Chapters 21, 22: The Language of QL ("Quantifier Logic")

Motivation:

(1) Fido is a cat.

(2) All cats are scary. *Valid argument!*

(3) Fido is scary.

*In **PL***: Let P = Fido is a cat.

Q = All cats are scary. P, Q ∴ R *Not tautologically valid!*

R = Fido is scary.

*Informal Introduction to Alphabet & Grammar of **QL***

1. Names and Predicates

(a) Use lower-case letters m, n, o, ..., to represent the names of individuals. Call these letters "***individual constants***".

> *ex.* "m" = "Socrates"
> "n" = "Plato"
> "o" = "Aristotle"

(b) Use capital letters to represent properties attributed to individuals. Call them "***predicate letters***".

Convention:

F, G, H, ...	one-place predicates
L, M, ...	two-place predicates
R, S ...	three-place predicates

ex.	"F"	means	"____is wise"
	"L"	means	"____loves____"
	"R"	means	"____prefers____to____"

First Grammatical rule for ***QL***

An n-place predicate combines with n names to form an atomic *wff*.

ex.	"Fn"	means	"Plato is wise."
	"Fo"	means	"Aristotle is wise."
	"Romo"	means	"Aritstole prefers Socrates to himself."
	"Lmm"	means	"Socrates loves himself."

2. Connectives

Use **PLC** connectives $\wedge$, $\vee$, $\neg$, $\supset$, $\equiv$ to form *wffs* from atomic *wffs*.

ex:	$(Fm \vee Fn)$	means	"Either Socrates or Plato is wise."
	and not $F(m \vee n)$		
	$\neg(Lnm \wedge Lmn)$		"It's not the case that Plato loves Socrates and Socrates loves Plato."
	$(Rnmo \supset (Lnm \wedge \neg Lno))$		"If Plato prefers Socrates to Aristotle, then he loves Socrates and doesn't love Aristotle."
	$(Fn \wedge Lnn)$		"Plato is wise and loves himself."

3. Quantifiers

(a) Use lower-case letters ..., x, y, z to represent pronouns. Call these letters "***individual variables***".

(b) Use the symbols ∀, ∃ to represent universal and existential quantifiers.

ex.	"∀x"	means	"For all x"	or	"Everyone"	or	"Everything"
	"∃x"	means	"There exists an x"	or	"Someone"	or	"Something"

Grammatical rule for wffs involving quantifiers

If $C(\ldots c \ldots c \ldots)$ is a **QL** *wff* containing at least one occurrence of the individual constant c, then the expression obtained by replacing all occurrences of c with some individual variable v, new to C, and prefixing the result with $\forall v$ or $\exists v$, is a *wff*.

So if $C(\ldots c \ldots c \ldots)$ is a **QL** *wff* containing at least one occurrence of the individual constant c, then $\forall v C(\ldots v \ldots v \ldots)$ and $\exists v C(\ldots v \ldots v \ldots)$ are both **QL** *wffs*.

Rmno is a **QL** *wff*	"Socrates prefers Plato to Aristotle."

<u>*Thus so are:*</u>

∀xRxno	"For all x, x prefers Plato to Aristotle." *or* "Everyone prefers Plato to Aristotle."
∃zRzno	"There exists a z such that z prefers Plato to Aristotle." *or* "Someone prefers Plato to Aristotle."
∀x∃zRznx	"For all x, there exists a z such that z prefers Plato to x." *or* "Everyone is such that someone prefers Plato to them."
∀y∀x∃zRzyx	"Anyone is preferred to everyone by someone."

And so are:

$\forall$xRxno

$\forall$x$\forall$xRxnx

$\forall$x$\forall$x$\forall$xRxxx

As well as:

Fn

Rmno

(Fn $\wedge$ Rmno)

$\exists$x(Fn $\wedge$ Rmnx)

$\forall$y$\exists$x(Fn $\wedge$ Rynx)

(Fn $\supset$ $\neg\forall$y$\exists$x(Fn $\wedge$ Rynx))

$\forall$z(Fz $\supset$ $\neg\forall$y$\exists$x(Fz $\wedge$ Ryzx))

The ***domain of discourse*** of a quantifier expression is the collection of individuals that expression quantifies over.

$\forall v C(\ldots v \ldots v \ldots)$ means "Everything in the domain of discourse has the property that $C(\ldots c \ldots c \ldots)$ attributes to the individual named by c".

$\exists v C(\ldots v \ldots v \ldots)$ means "At least one thing in the domain of discourse has the property that $C(\ldots c \ldots c \ldots)$ attributes to the individual named by c".

So again:

Let the domain of discourse be people.

∃xFx	"Someone is wise."
¬∃xFx	"No one is wise."
∃x¬Fx	"Someone isn't wise."
∀x(Fx ⊃ Lmx)	"For all people, if they are wise, then Socrates loves them." *or* "All wise people are loved by Socrates." *or* "Socrates loves anyone wise."

Named versus Nameless Individuals

Suppose: The following **QL** *wff* is true: Fn

Then: Is the following **QL** *wff* true as well? ∃xFx

- ∃xFx says "Something in the domain of discourse is an F".
- This will be true if the thing named by n is in fact in the domain of discourse.

So: *Given* that everything named by some individual constant is in the domain, then any *wff* of the form $C(...c...c...)$ entails a *wff* of the form $\exists vC(...v...v...)$.

But: $\exists vC(...v...v...)$ does not necessarily imply $C(...c...c...)$.

- There can be domains with *nameless* individuals in which
  - (a) No *named* individual has the property C.
  - (b) At least one *nameless* individual has the property C.
- In such a domain, $\exists vC(...v...v...)$ is true, but $C(...c...c...)$ is false, for *any* named individual c.

Similarly: *Given* that everything named by some individual constant is in the domain, then any *wff* of the form $\neg C(\ldots c \ldots c \ldots)$ entails a *wff* of the form $\neg\forall v C(\ldots v \ldots v \ldots)$.

But: $\neg\forall v C(\ldots v \ldots v \ldots)$ does not necessarily imply $\neg C(\ldots c \ldots c \ldots)$.

- There can be a domain in which all named things have the property C, but some nameless things *don't*.

So: It's *not* the case that

(a) An existential quantifier *wff* is true if and only if one of its instances is true.

(b) A universal quantifier *wff* is true if and only if all of its instances are true.