Chapter 34: Definite Descriptions

A ***definite description*** is a phrase that doesn't contain a name and that aims to designate a *particular thing.*

Ex.	"The Queen of England"	(*vs.* "Elizabeth")
	"The smallest prime number"	(*vs.* "1")
	"The trombone in my basement"	(*vs.* "Grontor")

Attributing properties to a particular thing using definite descriptions

Russell's translation scheme

A sentence of the type "The F is G" is translated into $\mathbf{QL}^{=}$ by the *wff*

(R) $\exists v((Fv \wedge \forall w(Fw \supset w = v)) \wedge Gv)$

Ex1. "The present king of France is bald." True or false?

F ⇒ _____is a present king of France   G ⇒ _____is bald

$$\exists x((Fx \wedge \forall y(Fy \supset y = x)) \wedge Gx)$$

There is a present king of France...

...and there's only one...

...and he's bald.

false!

Ex2. "The book in Dibner is a logic text."

F ⇒ ____is a book in Dibner   G ⇒ ____is a logic text

$$\exists x((Fx \wedge \forall y(Fy \supset y = x)) \wedge Gx)$$

There is a book in Dibner.

...and there's only one...

...and it's a logic text.

false!

Ex3. "The president of the U.S. is a grandparent."

F ⇒ ____is a president of the U.S.   G ⇒ ____is a grandparent

$$\exists x((Fx \wedge \forall y(Fy \supset y = x)) \wedge Gx)$$

There is a president of the U.S.

...and there's only one...

...and he's a grandparent.

false!

More Translations:

b ⇒ Bryn	G ⇒ ____is a girl
m ⇒ Mrs. Jones	L ⇒ ____loves____
F ⇒ ____speaks Welsh	M ⇒ ____is taller than____

Ex1. The Welsh speaker who loves Mrs. Jones is taller than her.

There's a Welsh speaker who loves Mrs. Jones, and there's only one, and that one is taller than her.

There's an x such that $\mathsf{Fx} \wedge \mathsf{Lxm}$, and for all y, $(\mathsf{Fy} \wedge \mathsf{Lym}) \supset \mathsf{y} = \mathsf{x}$, and Mxm.

$\exists \mathsf{x}(((\mathsf{Fx} \wedge \mathsf{Lxm}) \wedge \forall \mathsf{y}((\mathsf{Fy} \wedge \mathsf{Lym}) \supset \mathsf{y} = \mathsf{x})) \wedge \mathsf{Mxm})$

Ex2. Bryn loves the girl who loves him.

There's a girl who loves Bryn, and there's only one, and Bryn loves her.

There's an x such that $\mathsf{Gx} \wedge \mathsf{Lxb}$, and for all y, $(\mathsf{Gy} \wedge \mathsf{Lyb}) \supset \mathsf{y} = \mathsf{x}$, and Lbx.

$\exists \mathsf{x}(((\mathsf{Gx} \wedge \mathsf{Lxb}) \wedge \forall \mathsf{y}((\mathsf{Gy} \wedge \mathsf{Lyb}) \supset \mathsf{y} = \mathsf{x})) \wedge \mathsf{Lbx})$

Ex3. Bryn only loves the girl who loves him.

Bryn loves the girl who loves him (the x-girl), and for any other girl z, if Bryn loves z, then z is the x-girl.

$\exists \mathsf{x}((((\mathsf{Gx} \wedge \mathsf{Lxb}) \wedge \forall \mathsf{y}((\mathsf{Gy} \wedge \mathsf{Lyb}) \supset \mathsf{y} = \mathsf{x})) \wedge \mathsf{Lbx}) \wedge \forall \mathsf{z}((\mathsf{Gz} \wedge \mathsf{Lbz}) \supset \mathsf{z} = \mathsf{x}))$

More Translations:

b ⇒ Bryn	G ⇒ ____is a girl
m ⇒ Mrs. Jones	L ⇒ ____loves____
F ⇒ ____speaks Welsh	M ⇒ ____is taller than____

Ex4. The tallest girl speaks Welsh.

(a) x is a tallest girl:

x is a girl such that she is taller than all other girls

$Gx \wedge \forall y((Gy \wedge \neg y = x) \supset Mxy)$

(b) x is the tallest girl:

x is a tallest girl, and for any other tallest girl z, z = x

$$\exists x\{(Gx \wedge \forall y((Gy \wedge \neg y = x) \supset Mxy)) \\ \wedge \forall z[(Gz \wedge \forall v((Gv \wedge \neg v = z) \supset Mzv) \supset z = x]\}$$

There is an x, such that x is the tallest girl and x speaks Welsh.

$$\exists x\{\{(Gx \wedge \forall y((Gy \wedge \neg y = x) \supset Mxy)) \\ \wedge \forall z[(Gz \wedge \forall v((Gv \wedge \neg v = z) \supset Mzv) \supset z = x]\} \wedge Fx\}$$

Chapter 35: $QL^=$ Trees

New Tree Rule:

(L) Suppose W_1 is of the form $C(...m...)$ with one or more occurrences of the constant m, and W_2 is either of the form $m = n$ or $n = m$. Then we can add a *wff* $C(...n...)$, formed by substituting some or all occurrences of m in W_1 by occurrences of n, to any open branch containing both W_1 and W_2. Do not check off W_1 and W_2.

$$\begin{array}{c} C(...m...) \\ m = n \text{ or } n = m \\ | \\ C(...n...) \end{array}$$

- (L) takes two *wffs* as input (compare with other tree rules).
- $C(...n...)$ need not involve substituting *all* occurrences of m in $C(...m...)$ with n.

Ex1. $(\mathsf{Fm} \wedge \forall \mathsf{x}(\mathsf{Fx} \supset \mathsf{x} = \mathsf{m})), (\mathsf{Fn} \wedge \mathsf{Gn}) \therefore \mathsf{Gm}$

> Only Russell is a great philosopher.
> Bertie is a great philosopher who smokes.
> So Russell smokes.

(1) $(\mathsf{Fm} \wedge \forall \mathsf{x}(\mathsf{Fx} \supset \mathsf{x} = \mathsf{m}))$ ✓

(2) $(\mathsf{Fn} \wedge \mathsf{Gn})$ ✓

(3) $\neg \mathsf{Gm}$

(4) Fm — (b) on 1.

(5) $\forall \mathsf{x}(\mathsf{Fx} \supset \mathsf{x} = \mathsf{m})$

(6) Fn — (b) on 2.

(7) Gn

(8) $(\mathsf{Fn} \supset \mathsf{n} = \mathsf{m})$ ✓ — $(\forall')$ on 5, n/x

(9)	$\neg \mathsf{Fn}$	$\mathsf{n} = \mathsf{m}$	(g) on 8.
(10)	*	Gm	(L) on 7, 9.
		*	

Ex2. $\mathsf{m} = \mathsf{n},\ \mathsf{n} = \mathsf{o}\ \therefore\ \mathsf{m} = \mathsf{o}$

George is Mr. Orwell.
Mr. Orwell is Eric Blair.
So George is Eric Blair.

(1)	$\mathsf{m} = \mathsf{n}$	
(2)	$\mathsf{n} = \mathsf{o}$	
(3)	$\neg \mathsf{m} = \mathsf{o}$	
(4)	$\mathsf{m} = \mathsf{o}$	(L) on 1, 2.
	*	

Ex3. Show that transitivity of the identity relation is a *q*-logical truth.
$\forall x \forall y \forall z((x = y \wedge y = z) \supset x = z)$

(1)	$\neg\forall x \forall y \forall z((x = y \wedge y = z) \supset x = z)$ ✓	
(2)	$\exists x \neg\forall y \forall z((x = y \wedge y = z) \supset x = z)$ ✓	$(\neg\forall)$ on 1.
(3)	$\neg\forall y \forall z((m = y \wedge y = z) \supset m = z)$ ✓	$(\exists)$ on 2, m/x.
(4)	$\exists y \neg\forall z((m = y \wedge y = z) \supset m = z)$ ✓	$(\neg\forall)$ on 3.
(5)	$\neg\forall z((m = n \wedge n = z) \supset m = z)$ ✓	$(\exists)$ on 4, n/y.
(6)	$\exists z \neg((m = n \wedge n = z) \supset m = z)$ ✓	$(\neg\forall)$ on 5.
(7)	$\neg((m = n \wedge n = o) \supset m = o)$ ✓	$(\exists)$ on 6, o/z.
(8)	$(m = n \wedge n = o)$ ✓	(d) on 7.
(9)	$\neg m = o$	
(10)	$m = n$	(b) on 8.
(11)	$n = o$	
(12)	$m = o$	(L) on 10, 11.
	*	

Modification to Tree Construction Algorithm

If a path in a tree contains two wffs of the form A, $\neg A$, or a *wff* of the form $\neg c = c$, where c is a constant, then close the path with the symbol $*$.

Ex4. Show that reflexivity of the identity relation is a q-logical truth.

$\forall \mathsf{x}\mathsf{x} = \mathsf{x}$

(1) $\neg\forall \mathsf{x}\mathsf{x} = \mathsf{x}$ ✓

(2) $\exists \mathsf{x}\neg\mathsf{x} = \mathsf{x}$ ✓ $(\neg\forall)$ on 1.

(3) $\neg\mathsf{a} = \mathsf{a}$ $(\exists)$ on 2.

$*$

Ex5. Show that symmetry of the identity relation is a *q*-logical truth.

$\forall x \forall y(x = y \supset y = x)$

(1)	$\neg\forall x \forall y(x = y \supset y = x)$ ✓	
(2)	$\exists x \neg\forall y(x = y \supset y = x)$ ✓	$(\neg\forall)$ on 1.
(3)	$\neg\forall y(a = y \supset y = a)$ ✓	$(\exists)$ on 2, a/x.
(4)	$\exists y \neg(a = y \supset y = a)$ ✓	$(\neg\forall)$ on 3.
(5)	$\neg(a = b \supset b = a)$ ✓	$(\exists)$ on 4, b/y.
(6)	$a = b$	(d) on 5.
(7)	$\neg b = a$	
(8)	$\neg b = b$	(L) on 6, 7.
	*	

Ex6. $\exists x((Fx \wedge \forall y(Fy \supset y = x)) \wedge Gx),\ Fm \therefore Gm$

> The present king of France is bald.
> Louie is the present king of France.
> So Louie is bald.

(1)	$\exists x((Fx \wedge \forall y(Fy \supset y = x)) \wedge Gx)$ ✓		
(2)	Fm		
(3)	$\neg Gm$		
(4)	$((Fa \wedge \forall y(Fy \supset y = a)) \wedge Ga)$ ✓		$(\exists)$ on 1, a/x.
(5)	$(Fa \wedge \forall y(Fy \supset y = a))$ ✓		(b) on 4.
(6)	Ga		
(7)	Fa		(b) on 5.
(8)	$\forall y(Fy \supset y = a)$		
(9)	$(Fm \supset m = a)$ ✓		$(\forall')$ on 8, m/y.
(10)	$\neg Fm$	$m = a$	(g) on 9.
(11)	*	$\neg Gm$	(L) on 3, 10.
		*	