## Chapters 23, 24: QL Translations

<u>Let</u>: "F" means "\_\_\_\_is wise"

Domain = people

∀x¬Fx	"Everyone is unwise."
¬∀ <b>x</b> ¬Fx	"Not everyone is unwise."
	or

∃**xFx** "Someone is wise."

 $\neg \forall v \neg C(\dots v \dots v \dots)$  is true *iff*  $\exists v C(\dots v \dots v \dots)$  is true.

 $\exists \mathbf{x} \neg F \mathbf{x}$ "Someone is unwise." $\neg \exists \mathbf{x} \neg F \mathbf{x}$ "No one is unwise." $\forall \mathbf{x} F \mathbf{x}$ "Everyone is wise."

 $\neg \exists v \neg C(\dots v \dots v \dots)$  is true *iff*  $\forall v C(\dots v \dots v \dots)$  is true.

## <u>Motivation</u>:

 $\exists xFx$ means $(Fm \lor Fn \lor Fo \lor \cdots)$  $\forall xFx$ means $(Fm \land Fn \land Fo \land \cdots)$ 

(Provided that everything in our domain has a name!)

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<u>So</u>:
                  means \neg(\neg \mathsf{Fm} \lor \neg \mathsf{Fn} \lor \neg \mathsf{Fo} \lor \cdots)
\neg \exists x \neg Fx
                   or
                   (Fm \land Fn \land Fo \land \cdots)
                   or
                  ∀xFx
And:
\neg \forall x \neg Fx
                  means \neg(\neg \mathsf{Fm} \land \neg \mathsf{Fn} \land \neg \mathsf{Fo} \land \cdots)
                   or
                   (Fm \lor Fn \lor Fo \lor \cdots)
                   or
                   ∃xFx
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Translating Restricted Quantifications into **QL** 

"All $A$ are $B$ "	translates as	$\forall v(Av \supset Bv)$
"Some $A$ are $B$ "	translates as	$\exists v (Av \land Bv)$
"Some $A$ are not $B$ "	translates as	$\exists v (Av \land \neg Bv)$
"No $A$ are $B$ "	translates as	$\forall v(Av \supset \neg Bv)$

 $\underline{Ex1}$ : All logicians are rational.

G	means	"	is a logician"
Н	means	"	is rational"

 $\forall \mathbf{x}(\mathbf{G}\mathbf{x} \supset \mathbf{H}\mathbf{x})$  "For all things x, if x is a logician, then x is rational."

<u>Ex2</u>: Some philosophers are logicians.

 $\mathsf{F}$  mean "\_\_\_\_is a philosopher"

 $\exists \mathbf{x}(\mathbf{Fx} \wedge \mathbf{Gx})$  "There exists an x such that x is a philosopher and x is a logician."

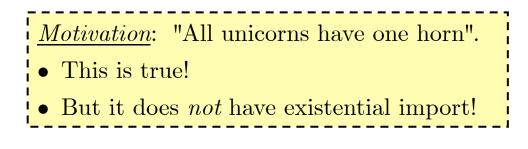
 $\underline{Ex3}: \quad \forall \mathbf{x}(\mathbf{Gx} \land \mathbf{Hx}) \qquad \text{"Everything is a logician-philosopher."}$ 

 $\exists x(Fx \supset Gx)$  "There's a thing such that, if it's a philosopher, then it's a logician."

$\exists x(Fx \supset Gx) \text{ is not the same as } \exists x(Fx \land Gx)!$				
Suppose:	There's a thing named $\mathbf{n}$ that is not a philosopher.			
<u>Then</u> :	$(Fn \supset Gn)$ is true!			
<u>So</u> :	$\exists \mathbf{x}(\mathbf{F}\mathbf{x} \supset \mathbf{G}\mathbf{x}) \text{ is true!}$			
<u>Now</u> :	Suppose no philosophers are logicians.			
<u>Then</u> :	$\exists \mathbf{x}(\mathbf{F}\mathbf{x} \wedge \mathbf{G}\mathbf{x}) \text{ is false!}$			
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<u>Note</u>: "All A are B" and "No A is B" do not make existential claims.

• To say "All logicians are rational" in **QL** is *not* to say there *are* such things as rational logicians.



## <u>More translation examples</u>

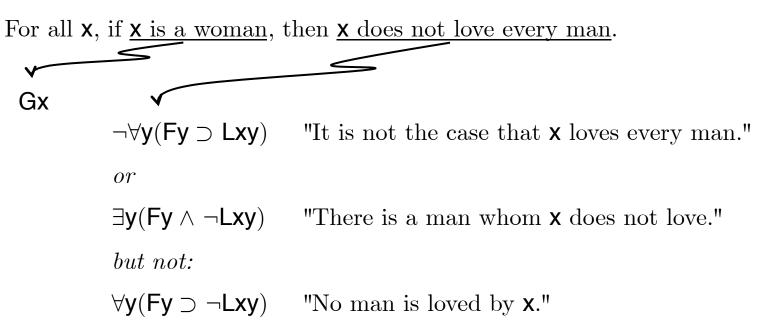
Translation Key:			
<b>m</b> means Maldwyn	F	means	is a man
n means Nerys	Μ	means	is married to
o means Owen	R	means	prefersto
	G	means	is a woman
Domain = people	L	means	loves

 $\underline{Ex1}$ .Whoever is loved by Owen is loved by Maldwyn, too.For all  $\mathbf{x}$ , if  $\underline{\mathbf{x}}$  is loved by Owen, then  $\underline{\mathbf{x}}$  is loved by Maldwyn.For all  $\mathbf{x}$ , if  $\mathbf{Lox}$ , then  $\mathbf{Lmx}$ . $\forall \mathbf{x}(\mathbf{Lox} \supset \mathbf{Lmx})$ 

<u>Ex2</u>.Every man loves someone.For all  $\mathbf{x}$ , if  $\underline{\mathbf{x}}$  is a man, then  $\underline{\mathbf{x}}$  loves someone.For all  $\mathbf{x}$ , if  $\mathbf{Fx}$ , then  $\exists \mathbf{y} \mathsf{Lxy}$ . $\forall \mathbf{x}(\mathsf{Fx} \supset \exists \mathbf{y} \mathsf{Lxy})$ 

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<u>Ex3</u>. No woman loves every man.



$$\label{eq:formula} \begin{split} &\operatorname{For all} x, \, \mathrm{if} \; Gx, \, \mathrm{then} \; \neg \forall y (Fy \supset Lxy). \\ &\forall x (Gx \supset \neg \forall y (Fy \supset Lxy)) \end{split}$$

Translation Key:			
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<u>Ex4</u>. Nerys loves any married man who prefers her to whomever they are married to. For all  $\mathbf{x}$ , if  $\underline{\mathbf{x}}$  is a married man who prefers Nerys to whomever  $\mathbf{x}$  is married to, then <u>Nerys loves  $\mathbf{x}$ </u>. Lnx

"<u>x is a married man</u>" means "<u>x is a man and there is someone to whom x is married</u>"  $(Fx \land \exists yMxy)$ 

"<u>x prefers Nerys to whomever x is married to</u>" means

For all **x**, if  $(Fx \land \exists yMxy)$  and  $\forall z(Mxz \supset Rxnz)$ , then Lnx.

 $\forall x(((Fx \land \exists yMxy) \land \forall z(Mxz \supset Rxnz)) \supset Lnx)$ 

<u>Case 1</u>:

 $\exists v \exists w C(\dots v \dots w \dots) \equiv \exists w \exists v C(\dots v \dots w \dots)$ 

 $\forall v \forall w C(\dots v \dots w \dots) \equiv \forall w \forall v C(\dots v \dots w \dots)$ 

Immediately adjacent quantifiers <u>of</u> <u>the same kind</u> can be interchanged.

<u>Case 2</u>:

$$(A \land \forall vB(...v..)) \equiv \forall v(A \land B(...v..))$$
$$(A \land \exists B(...v..)) \equiv \exists v(A \land B(...v..))$$
$$Provided \ v \ does \ not \ occur \ in \ A.$$

Exs:Nerys is a woman everyone loves. $(Gn \land \forall xLxn)$  same as  $\forall x(Gn \land Lxn)$ Someone who is married loves Nerys. $\exists x(\exists yMxy \land Lxn)$  same as  $\exists x \exists y(Mxy \land Lxn)$ 

$$\underline{Case 3}: \qquad (A \lor \forall vB(...v..)) \equiv \forall v(A \lor B(...v..))$$
$$(A \lor \exists vB(...v..)) \equiv \exists v(A \lor B(...v..))$$
$$Provided \ v \ does \ not \ occur \ in \ A.$$

<u>Case 4</u>:

$$(A \supset \forall B(...v..)) \equiv \forall v(A \supset B(...v..))$$
$$(A \supset \exists vB(...v..)) \equiv \exists v(A \supset B(...v..))$$
$$Provided \ v \ does \ not \ occur \ in \ A.$$

<u>Case 5</u>:

$$(\forall vB(...v..) \supset A) \equiv \exists v(B(...v..) \supset A)$$
$$(\exists vB(...v..) \supset A) \equiv \forall v(B(...v..) \supset A)$$
Provided v does not occur in A.

$$\underline{Check}: \quad (\forall \mathbf{x} \mathbf{F} \mathbf{x} \supset \mathbf{F} \mathbf{n}) \equiv (\neg \forall \mathbf{x} \mathbf{F} \mathbf{x} \lor \mathbf{F} \mathbf{n})$$
$$\equiv (\exists \mathbf{x} \neg \mathbf{F} \mathbf{x} \lor \mathbf{F} \mathbf{n})$$
$$\equiv \exists \mathbf{x} (\neg \mathbf{F} \mathbf{x} \lor \mathbf{F} \mathbf{n})$$
$$\equiv \exists \mathbf{x} (\mathbf{F} \mathbf{x} \supset \mathbf{F} \mathbf{n})$$