The formal language of statement calculus:

1. **Symbol alphabet:**
   - statement variables: $p_1, p_2, \ldots$
   - punctuation: (,), ,
   - connectives: $\sim, \rightarrow$
2. **Grammar:** A **well-formed formula (*wf*)** of statement calculus**:**
   - (i) p_i is a *wf*, for $i \geq 1$.
   - (ii) If $\mathcal{A}$ and $\mathcal{B}$ are *wf*s, then so are $(\sim\mathcal{A})$ and $(\mathcal{A} \rightarrow \mathcal{B})$.
   - (iii) Nothing else is a *wf*.

The formal system *L* of statement calculus: Consists of the formal language of statement calculus plus:

3. **Axioms:** For any *wf*s $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ of statement calculus,
   - (L1) $(\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{A}))$
   - (L2) $((\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C})) \rightarrow ((\mathcal{A} \rightarrow \mathcal{B}) \rightarrow (\mathcal{A} \rightarrow \mathcal{C})))$
   - (L3) $(((\sim\mathcal{A}) \rightarrow (\sim\mathcal{B})) \rightarrow (\mathcal{B} \rightarrow \mathcal{A}))$
4. **Rule of Deduction:** Modus Ponens (MP). From $\mathcal{A}$ and $(\mathcal{A} \rightarrow \mathcal{B})$, one can derive $\mathcal{B}$, where $\mathcal{A}$, $\mathcal{B}$ are *wf*s of *L*.

The formal language $\mathcal{L}$ of predicate calculus (or the first order language $\mathcal{L}$):

1. **Symbol alphabet:**
   - individual variables: $x_1, x_2, \ldots$
   - individual constants: $a_1, a_2, \ldots$
   - predicate letters: $A_1^1, \ldots, A_i^n$
   - function letters: $f_1^1, \ldots, f_i^n$
   - punctuation: (,), ,
   - connectives: $\sim, \rightarrow$
   - quantifier: $\forall$
2. **Grammar:**
   - (A) A **term** in $\mathcal{L}$:
     - (i) Variables and constants are terms in $\mathcal{L}$.
     - (ii) If f_i^n is a function letter in $\mathcal{L}$ and $t_1, \ldots, t_n$ are terms in $\mathcal{L}$, so is $f_i^n(t_1, \ldots, t_n)$.
     - (iii) Nothing else is a term in $\mathcal{L}$.
   - (B) An **atomic formula** in $\mathcal{L}$ is any expression of the form $A_i^n(t_1, \ldots, t_n)$, where $t_1, \ldots, t_n$ are terms in $\mathcal{L}$.
   - (C) A **well-formed formula (*wf*)** of $\mathcal{L}$:
     - (i) Any atomic formula is a *wf* of $\mathcal{L}$.
     - (ii) If $\mathcal{A}$ and $\mathcal{B}$ are *wf*s of $\mathcal{L}$, so are $(\sim\mathcal{A})$, $(\mathcal{A} \rightarrow \mathcal{B})$, and $(\forall x_i)\mathcal{A}$, where x_i is any variable.
     - (iii) Nothing else is a *wf* of $\mathcal{L}$.

The formal system *K* of predicate calculus: Consists of the formal language $\mathcal{L}$ plus:

3. **Axioms:** For any *wf*s $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{L}$,
   - (K1) $(\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{A}))$
   - (K2) $((\mathcal{A} \rightarrow (\mathcal{B} \rightarrow \mathcal{C})) \rightarrow ((\mathcal{A} \rightarrow \mathcal{B}) \rightarrow (\mathcal{A} \rightarrow \mathcal{C})))$
   - (K3) $(((\sim\mathcal{A}) \rightarrow (\sim\mathcal{B})) \rightarrow (\mathcal{B} \rightarrow \mathcal{A}))$
   - (K4) $(((\forall x_i)\mathcal{A} \rightarrow \mathcal{A})$, if x_i does not occur free in $\mathcal{A}$.
   - (K5) $(((\forall x_i)\mathcal{A}(x_i) \rightarrow \mathcal{A}(t))$, if $\mathcal{A}(x_i)$ is a *wf* of $\mathcal{L}$ and t is a term in $\mathcal{L}$ which is free for x_i in $\mathcal{A}(x_i)$.
   - (K6) $(\forall x_i)(\mathcal{A} \rightarrow \mathcal{B}) \rightarrow (\mathcal{A} \rightarrow (\forall x_i)\mathcal{B})$, if $\mathcal{A}$ contains no free occurrence of the variable x_i.
4. **Rules of Deduction:**

   Modus Ponens (MP): From $\mathcal{A}$ and $(\mathcal{A} \rightarrow \mathcal{B})$, one can derive $\mathcal{B}$, where $\mathcal{A}$, $\mathcal{B}$ are *wf*s of $\mathcal{L}$.

   Generalization (Gen): From $\mathcal{A}$, one can derive $(\forall x_i)\mathcal{A}$, where $\mathcal{A}$ is any *wf* of $\mathcal{L}$ and x_i is any variable.