Chapters 23, 24: QL Translations

Let: "F" means "____is wise"

Domain = people

$\forall x \neg Fx$	"Everyone is unwise."
$\neg \forall x \neg Fx$	"Not everyone is unwise."
	or
$\exists x Fx$	"Someone is wise."

$\neg \forall v \neg C(\ldots v \ldots v \ldots)$ is true *iff* $\exists v C(\ldots v \ldots v \ldots)$ is true.

$\exists x \neg Fx$	"Someone is unwise."
$\neg \exists x \neg Fx$	"No one is unwise."
	or
$\forall x Fx$	"Everyone is wise."

$\neg \exists v \neg C(\ldots v \ldots v \ldots)$ is true *iff* $\forall v C(\ldots v \ldots v \ldots)$ is true.

Motivation:

$\exists x Fx$	means	$(Fm \vee Fn \vee Fo \vee \cdots)$
$\forall x Fx$	means	$(Fm \wedge Fn \wedge Fo \wedge \cdots)$

(Provided that everything in our domain has a name!)

So:

$\neg\exists x\neg Fx$ means $\neg(\neg Fm \vee \neg Fn \vee \neg Fo \vee \cdots)$

or

$(Fm \wedge Fn \wedge Fo \wedge \cdots)$

or

$\forall x Fx$

And:

$\neg\forall x\neg Fx$ means $\neg(\neg Fm \wedge \neg Fn \wedge \neg Fo \wedge \cdots)$

or

$(Fm \vee Fn \vee Fo \vee \cdots)$

or

$\exists x Fx$

*Translating Restricted Quantifications into **QL***

"All A are B"	translates as	$\forall v(Av \supset Bv)$
"Some A are B"	translates as	$\exists v(Av \wedge Bv)$
"Some A are not B"	translates as	$\exists v(Av \wedge \neg Bv)$
"No A are B"	translates as	$\forall v(Av \supset \neg Bv)$

Ex1: All logicians are rational.

G	means	"____is a logician"
H	means	"____is rational"

$\forall \mathsf{x}(\mathsf{Gx} \supset \mathsf{Hx})$ "For all things x, if x is a logician, then x is rational."

Ex2: Some philosophers are logicians.

F	mean	"____is a philosopher"

$\exists \mathsf{x}(\mathsf{Fx} \wedge \mathsf{Gx})$ "There exists an x such that x is a philosopher and x is a logician."

Ex3: $\forall x(Gx \wedge Hx)$ "Everything is a logician-philosopher."

$\exists x(Fx \supset Gx)$ "There's a thing such that, if it's a philosopher, then it's a logician."

$\exists x(Fx \supset Gx)$ is not the same as $\exists x(Fx \wedge Gx)$!

Suppose: There's a thing named n that is not a philosopher.

Then: $(Fn \supset Gn)$ is true!

So: $\exists x(Fx \supset Gx)$ is true!

Now: Suppose no philosophers are logicians.

Then: $\exists x(Fx \wedge Gx)$ is false!

Note: "All *A* are *B*" and "No *A* is *B*" do *not* make existential claims.

- To say "All logicians are rational" in **QL** is *not* to say there *are* such things as rational logicians.

Motivation: "All unicorns have one horn".

- This is true!
- But it does *not* have existential import!

More translation examples

Translation Key:					
m	means	Maldwyn	F	means	____is a man
n	means	Nerys	M	means	____is married to____
o	means	Owen	R	means	____prefers____to____
			G	means	____is a woman
Domain = people			L	means	____loves____

Ex1. Whoever is loved by Owen is loved by Maldwyn, too.

For all x, if x is loved by Owen, then x is loved by Maldwyn.

For all x, if Lox, then Lmx.

$\forall x(Lox \supset Lmx)$

Ex2. Every man loves someone.

For all x, if x is a man, then x loves someone.

For all x, if Fx, then $\exists yLxy$.

$\forall x(Fx \supset \exists yLxy)$

Translation Key:					
m	means	Maldwyn	F	means	____is a man
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Ex3. No woman loves every man.

For all x, if x is a woman, then x does not love every man.

Gx

$\neg\forall y(Fy \supset Lxy)$ "It is not the case that x loves every man."

or

$\exists y(Fy \wedge \neg Lxy)$ "There is a man whom x does not love."

but not:

$\forall y(Fy \supset \neg Lxy)$ "No man is loved by x."

For all x, if Gx, then $\neg\forall y(Fy \supset Lxy)$.

$\forall x(Gx \supset \neg\forall y(Fy \supset Lxy))$

Translation Key:

m	means	Maldwyn	F	means	____is a man
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Ex4. Nerys loves any married man who prefers her to whomever they are married to.

For all x, if x is a married man who prefers Nerys to whomever x is married to, then Nerys loves x.

Lnx

"x is a married man" means "x is a man and there is someone to whom x is married"

$(Fx \wedge \exists yMxy)$

"x prefers Nerys to whomever x is married to" means

"For all z, if x is married to z, then x prefers Nerys to z"

$\forall z(Mxz \supset Rxnz)$

Note: This z *might be different from the* y *above!*

For all x, if $(Fx \wedge \exists yMxy)$ and $\forall z(Mxz \supset Rxnz)$, then Lnx.

$\forall x(((Fx \wedge \exists yMxy) \wedge \forall z(Mxz \supset Rxnz)) \supset Lnx)$

Special Cases of Mixed Quantifiers

Case 1:

$\exists v \exists w C(...v...w...) \equiv \exists w \exists v C(...v...w...)$

$\forall v \forall w C(...v...w...) \equiv \forall w \forall v C(...v...w...)$

Immediately adjacent quantifiers of the same kind can be interchanged.

Case 2:

$(A \wedge \forall v B(...v...)) \equiv \forall v (A \wedge B(...v...))$

$(A \wedge \exists B(...v...)) \equiv \exists v (A \wedge B(...v...))$

Provided v does not occur in A.

Exs: Nerys is a woman everyone loves.

(Gn ∧ ∀xLxn) same as ∀x(Gn ∧ Lxn)

Someone who is married loves Nerys.

∃x(∃yMxy ∧ Lxn) same as ∃x∃y(Mxy ∧ Lxn)

Case 3:

$$(A \vee \forall v B(\ldots v \ldots)) \equiv \forall v(A \vee B(\ldots v \ldots))$$

$$(A \vee \exists v B(\ldots v \ldots)) \equiv \exists v(A \vee B(\ldots v \ldots))$$

Provided v does not occur in A.

Case 4:

$$(A \supset \forall B(\ldots v \ldots)) \equiv \forall v(A \supset B(\ldots v \ldots))$$

$$(A \supset \exists v B(\ldots v \ldots)) \equiv \exists v(A \supset B(\ldots v \ldots))$$

Provided v does not occur in A.

Case 5:

$$(\forall v B(\ldots v \ldots) \supset A) \equiv \exists v(B(\ldots v \ldots) \supset A)$$

$$(\exists v B(\ldots v \ldots) \supset A) \equiv \forall v(B(\ldots v \ldots) \supset A)$$

Provided v does not occur in A.

Check:

$$\begin{aligned} (\forall x Fx \supset Fn) &\equiv (\neg \forall x Fx \vee Fn) \\ &\equiv (\exists x \neg Fx \vee Fn) \\ &\equiv \exists x(\neg Fx \vee Fn) \\ &\equiv \exists x(Fx \supset Fn) \end{aligned}$$