Chapter 25: Introducting QL Trees (Informally)

Recall the tree method:

To show that A_1, ..., $A_n \therefore C$ is tautologically valid, show that A_1, ..., A_n, $\neg C$ is tautologically inconsistent.

How will this work for QL:

Ex1: Fn, ∀x(Fx ⊃ Gx) ∴ Gn

F	means	"____is a philosopher"
G	means	"____is a jerk"
n	means	Jack

Fn — n is in the domain (and it's an F).

∀x(Fx ⊃ Gx) — Every F in the domain is a G.

¬Gn

(Fn ⊃ Gn) ✓

¬Fn | Gn

* | *

Note: If ∀x(Fx ⊃ Gx) is true, and n is in the domain, then (Fn ⊃ Gn) is true.

But: So might alot of other *wffs*, depending on how many other individuals are in the domain.

So: (Fn ⊃ Gn) doesn't *exhaust the truth* of ∀x(Fx ⊃ Gx)!

Thus: We should not check off at this point.

Ex2: $\forall x(Fx \supset Gx),\ (Fn \wedge \neg Hn)\ \therefore\ \neg\forall x(Gx \supset Hx)$

$$\forall x(Fx \supset Gx)$$

$$(Fn \wedge \neg Hn)\ \checkmark$$

$$\neg\neg\forall x(Gx \supset Hx)\ \checkmark$$

$$\forall x(Gx \supset Hx)$$

$$Fn$$

n is in the domain.

$$\neg Hn$$

$$(Fn \supset Gn)\ \checkmark$$

$$(Gn \supset Hn)\ \checkmark$$

$\neg Fn$	Gn
*	

$\neg Gn$	Hn
*	*

∀-Instantiation Rule

(∀) If $\forall v C(\ldots v \ldots v \ldots)$ appears on an open path, add $C(\ldots c \ldots c \ldots)$ to the path, where c is any constant that *already appears* on the path. *Do not check off* $\forall v C(\ldots v \ldots v \ldots)$.

Ex3: ∀x(Fx ⊃ Gx), (Fm ∨ Fn) ∴ (Gm ∨ Gn)

∀x(Fx ⊃ Gx)

(Fm ∨ Fn) ✓

¬(Gm ∨ Gn) ✓

¬Gm

¬Gn

Fm | Fn

(Fm ⊃ Gm) ✓ | (Fn ⊃ Gn) ✓

¬Fm | Gm | ¬Fn | Gn

* | * | * | *

Note: A non-primitive *wff* that remains unchecked! It's truth content has not been exhausted.

Negated Quantifier Rules

($\neg\forall$) If $\neg\forall v C(\ldots v \ldots v \ldots)$ appears on an open path, add $\exists v \neg C(\ldots v \ldots v \ldots)$ each open path *and check it off.*

($\neg\exists$) If $\neg\exists v C(\ldots v \ldots v \ldots)$ appears on an open path, add $\forall v \neg C(\ldots v \ldots v \ldots)$ each open path *and check it off.*

Ex4: $\neg\exists x(Fx \wedge Gx)$, $Fm \therefore \neg Gm$

$\neg\exists x(Fx \wedge Gx)$ ✓

Fm

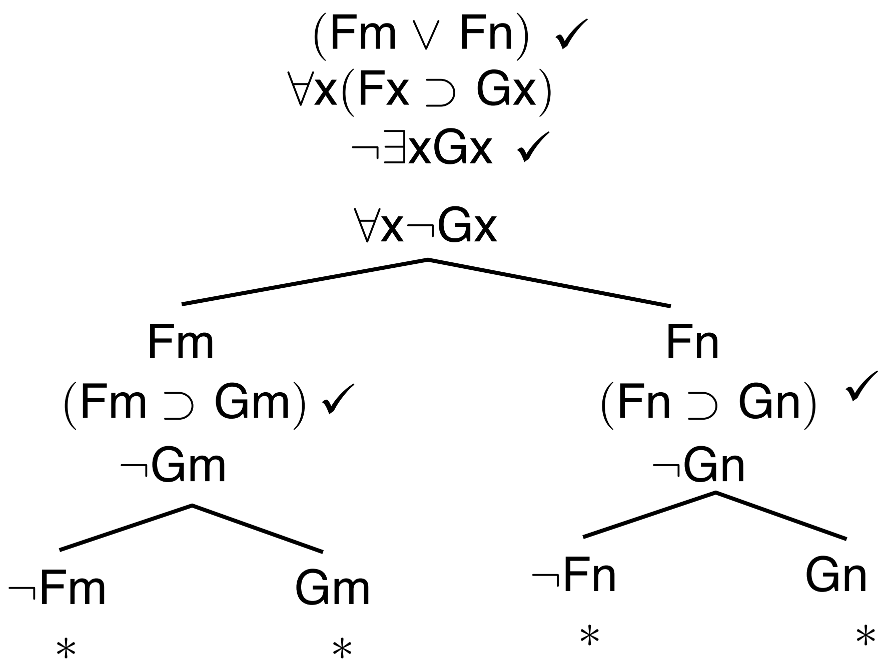
$\neg\neg Gm$

$\forall x \neg(Fx \wedge Gx)$

$\neg(Fm \wedge Gm)$ ✓

$\neg Fm$	$\neg Gm$
*	*

Ex5: $(Fm \vee Fn), \forall x(Fx \supset Gx) \therefore \exists xGx$



Ex6: $\exists x Fx, \forall x(Fx \supset Gx) \therefore \exists x Gx$

$\exists x Fx$ ✓

$\forall x(Fx \supset Gx)$

$\neg\exists x Gx$ ✓

$\forall x \neg Gx$

Fa

$(Fa \supset Ga)$ ✓

$\neg Ga$

$\neg Fa$ | Ga

* | *

At least one thing in the domain is an F.

Call it a. This exhausts the truth content of $\exists x Fx$!

Ex7: $\forall x(Fx \supset Gx), \forall x(Gx \supset Hx) \therefore \forall x(Fx \supset Hx)$

$$\forall x(Fx \supset Gx)$$

$$\forall x(Gx \supset Hx)$$

$$\neg\forall x(Fx \supset Hx) \checkmark$$

$$\exists x\neg(Fx \supset Hx) \checkmark$$

$$\neg(Fa \supset Ha) \checkmark$$

$$(Fa \supset Ga) \checkmark$$

$$(Ga \supset Ha) \checkmark$$

$$Fa$$

$$\neg Ha$$

$\neg Fa$	Ga
$*$	

$\neg Ga$	Ha
$*$	$*$

Ex8: $\exists x Fx \therefore Fn$

Tautologically invalid!
But: Without further do, we can construct a closed tree.

$\exists x Fx$ ✓

$\neg Fn$

Fn

*

Moral: When you instantiate an existential, don't use a constant that has already appeared in the tree!

Ex9: $\exists x Fx, \exists x Gx \therefore \exists x(Fx \wedge Gx)$

Tautologically invalid!

$\exists x Fx$ ✓

$\exists x Gx$ ✓

$\neg \exists x(Fx \wedge Gx)$ ✓

$\forall x \neg (Fx \wedge Gx)$

Fa

Ga

Suppose we instantiate Premise #2 using the constant a that has already appeared in the tree...

$\neg (Fa \wedge Ga)$ ✓

$\neg Fa$ | $\neg Ga$

* | *

... then we erroneously get a completed closed tree!

∃-Instantiation Rule

(∃) If $\exists v C(\ldots v \ldots v \ldots)$ appears on an open path, add $C(\ldots c \ldots c \ldots)$ to each open path, where c is *new* to the tree. *Check off* $\exists v C(\ldots v \ldots v \ldots)$.

Ex10: ∀x∃y(Fy ∧ Lxy), ∀x(Gx ⊃ ¬Fx), Gn ∴ ¬∀xGx

∀x∃y(Fy ∧ Lxy)
∀x(Gx ⊃ ¬Fx)
Gn
¬¬∀xGx ✓
∀xGx
∃y(Fy ∧ Lny) ✓ ← From Premise #1
(Fa ∧ Lna) ✓
Fa
Lna
(Ga ⊃ ¬Fa) ✓ ← From Premise #2
Ga

¬Ga	¬Fa
*	*

∃-Instantiation Rule

(∃) If $\exists vC(\ldots v \ldots v \ldots)$ appears on an open path, add $C(\ldots c \ldots c \ldots)$ to each open path, where c is *new* to the tree. *Check off* $\exists vC(\ldots v \ldots v \ldots)$.

Ex10: ∀x∃y(Fy ∧ Lxy), ∀x(Gx ⊃ ¬Fx), Gn ∴ ¬∀xGx

∀x∃y(Fy ∧ Lxy)

∀x(Gx ⊃ ¬Fx)

Gn

¬¬∀xGx ✓

∀xGx

∃y(Fy ∧ Lny) ✓ ← From Premise #1

(Fa ∧ Lna)

∃y(Fy ∧ Lay) ← From Premise #1

(Fb ∧ Lab)

∃y(Fy ∧ Lby) ← From Premise #1

(Fc ∧ Lbc)

⋮

Tree construction will never halt!

- For **QL** trees, there is no guarantee that tree construction will halt!
- Some **QL** trees will halt, others will not.
- There is no mechanical test (algorithm) for deciding whether **QL** arguments are tautologically valid.