from axis indicates motion with respect to absolute space.

"This ascent of the water shows its endeavor to recede from the axis of its motion; and the true and absolute circular motion of the water... becomes known, and may be measured by this endeavor."



Possible explanations in terms of relative motion

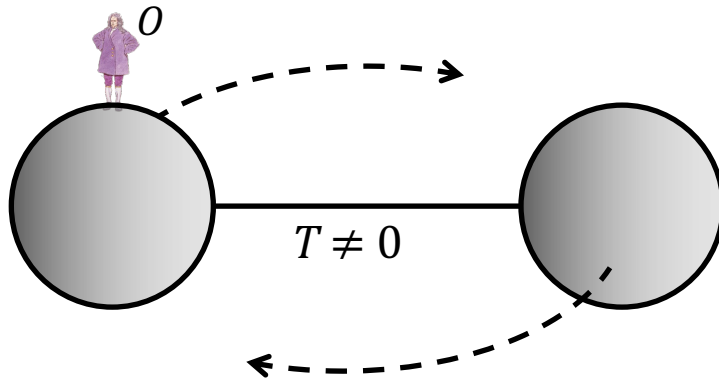
- (a) Suppose "true" motion is motion with respect to physical objects in immediate contact with the moving object and considered to be at rest.
- Then: The water is in "true" rotation in Stage 2, and "true" rest in Stages 1 and 3.
 - But: Stages 1 and 3 are physically distinct.
- (b) Suppose relative motion between water and some physical object other than the bucket explains the force of recession in Stage 3.
- But: There are many possible objects in many relative motions with respect to the water in Stage 3.
 - *Which* relative motion explains the force?



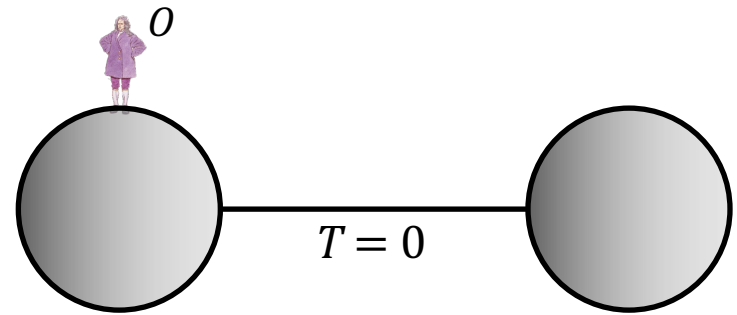
"There is only one real circular motion of any one revolving body, corresponding to only one power of endeavoring to recede from its axis of motion, as its proper and adequate effect; but relative motions, in one and the same body, are innumerable, according to the various relations it bears to external bodies, and, like other relations, are altogether destitute of any real effect, any otherwise than they may perhaps partake of that one only true motion."

The Rotating Globes Thought Experiment (Scholium to Principia)

- Set Up: Two globes connected by a string in an otherwise empty universe.
- Question: Can an observer O determine if the globes are rotating about an axis through the center of the string?



(a) rotating globes

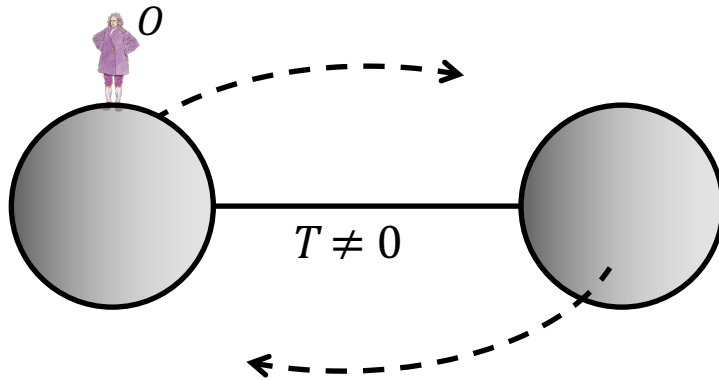


(b) non-rotating globes

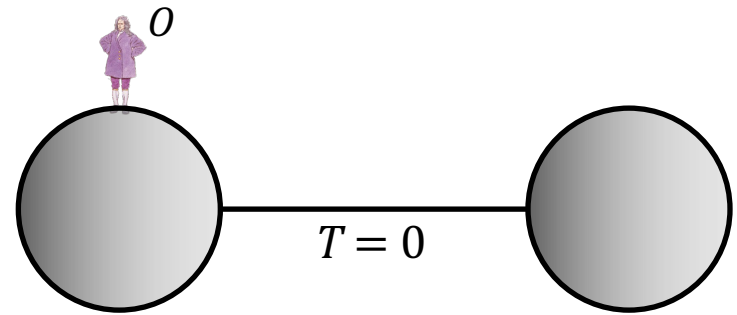
- Newton: Globes are rotating just when the tension in the string is non-zero.
 - And: This tension (inertial force) indicates the globes are rotating with respect to absolute space.

The Rotating Globes Thought Experiment (Scholium to Principia)

- Set Up: Two globes connected by a string in an otherwise empty universe.
- Question: Can an observer O determine if the globes are rotating about an axis through the center of the string?



(a) rotating globes

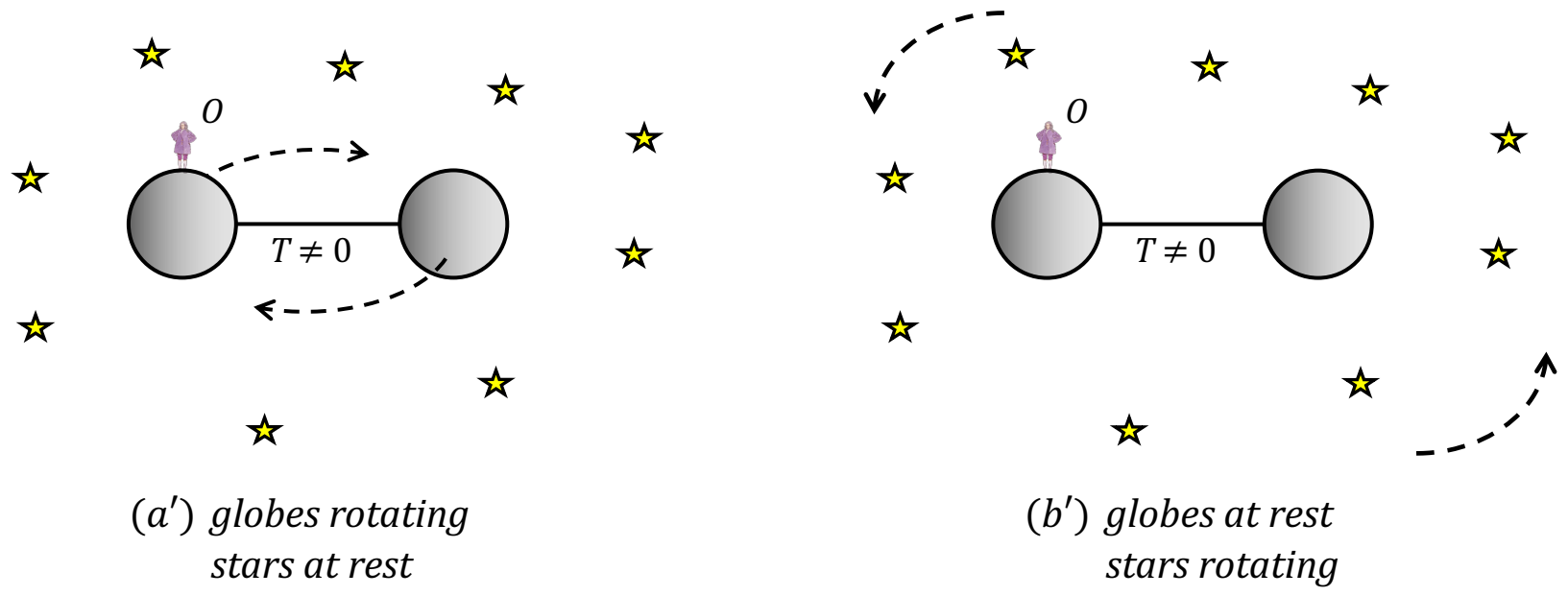


(b) non-rotating globes

- Can a relationalist explain the difference between (a) and (b)?
 - The tension in (a) is assumedly due to a motion of the globes.

But if all motion is relative, with respect to what are the globes moving in (a)?

- Suppose we add the fixed stars to (a).
 - Perhaps a universe that's empty except for 2 globes is unrealistic...



- But: For a relationalist, (a') is physically indistinguishable from (b').
- And: Is it plausible to suggest that the fixed stars, simply by rotating about the globes, cause a tension to appear in the string?

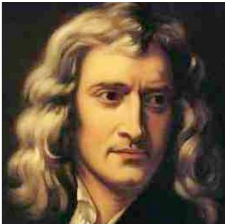
3. Newton's Cosmos (*Principia*)

(a) Characteristics:

- *Space and time are absolute (manifestation of God's omnipotence).*
- *Atomism: matter consists of atoms moving in a void.*
- *Universal action-at-a-distance force (gravity), penetrating matter.*

(b) 3 Laws of Motion

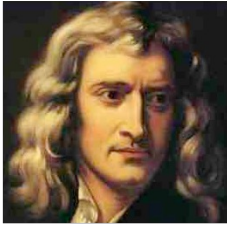
1st Law of Motion



"Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it."

- Describes *natural motion: uniform* motion (rest or constant velocity) in a *straight line*.
- *Encodes the Principle of Inertia* (taken directly from Descartes' Laws 1 & 2).
 - *Inertia* = tendency in an object to obey Law 1.
 - *Inertial motion* = rest or constant velocity in a straight line.

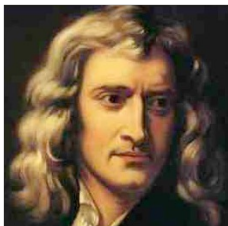
2nd Law of Motion



"The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

- Or: Acceleration (change in quantity of motion) is proportional to force ($F = ma$).
- Describes *Forced motion*.
 - *Accelerated motion* = *non-uniform* motion (non-constant velocity).
 - *Inertial mass* = m = measure of amount of inertia.

3rd Law of Motion



"To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts, and takes place in the direction of the straight line along which the force is impressed."

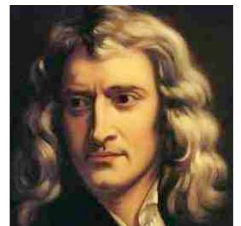
(c) Uniform motion is undetectable:



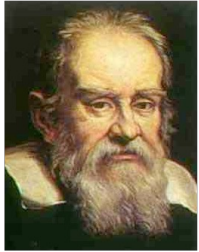
"The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion."

(Corollary V, Axioms, Laws of Motion.)

"For the differences of the motions tending towards the same parts, and the sums of those that tend towards contrary parts, are, at first (by supposition), in both cases the same; and it is from those sums and differences that the collisions and impulses do arise with which the bodies mutually impinge one upon another. Wherefore (by Law 2), the effects of those collisions will be equal in both cases; and therefore the mutual motions of the bodies among themselves in the one case will remain equal to the mutual motions of the bodies among themselves in the other. A clear proof of which we have from the experiment of a ship; where all motions happen after the same manner, whether the ship is at rest, or is carried uniformly forwards in a right line."



Recall Galileo:



"Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals... With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin... [Now] have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still... [T]he ship's motion is common to all the things contained in it, and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted."
(Dialogue Concerning the Two Chief World Systems 1632.)

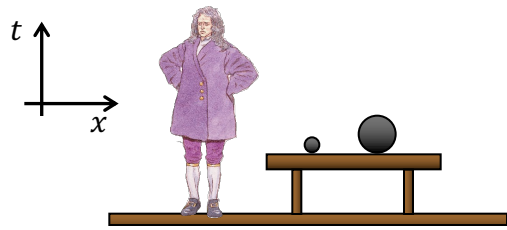
- No way to tell (by experiments with butterflies, *etc.*) from within a closed frame of reference whether a ship is at rest or in uniform motion.

Contemporary understanding

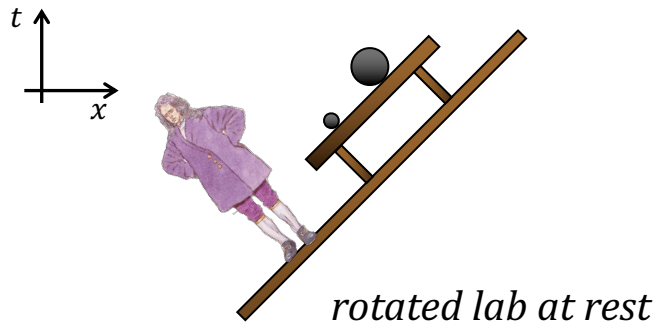
- Newton's laws of motion are the same in all "inertial reference frames" (*i.e.*, objects undergoing inertial motion).

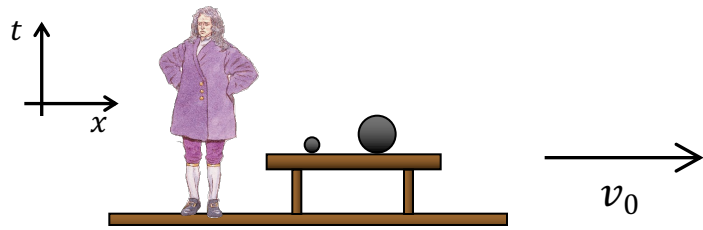
What this means: Any experiment involving moving objects subject to Newton's laws and performed with respect to one inertial reference frame, will produce the same results with respect to any other inertial reference frames: *Inertial reference frames cannot be distinguished by Newton's laws of motion.*

- Question: How are inertial reference frames related to each other?
 - Answer: By the symmetry transformations that leave invariant Newton's laws of motion (in particular, Law 2: $F = ma = md^2x/dt^2$).
 - And: These include rotations, velocity boosts, space translations, and time translations.

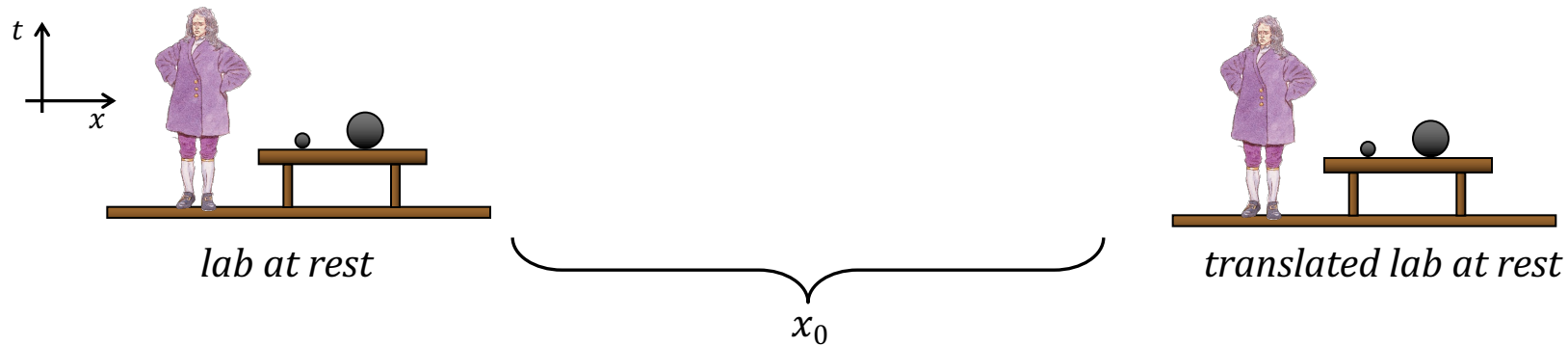


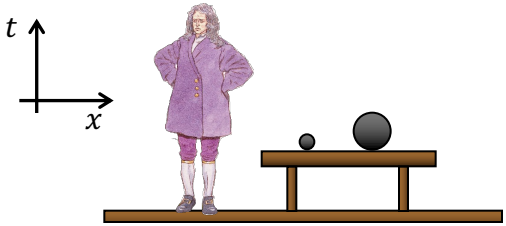
lab at rest



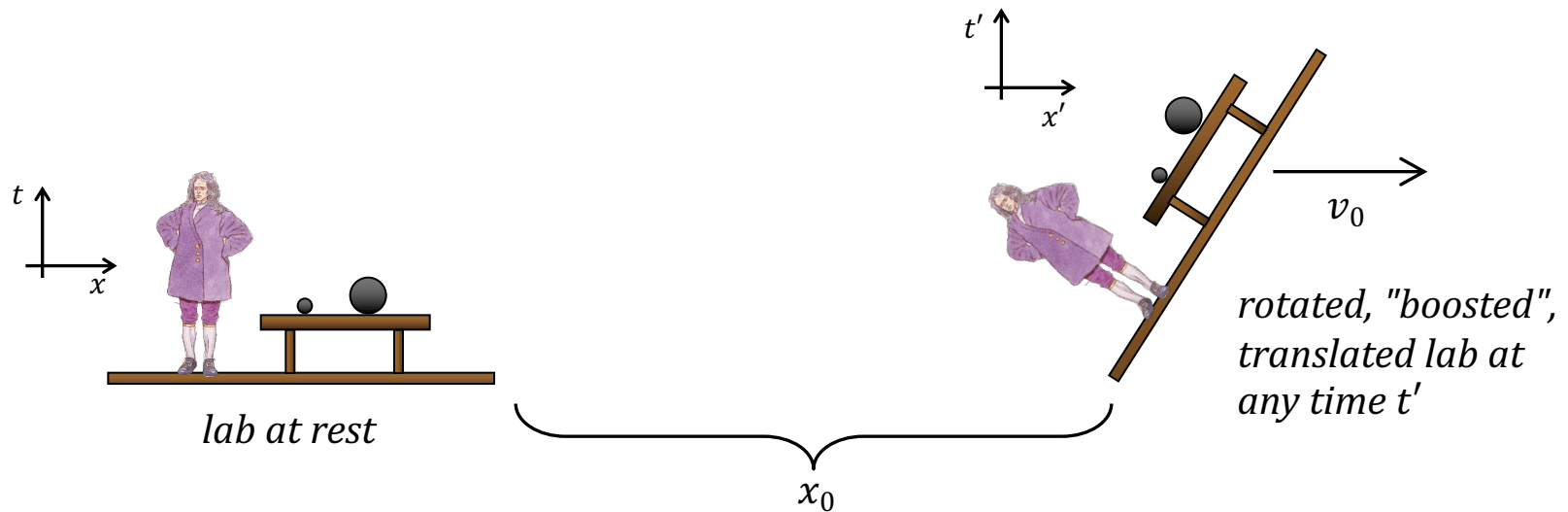


lab in constant velocity





*lab at rest tomorrow
at time t_0*



- Rest lab and rotated, spatially/temporally translated, constantly moving lab are indistinguishable according to Newton's Laws of Motion!

Consequences:

1. Velocity is relative! (No preferred, absolute velocities in nature.)
2. Position is relative! (No absolute positions in nature.)
3. Orientation is relative! (No absolute directions in nature.)
4. Acceleration is absolute! (Any given object has a unique value of acceleration.)

- Problem for Newton (Leibniz-Clarke correspondence 1715-16):
Absolute space entails both position and velocity are absolute!

(d) Universal force of gravity.

Contemporary understanding

- Particular type of forced motion (when force is due to gravity).

$$F = \frac{GM_g m_g}{r^2}$$

F = force of gravity on object with gravitational mass m_g due to object with gravitational mass M_g
 r = distance between m_g and M_g
 G = constant of nature (Newtonian gravitational constant)

- A *universal force*: acts on all objects *in exactly the same way*.
 - Thus: Unites Aristotles' *terrestrial and celestial realms*.

- For object with inertial mass m on Earth's surface in free fall:

$$F = GM_g m_g / r^2 = ma \Rightarrow GM_g / r^2 = a \quad , \text{ provided } m_g = m$$

where M_g is the gravitational mass of the Earth.

- *All objects near the Earth's surface fall with the same acceleration, regardless of their inertial mass m (provided an object's gravitational mass equals its inertial mass).*