Important Definitions and Propositions for Statement Calculus

Def. 2.12. A **valuation** of L is a function $v:$ $\{$*wfs* of $L\} \rightarrow \{T, F\}$ such that, for any *wfs* $\mathcal{A}$, $\mathcal{B}$ of L,
(i) $v(\mathcal{A}) \neq v(\sim\mathcal{A})$
(ii) $v(\mathcal{A} \rightarrow \mathcal{B}) = F$ *iff* $v(\mathcal{A}) = T$ and $v(\mathcal{B}) = F$.

Def. 2.13. A *wf* $\mathcal{A}$ of L is a **tautology of *L*** just when $v(\mathcal{A}) = T$ for all valuations of L.

Def. 2.2. A **proof in *L*** is a finite sequence of *wfs* $\mathcal{A}_1$, .., $\mathcal{A}_n$ of L such that any member is either an axiom of L or follows from previous members by MP.

If Γ is a set of *wfs* of L, a **deduction from Γ in *L*** is a proof in L in which any member of the sequence can also be an element of Γ.

A *wf* $\mathcal{A}$ of L is a **theorem of *L*** if it is the last member of a proof in L.

Notation: $\Gamma \vdash_L \mathcal{A}$ ("$\mathcal{A}$ is deducible from Γ in L" or "Γ syntactically implies $\mathcal{A}$ in L")
$\vdash_L \mathcal{A}$ ("$\mathcal{A}$ is a theorem of L")

Prop. 2.8. (Deduction Theorem for *L*) Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be *wfs* of L and let Γ be a (possibly empty) set of *wfs* of L. If $\Gamma \cup \{\mathcal{A}\} \vdash_L \mathcal{B}$, then $\Gamma \vdash_L (\mathcal{A} \rightarrow \mathcal{B})$.

Prop. 2.14. (The Soundness Theorem for *L*)
For any *wf* $\mathcal{A}$ of L, if $\mathcal{A}$ is a theorem of L, then $\mathcal{A}$ is a tautology of L.

Def. 2.15. An **extension** L^* of L is either L itself or a formal system obtained by adding to or modifying the axioms of L in such a way that each theorem of L is also a theorem of L^*.

Def. 2.16. An extension L^* of L is **consistent** if for no *wf* $\mathcal{A}$ are both $\mathcal{A}$ and $(\sim\mathcal{A})$ theorems of L^*.

Prop. 2.19. If L^* is a consistent extension of L, and $\nvdash_{L^*} \mathcal{A}$ for some *wf* $\mathcal{A}$ of L, then the extension L^{**} formed by adding $(\sim\mathcal{A})$ to the axioms of L^* is consistent.

Def. 2.20. An extension L^* of L is **complete** if for each *wf* $\mathcal{A}$, either $\vdash_L \mathcal{A}$ or $\vdash_L (\sim\mathcal{A})$.

Prop. 2.21. (Lindenbaum's Lemma for *L*) Let L^* be a consistent extension of L. Then there is a *consistent complete* extension of L^*.

Prop. 2.22. Let L^* be a consistent extension of L. then there is a valuation of L in which every theorem of L^* takes the value T.

Prop. 2.23. (The Adequacy Theorem for *L*)
If $\mathcal{A}$ is a tautology of L, then $\mathcal{A}$ is a theorem of L.

Prop. 2.24. L is decidable. There is an effective method for deciding, given any *wf* $\mathcal{A}$ of L, whether it's a theorem of L.