Chapter 26: The Syntax of QL

Alphabet:

a, b, c, m, n, o, ..., c_k	individual constants ($k \geq 0$)
w, x, y, z, ..., v_k	individual variables ($k \geq 0$)
A, B, C, ..., $P^0{}_k$	0-place predicates (propositional atoms) ($k \geq 0$)
F, G, H, ..., $P^1{}_k$	1-place predicates ($k \geq 0$)
L, M, ..., $P^2{}_k$	2-place predicates ($k \geq 0$)
R, S, ..., $P^3{}_k$	3-place predicates ($k \geq 0$)
$\vdots$	
$P^n{}_k$	n-place predicates ($k \geq 0$, $n \geq 0$)
$\wedge, \vee, \neg, \supset, \forall, \exists,$ (,)	connectives, quantifiers, punctuation
$\therefore$, $*$	argument symbols

Grammar:

*Term of **QL***

1. An individual constant or individual variable is a term of **QL**.
2. Nothing else is a term of **QL**.

*Atomic wff of **QL***

(A1) If $\mathsf{P}^n{}_k$ is an n-place predicate symbol, $n \geq 0$, and t_1, ..., t_n are terms of **QL**, then $\mathsf{P}^n{}_k t_1$, ..., t_n is an atomic *wff* of **QL**.

(A2) Nothing else is an atomic *wff* of **QL**.

*Wff of **QL***

(W1) Any atomic *wff* of **QL** is a *wff* of **QL**.

(W2) If A is a *wff* of **QL**, so is $\neg A$.

(W3) If A, B are *wffs* of **QL**, so is $(A \wedge B)$.

(W4) If A, B are *wffs* of **QL**, so is $(A \vee B)$.

(W5) If A, B are *wffs* of **QL**, so is $(A \supset B)$.

(W6) If A is a *wff* of **QL** and v is a variable which occurs in A (but neither $\forall v$ nor $\exists v$ occurs in A), then $\forall v A$ is a *wff* of **QL**.

(W7) If A is a *wff* of **QL** and v is a variable which occurs in A (but neither $\forall v$ nor $\exists v$ occurs in A), then $\exists v A$ is a *wff* of **QL**.

(W8) Nothing else is a *wff* of **QL**.

Ex1. $\neg\forall x((Fx \wedge \exists zMxz) \supset \forall y((Gy \wedge Lyx) \supset Lxy))$

Ex2. Fx

Ex3. $\forall x(\exists yMxy \supset \exists y(\neg Mxy \wedge Lxy))$

$$Mxy$$

$$\neg Mxy \qquad Lxy$$

$$Mxy \qquad (\neg Mxy \wedge Lxy)$$

$$\exists yMxy \qquad \exists y(\neg Mxy \wedge Lxy))$$

$$(\exists yMxy \supset \exists y(\neg Mxy \wedge Lxy))$$

$$\forall x(\exists yMxy \supset \exists y(\neg Mxy \wedge Lxy))$$

Ex4. $\forall x(Fx \supset \exists x(Gx \wedge Lxn))$

$$Gx \qquad Lxn$$

$$(Gx \wedge Lxn)$$

$$Fx \qquad \exists x(Gx \wedge Lxn)$$

$$(Fx \supset \exists x(Gx \wedge Lxn))$$

Note: $\exists x$ occurs in the *wff* at this stage of construction.
So: The result of attaching $\forall x$ in front is not a *wff*!

A *wff* A is a ***subformula*** fo a *wff* B if A appears anywhere on the construction tree of B.

The ***main operator*** of A is the operator (quantifier or connective) introduced in the final step in the construction tree of A.

The ***scope*** of a quantifier in a **QL** *wff* is the *wff* that occurs at the step in the construction tree where the quantifier is introduced.

Ex3. $\forall x(\exists yMxy \supset \exists y(\neg Mxy \wedge Lxy))$

Mxy

$\neg Mxy \qquad Lxy$

$Mxy \qquad (\neg Mxy \wedge Lxy)$

$\exists yMxy \qquad \exists y(\neg Mxy \wedge Lxy))$

$(\exists yMxy \supset \exists y(\neg Mxy \wedge Lxy))$

$\forall x(\exists yMxy \supset \exists y(\neg Mxy \wedge Lxy))$

- Scope of first $\exists y$ is $\exists yMxy$
- Scope of second $\exists y$ is $\exists y(\neg Mxy \wedge Lxy)$
- Scope of $\forall x$ is $\forall x(\exists yMxy \supset \exists y(\neg Mxy \wedge Lxy))$

An ***occurrence*** of a variable v in a *wff* A is ***bound*** just when it is in the scope of a quantifier $\forall v$ or $\exists v$ that occurs in A.

An ***occurrence*** of a variable in a *wff* A is ***free*** just when it is not bound.

Ex1.
- (Mxy ⊃ Rxny) — x is free, y is free
- ∀y(Mxy ⊃ Rxny) — x is free, y is bound
- ((Fx ∧ ∃zMxz) ∧ ∀y(Mxy ⊃ Rxny)) — x is free, y is bound, z is bound

Ex2. (Mxy ⊃ ∃y(Mxy ∧ Lxy))

- y is free in Mxy
- y is bound in ∃y(Mxy ∧ Lxy)

A variable is ***free*** (***bound***) in a *wff* just when *all* of its occurrences are free (bound).

A ***sentence*** in **QL**, or a ***closed wff***, is a *wff* with no free occurrences of variables.

An ***open wff*** in **QL** is a *wff* that is not closed.

An ***instance*** $C(\ldots c \ldots c \ldots)$ of a quantified sentence $\forall v C(\ldots v \ldots v \ldots)$ or $\exists v C(\ldots v \ldots v \ldots)$, is the result of stripping off the quantifier and substituting an individual constant c for all occurrences of the variable v.

Ex. $\exists$x(Fx $\land$ Gn)

Instances:

- (Fn $\land$ Gn)
- (Fo $\land$ Gn)
- (Fm $\land$ Gn)

$\vdots$