# Chapter 28: Q-Validity

In **PL**:

The **PL** *wffs* $A_1$, ..., $A_n$ ***tautologically entail*** the **PL** *wff* $C$ just when there is no **PL** valuation that makes $A_1$, ..., $A_n$ true and $C$ false.

An argument in **PL** is ***tautologically valid*** just when its premises tautologically entail its conclusion.

In **QL**:

The **QL** *closed wffs* $A_1$, ..., $A_n$ ***q-entail*** the **QL** closed *wff* $C$ just when there is no *q*-valuation of the vocabulary $V$ associated with these *wff* that makes $A_1$, ..., $A_n$ true and $C$ false.

An argument in **QL** is ***q-valid*** just when its premises *q*-entail its conclusion.

A $q$-valuation $q$ ***satisfies*** a set of *closed wffs* (or is a ***model*** of this set) just when $q$ makes all the *wffs* true.

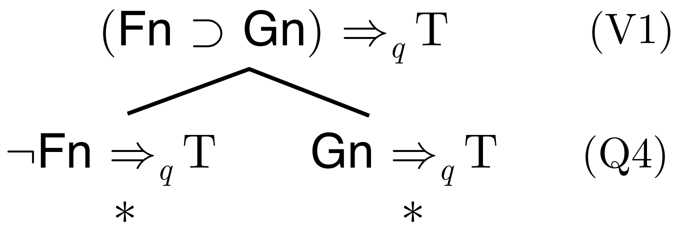
*So*: An argument in **QL** is $q$-valid just when there is no model of its premises and negated conclusion.

A ***countermodel*** of an argument in **QL** is a $q$-valuation that makes its premises true and its conclusion false.

A ***q-logical truth*** is a closed **QL** *wff* that is true on all $q$-valuations of its vocabulary.

*Ex1*: $\forall x(Fx \supset Gx), Fn \therefore Gn$ $q$-valid?

$$\left.\begin{array}{c} \forall x(Fx \supset Gx) \Rightarrow_q T \\ Fn \Rightarrow_q T \\ \neg Gn \Rightarrow_q T \end{array}\right\} \textit{Assume a countermodel exists.}$$



$(Fn \supset Gn) \Rightarrow_q T$ (V1)

$\neg Fn \Rightarrow_q T$ $Gn \Rightarrow_q T$ (Q4)

$*$ $*$

No counterexample exists.

*So:* **QL** argument is $q$-valid

*Ex2*: $\exists \mathrm{xFx}, \forall \mathrm{x}(\mathrm{Fx} \supset \mathrm{Gx}) \therefore \exists \mathrm{xGx}$

$$\left.\begin{array}{ll} (1) & \exists \mathrm{xFx} \Rightarrow_q \mathrm{T} \\ (2) & \forall \mathrm{x}(\mathrm{Fx} \supset \mathrm{Gx}) \Rightarrow_q \mathrm{T} \\ (3) & \neg\exists \mathrm{xGx} \Rightarrow_q \mathrm{T} \end{array}\right\} \textit{Assume a countermodel exists.}$$

(4) $\forall \mathrm{x}\neg \mathrm{Gx} \Rightarrow_q \mathrm{T}$ (V4) on 3.

(5) $\mathrm{Fa} \Rightarrow_{q+} \mathrm{T}$ (V2) on 1.

(6) $\forall \mathrm{x}(\mathrm{Fx} \supset \mathrm{Gx}) \Rightarrow_{q+} \mathrm{T}$ (V5) on 2.

(7) $(\mathrm{Fa} \supset \mathrm{Ga}) \Rightarrow_{q+} \mathrm{T}$ (V1) on 6.

(8) $\forall \mathrm{x}\neg \mathrm{Gx} \Rightarrow_{q+} \mathrm{T}$ (V5) on 4.

(9) $\neg \mathrm{Ga} \Rightarrow_{q+} \mathrm{T}$ (V1) on 8.

(10) $\neg \mathrm{Fa} \Rightarrow_{q+} \mathrm{T}$ $\quad$ $\mathrm{Ga} \Rightarrow_{q+} \mathrm{T}$ (Q4) on 7.

$*$ $\quad$ $*$

No counterexample exists.
*So:* **QL** argument is *q*-valid.

*Ex3*: $(\exists x(Fx \wedge \neg Gx) \supset \neg Gn), Gn \therefore \forall x(Fx \supset Gx)$

(1) $(\exists x(Fx \wedge \neg Gx) \supset \neg Gn) \Rightarrow_q T$
(2) $Gn \Rightarrow_q T$
(3) $\neg\forall x(Fx \supset Gx) \Rightarrow_q T$

*Assume a countermodel q exists.*

(4) $\exists x \neg(Fx \supset Gx) \Rightarrow_q T$ (V3) on 3.

(5) $\neg\exists x(Fx \wedge \neg Gx) \Rightarrow_q T$ $\neg Gn \Rightarrow_q T$ (Q4) on 1.
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(6) $\forall x \neg(Fx \wedge \neg Gx) \Rightarrow_q T$ (V4) on 5.
(7) $\neg(Fa \supset Ga) \Rightarrow_{q+} T$ (V2) on 4.
(8) $\forall x \neg(Fx \wedge \neg Gx) \Rightarrow_{q+} T$ (V5) on 6.
(9) $\neg(Fa \wedge \neg Ga) \Rightarrow_{q+} T$ (V1) on 8.

(10) $Fa \Rightarrow_{q+} T$
(11) $\neg Ga \Rightarrow_{q+} T$

From (7).

(12) $\neg Fa \Rightarrow_{q+} T$ $\neg\neg Ga \Rightarrow_{q+} T$ From 9.
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No counterexample exists.

*<u>So</u>:* **QL** argument is *q*-valid.